

# BOOK OF ABSTRACTS

## VI Workshop on Fluids and PDE

Celebrating the 60th birthdays of  
Helena J. Nussenzeig Lopes and Milton C. Lopes Filho



**Campinas, Brazil, 23rd - 27th October 2023**

### SCIENTIFIC COMMITTEE

David Ambrose (Drexel Univ., USA)  
Anne Bronzi (Unicamp, Brazil)  
Dragos Iftimie (Univ. de Lyon 1, France)  
Anna Mazzucato (Penn State Univ., USA)  
Gabriela Planas (Unicamp, Brazil)  
Roman Shvydkoy (Univ. Illinois, USA)

### ORGANIZING COMMITTEE

Anne Bronzi (Unicamp)  
Gabriela Planas (Unicamp)  
Marcelo Santos (Unicamp)

[www.ime.unicamp.br/~viwfpde](http://www.ime.unicamp.br/~viwfpde)





# Welcome

It is a pleasure to welcome you to the V Workshop on Fluids and PDE - *Celebrating the 60th birthdays of Helena J. Nussenzweig Lopes and Milton C. Lopes Filho*. In this conference, we will celebrate the 60th birthdays of Helena J. Nussenzweig Lopes and Milton C. Lopes Filho focusing on the major contributions of their work in the field of mathematical modeling and rigorous analysis of fluid dynamics problems, particularly incompressible flows with little regularity (non-smooth) and turbulent flows.

This is the sixth edition in a series of workshops aimed at fostering collaboration among the growing community of researchers working in Brazil on mathematical fluid dynamics, in particular on the incompressible Navier-Stokes and Euler equations, as well as stimulating further collaboration with the international community. The first and third editions took place at Universidade Estadual de Campinas (Unicamp), on August 16-17, 2007 and June 27 - July 1, 2011, respectively; the second and fourth editions were held at Rio de Janeiro, at the Universidade Federal do Rio de Janeiro (UFRJ), on August 13-15, 2008 and at Instituto de Matemática Pura e Aplicada (IMPA) on May 26 – 30, 2014, and the fifth edition was held remotely due to the coronavirus pandemic from September 20 – October 1, 2021.

## **Scientific Committee**

David Ambrose (Drexel University, USA)

Anne Bronzi (Universidade Estadual de Campinas, Brazil)

Dragos Iftimie (Université de Lyon 1, France)

Anna Mazzucato (Penn State University, USA)

Gabriela Planas (Universidade Estadual de Campinas, Brazil)

Roman Shvydkoy (University of Illinois at Chicago, USA)

## **Organizing Committee**

Anne Bronzi (Universidade Estadual de Campinas, Brazil)

Gabriela Planas (Universidade Estadual de Campinas, Brazil)

Marcelo Santos (Universidade Estadual de Campinas, Brazil)



# Contents

Welcome	i
Program	1
Lectures	3
<b>Eduardo Abreu</b> <i>A forward Lagrangian-Eulerian approach for PDEs in fluid dynamic models</i>	3
<b>Serguei Agafonov</b> <i>Darboux integrability for diagonal systems of hydrodynamic type</i>	4
<b>Sultan Aitzhan</b> <i>Cauchy problem for Benjamin-Ono equation with quasiperiodic data</i>	4
<b>Marcelo Fernandes de Almeida</b> <i>Global existence of solutions for Boussinesq system with energy dissipation</i>	5
<b>David Ambrose</b> <i>Dependence on the domain in parabolic dynamics</i>	5
<b>Claude Bardos</b> <i>Boundary effects in the zero viscosity limit of solutions of Navier Stokes equations with no slip boundary condition</i>	6
<b>Nikolai Vasilievich Chemetov</b> <i>Weak solution for Stochastic Degasperis-Procesi Equation</i>	6
<b>Peter Constantin</b> <i>TBA</i>	6
<b>Gianluca Crippa</b> <i>Anomalous dissipation in fluid dynamics</i>	7
<b>Gilberlandio J. Dias</b> <i>Interior Estimates for Power-law Shear-thickening Fluids</i>	7
<b>Bérengère Dubrulle</b> <i>Tracking fluid singularities on log-lattices</i>	8
<b>Josiane Cristina de Oliveira Faria</b> <i>Well-posedness and exponential stability for a Klein-Gordon system with locally distributed viscoelastic dampings in a past-history framework</i>	9
<b>Lucas Catão de Freitas Ferreira</b> <i>Long-time solvability for the 2D inviscid Boussinesq equations with borderline regularity and dispersive effects</i>	9
<b>Ricardo Martins Mendes Guimaraes</b> <i>On the locally self-similar blowup for the generalized SQG equation</i>	9
<b>Matthias Hieber</b> <i>Analysis of Coupled Atmosphere-Ocean Models</i>	10
<b>Edison Fausto Cuba Huamani</b> <i>Existence of solutions to gSQG in dimension two</i>	10
<b>Júlia Domingues Lemos</b> <i>Data-based approach for time-correlated closures of turbulence models</i>	10
<b>Lidiane dos Santos Monteiro Lima</b> <i>Uniform global well-posedness of the Navier–Stokes–Coriolis system in a new critical space</i>	11

