Polar Codes over q-ary Alphabets and Polynomially Fast Convergence to Shannon Capacity

Ameya Velingker

Carnegie Mellon University

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Based on joint work with **Venkatesan Guruswami**

Carnegie Mellon University

Channel Capacity

Theorem (*Shannon's Noisy-Channel Coding Theorem*, 1948). For any discrete memoryless channel W, there exists an associated nonnegative constant I(W), known as the <u>channel capacity</u> such that:

- ✔ For any $\varepsilon > 0$, one <u>can communicate</u> at asymptotic rate $\mathbf{R = I(W) - \varepsilon}$ with vanishing probability of miscommunication.
- ✖ For any $\mathbf{R > I(W)}$, it is <u>impossible to communicate</u> at a rate R without non-negligible probability of miscommunication.

For rate $R = I(W) - \varepsilon$, the **gap to capacity** is ε.

<u>Challenge</u>: Construct **explicit** coding schemes (with **efficient** encoding and decoding procedures) that obtain arbitrarily small gap to capacity!

Binary Polar Codes

- Polar codes give the **first known** construction to achieve capacity with the aforementioned guarantees
- **Rate** approaches I(W) as $N \to \infty$
- O(N log N) **complexity**
- For polar codes over **binary** alphabet, [Guruswami-Xia '13] and [Hassani-Alishahi-Urbanke '13] independently show the *speed* of convergence to capacity
  - There is an absolute constant c such that any block length $N \geq (1/\varepsilon)^c$ will yield a rate of at least $I(W) - \varepsilon$

Question: What about polar codes over **non-binary** alphabets?

Main Result

- We extend the work of [Guruswami-Xia '13] to **q-ary polar codes** for any $q > 2$
- q-ary polar codes are the **first explicit deterministic construction** of **q-ary** codes approaching the capacity of q-ary symmetric DMCs with provable guarantees
  - Encoding and decoding complexity polynomial in block length N
  - Constant $c = c(q)$ such that rate $\geq I(W) - \varepsilon$ for $N \geq (1/\varepsilon)^c$
  - Decoding error probability $\exp(-N^{0.49})$