Geometrically Uniform Subspace Codes

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Research

The aim of our work is to construct a class of geometrically uniform subspace codes. We establish a relationship between this class of codes with that of classical coding theory. Subspace codes are used in the context of network coding, which is a new way to approach the problem of efficient transmission of information.

Research

Definition

The **subspace distance** *between U and V is defined as:*

$$d(U, V) = dim(U) + dim(V) - 2dim(U \cap V)$$

where U and V are vector subspaces.

The codes that we construct have constant dimension and constant distance.

Example

Take the vector space $\mathbb{F}_2^4$, and consider the following codewords

$$S_1 = \{0000, 1000, 0010, 0001, 1010, 1001, 0011, 1011\}$$
$$S_2 = \{0000, 0100, 0001, 1000, 0101, 1100, 1001, 1101\}$$
$$S_3 = \{0000, 0010, 1000, 0100, 1010, 0110, 1100, 1110\}$$
$$S_4 = \{0000, 0001, 0100, 0010, 0101, 0011, 0110, 0111\}$$

The distance between the codewords is 2.

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Take the vector space $\mathbb{F}_2^7$. Code C with parameters $(7,3,4)$ has the following codewords:

$$\begin{aligned} S_1 &= \langle 0000000 \rangle \\ S_2 &= \langle 1000000, 0100000, 0001000 \rangle \\ S_3 &= \langle 0100000, 0010000, 0000100 \rangle \\ S_4 &= \langle 0010000, 0001000, 0000010 \rangle \\ S_5 &= \langle 0001000, 0000100, 0000001 \rangle \\ S_6 &= \langle 0000100, 0000010, 1000000 \rangle \\ S_7 &= \langle 0000010, 0000001, 0100000 \rangle \\ S_8 &= \langle 0000001, 1000000, 0010000 \rangle \end{aligned}$$

The distance between the codewords is 4.

This code can be identified with the simplex code P of parameters $(7,3,4)$, whose generator matrix is

$$G = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

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References

[1] A. Khaleghi, D. Silva, and F. R. Kschischang. *Subspace codes*. Pages 1-21, 2009.
[2] G. D. Forney, Jr. *Geometrically uniform codes*. IEEE Trans. Inf. Theor., 37(5):1241-1260, September, 2006.
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Thank You!