<b>Juliana Honda Lopes</b> <i>Existence of weak solutions for a nonhomogeneous incompressible cell-fluid Navier-Stokes model with chemotaxis</i> . . . . .	11
<b>Alexei Mailybaev</b> <i>Hidden scale invariance in Navier–Stokes intermittency</i> . . . . .	11
<b>Anna Mazzucato</b> <i>On Euler equations with in-flow and out-flow boundary conditions.</i>	12
<b>Cecilia Freire Mondaini</b> <i>Long-time statistics of SPDEs: mixing and numerical approximation</i> . . . . .	12
<b>Cesar Niche</b> <i>Strong alignment of micro-rotation and vorticity in 3D micropolar flows</i>	12
<b>Cilon Valdez Ferreira Perusato</b> <i>On the topological size of the class of Leray solutions with algebraic decay</i> . . . . .	13
<b>Yulia Petrova</b> <i>Vanishing adsorption admissibility criterion for contact discontinuities in the polymer model</i> . . . . .	13
<b>Juliana Conceicao Precioso</b> <i>On the aggregation equations in Besov-Morrey space</i> .	13
<b>Fábio Ramos</b> <i>Transition of Friction Similarity Laws in Wall-Bounded Flows at Extreme Reynolds Numbers</i> . . . . .	14
<b>Ricardo M. M. Rosa</b> <i>Improved error estimate for the order of strong convergence of the Euler method for random differential equations</i> . . . . .	14
<b>Christian Seis</b> <i>Steady vortex rings with surface tension</i> . . . . .	15
<b>Roman Shvydkoy</b> <i>Volumetric approach to intermittency in fully developed turbulence</i>	15
<b>Henrique Borrin de Souza</b> <i>Regular flow for relativistic Vlasov-Maxwell system</i> . .	15
<b>Sergey Tikhomirov</b> <i>Viscous fingers in miscible displacement in porous media: theoretical estimates and applications in polymer flooding</i> . . . . .	16
<b>Edriss S. Titi</b> <i>Hydrostatic Euler Equations</i> . . . . .	16
<b>Zhouping Xin</b> <i>On Prandtl’s boundary layers for steady flows in infinite convergent nozzles</i> . . . . .	16
<b>Poster Session</b>	<b>17</b>
<b>Fidel Cuba Balvin</b> <i>Global well-posedness for some coupled system of mkdv type equations in modulation spaces</i> . . . . .	17
<b>Erika Paola Ortiz Bernal</b> <i>Blowup in 3D incompressible Euler equations on a logarithmic lattice</i> . . . . .	18
<b>Mikhail Glazkov</b> <i>On the uniqueness for parabolic equations with a drift term from <math>L_2</math></i>	18
<b>David Antonio Paternina Salgado</b> <i>Example of Anomalous Dissipation in the Vorticity Power Equation for Second-Grade Fluids</i> . . . . .	18
<b>Alexandre Batista de Souza</b> <i>Stochastic approximation for partial differential equations</i> . . . . .	18
<b>Author Index</b>	<b>19</b>

# Program

	10/23	10/24	10/25	10/26	10/27
8h30-9h	Registration				
9h-9h50	Opening	C. Niche	A. Mazzucato	Z. Xin	D. Ambrose
9h50-10h20	Coffee-break	Coffee-break/ Poster session	Coffee-break	Coffee-break	Coffee-break
10h20-10h50	J. Faria	E. Abreu	S. Agafonov	J. Precioso	J. Lopes
10h50-11h20	S. Tikhomirov	Y. Petrova	C. Seis	F. Ramos	G. Dias
11h20-12h	C. Bardos	G. Crippa	B. Dubrulle	P. Constantin	Closing
12h-14h	Lunch	Lunch	Lunch	Lunch	Lunch
14h-14h50	M. Hieber	L. Ferreira		R. Rosa	
14h50-15h20	C. Mondaini	L. Lima		C. Perusato	
15h20-15h40	S. Aitzhan	R. Guimarães		J. Lemos	
15h40-16h10	Coffee-break/ Poster session	Coffee-break		Coffee-break	
16h10-17h	A. Mailybaev	N. Chemetov		E. Huamani	
				H. Souza	
17h-17h40	E. Titi	R. Shvydkoy		M. Almeida	
19h-22h				Conference dinner	

 Online lecture





# Lectures

## A forward Lagrangian-Eulerian approach for PDEs in fluid dynamic models

Eduardo Abreu

Universidade Estadual de Campinas, Brazil

We will discuss a forward Lagrangian-Eulerian approach to undertake a numerical-analytical study of inherent properties of multidimensional nonlinear hyperbolic conservation laws and nonlocal transport models in fluid dynamics: wave-breaking formation (shocks), blowing up phenomena, weak-entropy solutions and flows of irregular vector fields in the context of weak solutions related to incompressible Darcy-Muskat porous media equation with spatially discontinuous coefficients from geologic heterogeneities that yield nontrivial fluid flow conduits. The novel forward tracking Lagrangian-Eulerian formulation is based on the improved concept of no-flow curves [1-8]. In the context of multidimensional hyperbolic systems of conservation laws, the resulting Lagrangian-Eulerian method satisfies a weak positivity principle [1] in view of results of P. Lax and X.-D. Liu [Computational Fluid Dynamics Journal, 5(2) (1996) 133-156 and [Journal of Computational Physics, 187 (2003) 428-440]. Indeed, we also found a connection between the notion of no-flow curves [2] (viewed as a vector field with locally bounded variation) and the results of A. Bressan in the context of (local) existence and continuous dependence for discontinuous O.D.E. 's as introduced by A. Bressan (1988) [Proc. Amer. Math. Soc. 104, 772-778]. We have tested the forward Lagrangian-Eulerian approach for a fairly complex fluid dynamics [1,2,3,4,5,6,7], for instance, 4 by 4 compressible Euler equations (Double Mach Reflection problem and Mach 3 wind tunnel flow), the 3 by 3 shallow-water system (with and without bottom topography), and also 8 by 8 Orszag-Tang vortex system in magnetohydrodynamics and more recently (whose studies are still in progress) on the study of inviscid (non-dissipative) two-dimensional (2D) quasi-geostrophic equations and a nonclassical 2 by 2 three-phase flow system of non strictly hyperbolic conservation laws (with a resonance/umbilic point) coupled with a incompressible Darcy-Muskat porous media equation [9].

