Polar Codes over q-ary Alphabets and Polynomially Fast Convergence to Shannon Capacity

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Based on joint work with Venkatesan Guruswami
Channel Capacity

**Theorem** (*Shannon's Noisy-Channel Coding Theorem*, 1948). For any discrete memoryless channel $W$, there exists an associated nonnegative constant $I(W)$, known as the **channel capacity** such that:

- ✔ For any $\varepsilon > 0$, one can communicate at asymptotic rate $R = I(W) - \varepsilon$ with vanishing probability of miscommunication.
- ✗ For any $R > I(W)$, it is impossible to communicate at a rate $R$ without non-negligible probability of miscommunication.

For rate $R = I(W) - \varepsilon$, the **gap to capacity** is $\varepsilon$.

**Challenge**: Construct explicit coding schemes (with efficient encoding and decoding procedures) that obtain arbitrarily small gap to capacity!
Binary Polar Codes

- Polar codes give the **first known** construction to achieve capacity with the aforementioned guarantees
- **Rate** approaches $I(W)$ as $N \to \infty$
- $O(N \log N)$ complexity
- For polar codes over **binary** alphabet, [Guruswami-Xia '13] and [Hassani-Alishahi-Urbanke '13] independently show the **speed** of convergence to capacity
  - There is an absolute constant $c$ such that any block length $N \geq (1/\varepsilon)^c$ will yield a rate of at least $I(W) - \varepsilon$

**Question**: What about polar codes over **non-binary** alphabets?
Main Result

- We extend the work of [Guruswami-Xia '13] to **q-ary polar codes** for any $q > 2$

- q-ary polar codes are the **first explicit deterministic construction** of q-ary codes approaching the capacity of q-ary symmetric DMCs with provable guarantees
  - Encoding and decoding complexity polynomial in block length $N$
  - Constant $c = c(q)$ such that rate $\geq I(W) - \varepsilon$ for $N \geq (1/\varepsilon)^c$
  - Decoding error probability $\exp(-N^{0.49})$