Number of elements with prescribed Norm and Trace

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Let p be a prime and $\mathbb{F}_q$ be the finite field of q elements of characteristic p. Given $a, b \in \mathbb{F}_q$, and positive integer $n \geq 2$, let

$$N_n(a,b) = \#\{\alpha \in \mathbb{F}_{q^n} \mid Trace_{\mathbb{F}_{q^n}/\mathbb{F}_q}(\alpha) = a, Norm_{\mathbb{F}_{q^n}/\mathbb{F}_q}(\alpha) = b\}.$$

Motivated by various applications, many authors have investigated sharp estimates for the number $N_n(a,b)$. In 2010, after associating $N_n(a,b)$ with the number of rational points on certain toric Calabi-Yau hypersurface, Moisio and Wan [1] proved that the following:

Theorem 0.1.

$$\mid N_n(a,b) - \frac{q^n - 1}{q-1} \mid \leq (n-1)q^{\frac{n-2}{2}}.$$

In this talk, we will discuss how to associate $N_n(a,b)$ with the number of rational points on certain algebraic curves and point out improvements on Moisio-Wan's bound for a certain range of q.

References

[1] M. Moisio, D. Wan, *On Katz's bound for the number of elements with given trace and norm*, J. Reine Angew. Math. **638** (2010), 69–74.