Some references:

[1] EA, J. François, W. Lambert and J. Pérez, Journal of Scientific Computing (2022); <https://link.springer.com/article/10.1007/s10915-021-01712-8>

[2] EA, J. E. Agudelo, W. J. Lambert and J. Pérez, Journal of Dynamics and Differential Equations (2023); <https://doi.org/10.1007/s10884-023-10283-1>.

- [3] EA, E. Bachini, J. P. and M. Putti, Applied Mathematics and Computation (2023); <https://doi.org/10.1016/j.amc.2022.127776>
- [4] EA, R. A. De la Cruz, J. C. Juajibioy and W. J. Lambert, Journal of Dynamics and Differential Equations (2022); <https://link.springer.com/article/10.1007/s10884-022-10193-8>
- [5] EA, L. C. F. Ferreira, J. G. G. Delgado and J. Pérez, On a 1D model with nonlocal interactions and mass concentrations: an analytical-numerical approach. Nonlinearity (2022); <https://iopscience.iop.org/article/10.1088/1361-6544/ac5097>
- [6] EA, J. E. Agudelo and J. Pérez, Journal of Computational and Applied Mathematics (2024), <https://doi.org/10.1016/j.cam.2023.115465>.
- [7] EA, C. Díaz, J. Galvis and J. Pérez, Multiscale Modeling and Simulation (2020); <https://epubs.siam.org/doi/10.1137/20M1320250>
- [8] E. Abreu, M. Colombeau and E. Y. Panov (2016); <https://doi.org/10.1016/j.jmaa.2016.06.047>
- [9] EA, Mathematics and Computers in Simulation (2014); <https://doi.org/10.1016/j.matcom.2013.09.010>

---

## Darboux integrability for diagonal systems of hydrodynamic type

Serguei Agafonov

Universidade Estadual Paulista Júlio de Mesquita Filho, Brazil

We prove that 1) diagonal systems of hydrodynamic type are Darboux integrable if and only if the corresponding systems for commuting flows are Darboux integrable, 2) systems for commuting flows are Darboux integrable if and only if the Laplace transformation sequences terminate, 3) Darboux integrable systems are necessarily semihamiltonian. We give geometric interpretation for Darboux integrability of such systems in terms of congruences of lines and in terms of solution orbits with respect to symmetry subalgebras, discuss known and new examples.

---

## Cauchy problem for Benjamin-Ono equation with quasiperiodic data

Sultan Aitzhan

Drexel University, USA

We consider the well-posedness of Benjamin-Ono equation when initial data is quasi-periodic. Namely, for a fixed natural number  $N > 1$ , we have

$$u_0(x) = f_1(x) + \dots + f_N(x),$$

where each  $f_i(x)$  is periodic. While we mainly treat the case of smooth initial data, we also highlight the differences with the low regularity problem. We also discuss difficulties in upgrading local well-posedness to global one.

---

## **Global existence of solutions for Boussinesq system with energy dissipation**

Marcelo Fernandes de Almeida

Universidade Federal de Sergipe, Brazil

As is well known, the Boussinesq system coupled by energy dissipation brings new challenges in the study of the global existence of solutions, for example, this system does not have scale invariance which makes it difficult to show the existence of smooth solutions when the initial data belong to scale invariant function spaces. In this talk, we are interested in showing the main difficulties in showing the global existence of mild solutions for this system in bounded domain with smooth boundary or in a whole Euclidean space, when the initial velocity data are small and belong to certain Sobolev space. Our results on global and local well-posedness are the first to address an  $L^p$ -theory when the initial velocity data belongs to scaling invariant space, namely, the Sobolev space  $\dot{W}_\sigma^{1,n/2}(\Omega)$ . This is a joint work with Prof. Amorim, C. Braga (UFS) and Prof. Eder Mateus (UFS).

---

## **Dependence on the domain in parabolic dynamics**

David Ambrose

Drexel University, USA

We will address three different applied systems of nonlinear parabolic equations: the Kuramoto-Sivashinsky equation, the Constantin-Lax-Majda equation, and the Navier-Stokes equations. In each case we will give results indicating significantly different behavior depending on the spatial domain. For the Kuramoto-Sivashinsky equation, this is to be expected as it is apparent from linear theory, with different sizes of domain corresponding to different numbers of linearly growing Fourier modes. However for the Constantin-Lax-Majda equation and the Navier-Stokes equations, the differences we highlight for solutions on free space versus on the torus are less obvious. This includes joint work with Anna Mazzucato, Helena Nussenzveig Lopes, Milton Lopes Filho, Michael Siegel, Denis Silantyev, and Pavel Lushnikov.

