Geometrically Uniform Subspace Codes

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The aim of our work is to construct a class of geometrically uniform subspace codes. We establish a relationship between this class of codes with that of classical coding theory. Subspace codes are used in the context of network coding, which is a new way to approach the problem of efficient transmission of information.
The subspace distance between $U$ and $V$ is defined as:

$$d(U, V) = \dim(U) + \dim(V) - 2\dim(U \cap V)$$

where $U$ and $V$ are vector subspaces.

The codes that we construct have constant dimension and constant distance.

Example

Take the vector space $\mathbb{F}_2^4$, and consider the following codewords

- $S_1 = \{0000, 1000, 0010, 0001, 1010, 1001, 0011, 1011\}$
- $S_2 = \{0000, 0100, 0001, 1000, 0101, 1100, 1001, 1101\}$
- $S_3 = \{0000, 0010, 1000, 0100, 1010, 0110, 1100, 1110\}$
- $S_4 = \{0000, 0001, 0100, 0010, 0101, 0011, 0110, 0111\}$

The distance between the codewords is 2.
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*The distance between the codewords is 2.*
Take the vector space $\mathbb{F}_2^7$. Code $C$ with parameters $(7, 3, 4)$ has the following codewords:

- $S_1 = \langle 0000000 \rangle$
- $S_2 = \langle 1000000, 0100000, 0001000 \rangle$
- $S_3 = \langle 0100000, 0010000, 0000100 \rangle$
- $S_4 = \langle 0010000, 0001000, 0000010 \rangle$
- $S_5 = \langle 0001000, 0000100, 0000001 \rangle$
- $S_6 = \langle 0000100, 0000010, 1000000 \rangle$
- $S_7 = \langle 0000010, 0000001, 0100000 \rangle$
- $S_8 = \langle 0000001, 1000000, 0010000 \rangle$

The distance between the codewords is 4.
This code can be identified with the simplex code $P$ of parameters $(7, 3, 4)$, whose generator matrix is

\[
G = \begin{bmatrix}
0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 0 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 1 & 0 & 1
\end{bmatrix}
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Thank You!