---

---

## Boundary effects in the zero viscosity limit of solutions of Navier Stokes equations with no slip boundary condition

Claude Bardos

France

In this talk I consider the  $\nu \rightarrow 0$  limit of solutions of the  $2d$  Navier-Stokes equation with no slip boundary condition and will elaborate on two complementary problems:

- The convergence to the solution of the Euler equations under strong analyticity hypothesis during a short time  $0 < t < T$  to emphasize the role of the curvature of the boundary on this time  $T$  of validity in connection with the size of Görtler vortices.
- To prove that the Onsager  $\frac{1}{3}$  on the solution of the  $u(x, t)$  of the Euler equation implies the same regularity for the pressure and then use this remark to prove that in the zero viscosity limit of solutions  $u_\nu$  bounded in  $L^\infty((0, T), C^{0, \alpha})$  with  $\alpha > \frac{1}{3}$  there is no anomalous energy dissipation.

These observations are part of a programme initiated with E. Titi around 2007 and continuing with the contribution other colleagues in particular presently To Nguyen, Tr. Nguyen and D. Boutros.

---

## Weak solution for Stochastic Degasperis-Procesi Equation

Nikolai Vasilievich Chemetov

Universidade de São Paulo, Brazil

This work is concerned with the existence of solution to the stochastic Degasperis-Procesi equation on  $\mathbb{R}$  with an infinite dimensional multiplicative noise and integrable initial data. Writing the equation as a system composed of a stochastic nonlinear conservation law and an elliptic equation, we are able to develop a method based on the conjugation of kinetic theory with stochastic compactness arguments. More precisely, we apply the stochastic Jakubowski-Skorokhod representation theorem to show the existence of a weak kinetic martingale solution. In this framework, the solution is a stochastic process with sample paths in Lebesgue spaces, which are compatible with peakons and wave breaking physical phenomenon.

---

**TBA**

Peter Constantin

Princeton University, USA

TBA

---

## **Anomalous dissipation in fluid dynamics**

Gianluca Crippa

Universität Basel, Switzerland

Kolmogorov's K41 theory of fully developed turbulence advances quantitative predictions on anomalous dissipation in incompressible fluids: although smooth solutions of the Euler equations conserve the energy, in a turbulent regime information is transferred to small scales and dissipation can happen even without the effect of viscosity, and it is rather due to the limited regularity of the solutions. In rigorous mathematical terms, however, very little is known. In a recent work in collaboration with M. Colombo and M. Sorella we consider the case of passive-scalar advection, where anomalous dissipation is predicted by the Obukhov-Corrsin theory of scalar turbulence. In my talk, I will present the general context and illustrate the main ideas behind our construction of a velocity field and a passive scalar exhibiting anomalous dissipation in the supercritical Obukhov-Corrsin regularity regime. I will also describe how the same techniques provide an example of lack of selection for passive-scalar advection under vanishing diffusivity, and an example of anomalous dissipation for the forced Euler equations in the supercritical Onsager regularity regime (this last result has been obtained in collaboration with E. Bruè, M. Colombo, C. De Lellis, and M. Sorella).

---

## **Interior Estimates for Power-law Shear-thickening Fluids**

Gilberlandio J. Dias

Universidade Federal do Amapá, Brazil

In this work we establish interior estimates  $L^\infty$  for the symmetric gradient,  $D(\vec{v}) = \frac{1}{2}(\nabla\vec{v} + (\nabla\vec{v})^t)$ , of local weak solutions  $\vec{v}$  to equation

$$\operatorname{div} \left\{ |D(\vec{v})|^{p-2} D(\vec{v}) \right\} = 0, \quad x \in \Omega,$$

where  $p > 2$  and  $\Omega \subset^n$  is a domain. More precisely, for  $x_0 \in \Omega$  and  $R > 0$  so that  $B_R(x_0) = \{x \in^n; |x - x_0| < R\} \Subset \Omega$ , we search the validity of the estimate

$$\|D(\vec{v})\|_{\infty, B_{\frac{R}{2}}(x_0)} \leq \frac{\gamma}{R^{\frac{n}{p}}} \left( \int_{B_R(x_0)} |D(\vec{v})|^p dx \right)^{\frac{1}{p}},$$

with  $\gamma = \gamma(n, p)$ .

---

---

## Tracking fluid singularities on log-lattices

Bérengère Dubrulle

SPEC/IRAMIS/DSM, CEA, CNRS, University Paris-Saclay, CEA Saclay, 91191 Gif-sur- Yvette,  
France

Work done in collaboration with Quentin Pikeroen, Amaury Barral, Guillaume Costa, Ciro Campolina, Alexei Mailybaev, Simon Thalabard, V. Shukla and Giorgio Krstulovic. Development of finite-time singularities in real space, existence of complex singularities and construction of dissipative weak solutions in Euler flows is now an active line of research while the same questions formulated for the Navier-Stokes equations are among the open Millennium Prize Problems of the Clay Mathematics Institute. Present progress on this matter is hindered by the computational burden involved in simulations of the Euler equations or the Navier-Stokes equations at high Reynolds numbers.

Motivated by this observation, we investigate these issues on log-lattices, where the computational burden is logarithmic concerning ordinary fluid simulations. We first analyze properties of potential complex singularities in both 1D and 3D models for lattices of different spacings. Dominant complex singularities are tracked using the singularity strip method to obtain new scaling regarding the approach to the real axis and the influence of normal, hypo and hyper dissipation. We prove existence of finite-time blow-up for Euler equations and hypo-viscous Navier-Stokes solutions when the exponent of the Laplacian is small enough.

We then investigate the problem of constructing weak dissipative solution of Euler equation. Current construction of such solutions is usually done via convex integration methods, which are delicate to implement, especially on log-lattices. We then consider solutions of Navier-Stokes equations on log-lattice at fixed energy, variable viscosity. This protocol allows us to explore a new region of parameter space that was not accessible before. In this region, at finite resolution, we find solutions that are mixture of scale invariant solutions at small wavenumber, and thermalized solution at high wavenumber. In the limit of infinite resolution, we conjecture that these solutions converge to dissipative weak solutions of Euler equation. If the conjecture holds true, it means that our new protocol could provide a new way of constructing those solutions through a controlled limit procedure. As such, it would probably pave the way to new exact results regarding the existence and properties of dissipative weak solutions of Euler.

---

## **Well-posedness and exponential stability for a Klein-Gordon system with locally distributed viscoelastic dampings in a past-history framework**

Josiane Cristina de Oliveira Faria

Universidade Estadual de Maringá, Brazil

The main goal in the present talk is to discuss the well-posedness as well as the exponential stability for a strongly coupled Klein-Gordon system posed on a bounded domain of  $\mathbb{R}^2$  with smooth boundary, subject to locally distributed viscoelastic effects driven by nonnegative functions  $a(x)$  and  $b(x)$  acting on  $\Omega$ , where  $a = b = 0$  in a region  $A \subset \Omega$ .

---

## **Long-time solvability for the 2D inviscid Boussinesq equations with borderline regularity and dispersive effects**

Lucas Catão de Freitas Ferreira

Universidade Estadual de Campinas, Brazil

We are concerned with the long-time solvability for 2D inviscid Boussinesq equations for a larger class of initial data which covers the case of borderline regularity. First we show the local solvability in Besov spaces uniformly with respect to a parameter  $\kappa$  associated with the stratification of the fluid. Afterwards, employing a blow-up criterion and Strichartz-type estimates, the long-time solvability is obtained for large  $\kappa$  regardless of the size of initial data. Joint work with Prof. Vladimir Angulo-Castillo (UNAL, Colombia) and Prof. L. Kosloff (Unicamp, Brazil).

---

## **On the locally self-similar blowup for the generalized SQG equation**

Ricardo Martins Mendes Guimaraes

Universidade Estadual de Campinas, Brazil

We consider locally self-similar solutions for the inviscid generalized SQG equation in two-dimensional and under a  $L^r$  growth assumption on the self-similar profile and its gradient, we identify appropriate ranges of the self-similar parameter where the profile is either identically zero or its  $L^p$  asymptotic behavior can be characterized, for suitable  $p$ .

---

---

## Analysis of Coupled Atmosphere-Ocean Models

Matthias Hieber

Technische Universität Darmstadt, Germany

In this talk we consider various coupled atmosphere-ocean models as introduced by Lions, Temam and Wang. We analyze three circles of problems: coupling by nonlinear and so-called wind driven boundary conditions, coupling by Hibler's sea ice equations and, in the presence of stochastic forces, effects of transport noise. This is joint work with A. Agresti, T. Binz, F. Brandt, A. Hussein and T. Zoechling.

---

## Existence of solutions to gSQG in dimension two

Edison Fausto Cuba Huamani

Pontifícia Universidade Católica do Rio de Janeiro, Brazil

In this talk, we construct non-trivial vortex solutions to the gSQG. The proof is carried out by means of a combination of a desingularization argument with the implicit function theorem on the linearization of contour dynamics equation.

---

## Data-based approach for time-correlated closures of turbulence models

Júlia Domingues Lemos

Universidade Federal do Rio de Janeiro, Brazil

Developed turbulent motion of fluid still lacks an analytical description despite more than a century of active research. Nowadays phenomenological ideas are widely used in practical applications, such as small-scale closures for numerical simulations of turbulent flows. In the present work, we use a shell model of turbulence to construct a closure intended to have a solid theoretical background and to capture intrinsic probabilistic features of turbulence. Shell models of turbulence are dynamical deterministic systems used to model energy cascade and other key aspects of the Navier-Stokes such as intermittency. We rescale the variables of the Sabra model in a way which leads to hidden symmetries and universal distributions. We then use such fine distributions to write closures, i.e., missing expressions for some of the Sabra variables. Our closures rely on approximating probability density functions using a Gaussian Mixture Model, which makes them probabilistic by nature and allows us to write time-correlated closures. We also provide a framework where other Machine Learning tools can be employed with reduced black-box aspects. This work is joint with Alexei Mailybaev.

---



## **Uniform global well-posedness of the Navier–Stokes–Coriolis system in a new critical space**

Lidiane dos Santos Monteiro Lima

Universidade Federal de Goiás, Brazil

We present some results for Coriolis-Navier-Stokes system with initial data in a space based on Fourier transform, namely, the Fourier-Besov-Morrey space. Our approach allows us to prove the results uniformly in relation to angular velocity and covers some recent works in Coriolis-Navier-Stokes equations. We analyzed the existence of nonstationary and stationary solutions and, for nonstationary solutions, we also analysed the asymptotic behavior of solutions.

---

## **Existence of weak solutions for a nonhomogeneous incompressible cell-fluid Navier-Stokes model with chemotaxis**

Juliana Honda Lopes

Universidade de São Paulo, Brazil

This work is concerned with the mathematical analysis of a general cell-fluid Navier-Stokes model with the inclusion of chemotaxis proposed by Y. Qiao and S. Evje (2020). This general model relies on a mixture theory multiphase formulation. It consists of two mass balance equations and two general momentum balance equations, respectively, for the cell and fluid phase, combined with a convection-diffusion-reaction equation for oxygen. We investigate the existence of weak solutions in a two or three-dimensional bounded domain when the fluids are assumed to be incompressible with constant volume fraction.

JHL was financed by FAPESP-Brazil grant 2020/14206-3.

---

## **Hidden scale invariance in Navier–Stokes intermittency**

Alexei Mailybaev

Instituto de Matemática Pura e Aplicada, Brazil

We expose a hidden scaling symmetry of the Navier–Stokes equations in the limit of vanishing viscosity, which stems from dynamical space–time rescaling around suitably defined Lagrangian scaling centres. At a dynamical level, the hidden symmetry projects solutions which differ up to Galilean invariance and global temporal scaling onto the same representative flow. At a statistical level, this projection repairs the scale invariance, which is broken by intermittency in the original formulation. This is a joint work with Simon Thalabard.

---

---

## On Euler equations with in-flow and out-flow boundary conditions.

Anna Mazzucato

Penn State University, USA

I will discuss recent results concerning the well-posedness and regularity for the incompressible Euler equations when in-flow and out-flow boundary conditions are imposed on parts of the boundary. This is joint work with Gung-Min Gie (U. Louisville, USA) and James Kelliher (UC Riverside, USA).

---

## Long-time statistics of SPDEs: mixing and numerical approximation

Cecilia Freire Mondaini

Drexel University, USA

In analyzing complex systems modeled by stochastic partial differential equations (SPDEs), such as certain turbulent fluid flows, an important question concerns their long-time behavior. In particular, one is typically interested in determining how long it takes for the system to settle into statistical equilibrium, and in investigating efficient numerical schemes for approximating such long-time statistics. In this talk, I will present two general results in this direction, and illustrate them with an application to the 2D stochastic Navier-Stokes equations. Most importantly, our approach does not require gradient bounds for the underlying Markov semigroup as in previous works, and thus provides a flexible formulation for further applications. This is based on joint work with Nathan Glatt-Holtz (Tulane U).

---

## Strong alignment of micro-rotation and vorticity in 3D micropolar flows

Cesar Niche

Universidade Federal do Rio de Janeiro, Brazil

Small rigid particles suspended on a micropolar fluid provide microstructure that coexists and interacts with the local rotation of the fluid given by the vorticity. In this work we prove that the particles' angular velocity and the vorticity strongly align in 3D for large times. We provide average and pointwise estimates for the decay rate of the difference between these two vectors, which measures the alignment. This is joint work with Robert Guterres, Wilberclay Melo, Cilon Perusato and Paulo Zingano.

---

## **On the topological size of the class of Leray solutions with algebraic decay**

Cilon Valdez Ferreira Perusato

Universidade Federal de Pernambuco, Brazil

In 1987, Michael Wiegner in his seminal paper provided an important result regarding the energy decay of Leray solutions  $\mathbf{u}(\cdot, t)$  to the incompressible Navier-Stokes in  $\mathbb{R}^n$ : if the associated Stokes flows had their  $L^2$  norms bounded by  $O(1+t)^{-\alpha}$  for some  $0 < \alpha \leq (n+2)/4$ , then the same would be true of  $\|\mathbf{u}(\cdot, t)\|_{L^2(\mathbb{R}^n)}$ . The converse also holds, as shown by Z. Skalák, and by our analysis, which uses a more straightforward argument. As an application of these results, we discuss the genericity problem of algebraic decay estimates for Leray solutions of the unforced Navier–Stokes equations. In particular, we prove that Leray solutions with algebraic decay generically satisfy two-sided bounds of the form  $(1+t)^{-\alpha} \lesssim \|\mathbf{u}(\cdot, t)\|_{L^2(\mathbb{R}^n)} \lesssim (1+t)^{-\alpha}$ .

---

## **Vanishing adsorption admissibility criterion for contact discontinuities in the polymer model**

Yulia Petrova

Pontifícia Universidade Católica do Rio de Janeiro, Brazil

In the talk we discuss the admissibility criteria for solutions to a Riemann problem of a non-strictly hyperbolic system of conservation laws modelling chemical flooding process in oil recovery. We introduce the vanishing adsorption criterion for contact discontinuities and prove that this criterion, which derives from a physical effect, justifies the admissibility criteria adopted previously by Keyfitz-Kranzer, Isaacson-Temple, and de Souza-Marchesin for models such that the fractional flow function depends monotonically on chemical concentration. Another interesting feature is that the adsorption criterion selects the undercompressive contact discontinuities required to solve the general Riemann problem in an example model with non-monotone dependence. The talk is based on joint work with D. Marchesin and B. Plohr (arxiv:2211.10326).

---

## **On the aggregation equations in Besov-Morrey space**

Juliana Conceicao Precioso

Universidade Estadual Paulista Júlio de Mesquita Filho, Brazil

In this work, we present conditions to obtain a global-in-time existence of solutions to a class of nonlinear viscous transport equations describing aggregation phenomena in biology with sufficiently small initial data in Besov-Morrey spaces and gradient potential as a Radon measure. We also study the self-similarity and asymptotic stability of solutions at large times.

---

---

## Transition of Friction Similarity Laws in Wall-Bounded Flows at Extreme Reynolds Numbers

Fábio Ramos

Universidade Federal do Rio de Janeiro, Brazil

This talk delves into the transition of friction similarity laws in wall-bounded turbulent flows at extreme Reynolds numbers. We present novel power-law formulas for the friction factor of incompressible Newtonian fluidflows in pipe and flat plate zero-pressure gradient turbulent boundary layers, founded on a new phenomenology for coherent structures dominating the momentum exchange. Supported by recent experimental and Direct Numerical Simulation (DNS) data, the proposed asymptotic near-wall momentum exchange mechanism reveals a transition in the scaling of the mean velocity profile (MVP) of turbulent pipe and channel flows at extreme Reynolds numbers. These findings indicate a possible shift in the turbulent momentum transfer mechanism for wall-bounded flows at extreme Reynolds numbers, with implications for understanding and modeling such flows.

---

## Improved error estimate for the order of strong convergence of the Euler method for random differential equations

Ricardo M. M. Rosa

Universidade Federal do Rio de Janeiro, Brazil

The Euler method for approximating an ODE is known to be of order 1. For Stochastic ODEs with multiplicative noise, however, it drops to  $1/2$ . What about for Random ODEs? Current works tell us it is also order  $1/2$ , or even less, depending on the Holder exponent of the noise sample paths. Here we show that, in many typical situations, it is actually of order 1. This applies to a variety of noises, such as additive or multiplicative Itô processes, transport processes with sample paths of bounded variation, and even processes with discontinuous sample paths, as in point-process noises. For fractional Brownian motion noises, we may not reach order 1, depending on the Hurst parameter, but we still improve the order compared with the current belief. The proofs rely on writing a global error formula instead of estimating the local error; using Fubini to move the critical regularity term from the small mesh scale to the large scales; and using the Itô isometry or some other form of global estimate to control that critical term. In this talk, we discuss these improvements, sketch the proofs, and illustrate the results numerically with a number of interesting models. This is a joint work with Peter Kloeden (University of Tübingen, Germany).

---

## **Steady vortex rings with surface tension**

Christian Seis

Universität Münster, Germany

The existence of steady vortex rings for the two-phase Euler equations with surface tension is studied, describing the evolution of a perfect bubble air ring in water. Such objects are created in nature by cetaceans such as dolphins or beluga whales, and they appear to be surprisingly stable configurations. The mathematical model features a vortex sheet on the surface of the air bubble. We construct such vortex rings with small cross sections with the help of an implicit function theorem and derive the asymptotics of various quantities for small cross sections. Joint work with David Meyer (Münster).

---

## **Volumetric approach to intermittency in fully developed turbulence**

Roman Shvydkoy

University of Illinois, USA

We will discuss a set of concepts based on volumetric characteristics of a turbulence field that allows to see the Frisch-Parisi multifractal formalism from the perspective of information theory. We will discuss multiple uses of this framework including regularity of intermittent solutions to the NSE. This is a joint work with A. Cheskidov.

---

## **Regular flow for relativistic Vlasov-Maxwell system**

Henrique Borrin de Souza

Universidade Estadual de Campinas, Brazil

We study the Lagrangian structure of Vlasov-Maxwell system, that is, by using a suitable notion of flow, we prove that if the densities are bounded in time integrable in space functions, and the charge acceleration and its time derivative are integrable functions in spacetime, renormalized and distributional solutions of the system are the transport of the initial condition by its flow. The result is an extension of those obtained by Ambrosio, Colombo, and Figalli (Duke, 2017) for the Vlasov-Poisson system, and by the author and Marcon for relativistic Vlasov-systems with quasistatic approximations of Maxwell's equations (Math. Meth. Appl. Sci., 2023).

---

---

## **Viscous fingers in miscible displacement in porous media: theoretical estimates and applications in polymer flooding**

Sergey Tikhomirov

Pontificia Universidade Catolica do Rio de Janeiro, Brazil

We study the motion of viscous, miscible liquids in porous media. Injection of a less viscous fluid to a more viscous one leads to the growth of an instability often refereed as viscous fingering. Such a physical mechanism has a negative impact on various flooding schemes in oil fields. The model describing this phenomenon is a system of multi-dimensional PDEs consisting of conservation of mass, incompressibility condition and Darcy's law. The main goal is finding rigorous estimates for the size of the mixing zone containing the instabilities.

---

## **Hydrostatic Euler Equations**

Edriss S. Titi

Univeristy of Cambridge, UK

We will show the ill-posedness of the hydrostatic Euler equations in all Sobolev spaces and the formation of finite time singularity. In addition, we will discuss the effect of fast rotation on the life-span of spatial analytic solutions. Furthermore, we will discuss the notion of weak solutions for this system utilizing the Bony paradifferential calculus, and state analogues of the Onsager conjecture for conservation of the horizontal energy.

---

## **On Prandtl's boundary layers for steady flows in infinite convergent nozzles**

Zhouping Xin

The Chinese University of Hong Kong, Hong Kong

In this talk, I will present some results on the large Reynold number limits and asymptotic behaviors of solutions to the steady incompressible Navier-Stokes equations in 2-D infinitely long convergent nozzles. The main results show that the Prandtl's boundary layer theory can be rigorously established and the sink-type Euler flow superimposed with a self-similar Prandtl's flow is shown to be uniformly structurally stable provided that the nozzle boundaries satisfy curvature-decreasing condition with given negative mass flux. Furthermore, the asymptotic behaviors of the solutions at both the vertex and infinity can be determined uniquely. Some key analysis will be discussed. This is a joint work with Dr. Chen Gao.

---

# Poster Session

## Global well-posedness for some coupled system of mkdv type equations in modulation spaces

Fidel Cuba Balvin

Universidade Federal do Rio de Janeiro, Brazil

We consider the initial value problem (IVP) associated to a system consisting modified Korteweg-de Vries (mKdV) type equations

$$(I) \begin{cases} \partial_t v + \partial_x^3 v + \partial_x(vw^2) = 0, & v(x, 0) = \phi(x), \\ \partial_t w + \alpha \partial_x^3 w + \partial_x(v^2 w) = 0, & w(x, 0) = \psi(x), \end{cases}$$

and using the theory of [1] we derive trilinear estimates and use it in the contraction argument [3] in IVP (I), so we study the local well-posedness (LWP) [2]. Also, establish global well-posedness for given data in modulation spaces  $M_s^{2,p}(\mathbb{R}) \times M_s^{2,p}(\mathbb{R})$ ,  $s \geq \frac{1}{4}$ ,  $2 \leq p < \infty$  and for  $\alpha \neq 0$ .

## References

- [1] T. OH, Y. WANG, *On global well-posedness of the modified KdV equation in Modulation spaces*, Discrete and continuous Dynamical Systems, 41: 2971-2992, 2021.
  - [2] X. CARVAJAL, M. PANTHEE, *Nonlinear Schrodinger equations with the third order dispersion on Modulation spaces*, Partial Differential Equations and Applications, v.3, p. 59, 2022.
  - [3] A. GRUNROCK, *An improved local well-posedness results for the modified KdV equation*, Int. Math. Res. Not., 3287-3308, 2004.
-

---

## Blowup in 3D incompressible Euler equations on a logarithmic lattice

Erika Paola Ortiz Bernal

Instituto Nacional de Matemática Pura e Aplicada, Brazil

We implement a model on logarithmic lattices that allows us to study from a numerical point of view singular solutions of 3D incompressible Euler equations. Our goal is to follow recent results of T. Elgindi that local solutions of the 3D incompressible Euler equations can develop singularities in finite time for initial conditions of  $C^{1,\alpha}$  regularity. Our intention is to study numerically the structure of such singularities using the log-lattice model depending on the regularity parameter  $\alpha$ . We will report the numerical results obtained in this direction. This is a joint work with Ciro S. Campolina and Alexei A. Mailybaev.

---

## On the uniqueness for parabolic equations with a drift term from $L_2$

Mikhail Glazkov

Instituto Nacional de Matemática Pura e Aplicada, Brazil

In this work we prove uniqueness of weak solutions in the energy class for the initial boundary value problem for the equation  $\partial_t u - \Delta u + b \cdot \nabla u = f_0 - \operatorname{div} f$  with a drift  $b \in L_2$  satisfying the condition  $\operatorname{div} b \leq 0$ .

---

## Example of Anomalous Dissipation in the Vorticity Power Equation for Second-Grade Fluids

David Antonio Paternina Salgado

Universidade Federal do Rio de Janeiro, Brazil

In this study, we will develop a solution for the vorticity potential equation for second-grade fluids, which exhibit the phenomenon of anomalous dissipation, more specifically, infinite dissipation.

---

## Stochastic approximation for partial differential equations

Alexandre Batista de Souza

Universidade Estadual de Campinas, Brazil

In this work we study the explicit rate of convergence for mean field games where the player interacting in a suitable moderate way. In other words we provide a stochastic approximation for the Kolmogorov partial differential equation.

---



# Author Index

Abreu  
    Eduardo, 3

Agafonov  
    Serguei, 4

Aitzhan  
    Sultan, 4

Almeida  
    Marcelo, 5

Ambrose  
    David, 5

Balvin  
    Fidel, 17

Bardos  
    Claude, 6

Bernal  
    Erika, 18

Chemetov  
    Nikolai, 6

Constantin  
    Peter, 6

Crippa  
    Gianluca, 7

Dias  
    Gilberlandio, 7

Dubrulle  
    Bérengère, 8

Faria  
    Josiane, 9

Ferreira  
    Lucas, 9

Glazkov  
    Mikhail, 18

Guimaraes  
    Ricardo, 9

Hieber  
    Matthias, 10

Huamani  
    Edison, 10

Lemos  
    Júlia, 10

Lima  
    Lidiane, 11

Lopes  
    Juliana, 11

Mailybaev  
    Alexei, 11

Mazzucato  
    Anna, 12

Mondaini  
    Cecilia, 12

Niche  
    Cesar, 12

Perusato  
    Cilon, 13

Petrova  
    Yulia, 13

Precioso  
    Juliana, 13

Ramos  
    Fábio, 14

Rosa  
    Ricardo, 14

Salgado

David, 18

Seis

Christian, 15

Shvydkoy

Roman, 15

Souza

Alexandre, 18

Henrique, 15

Tikhomirov

Sergey, 16

Titi

Edriss, 16

Xin

Zhouping, 16