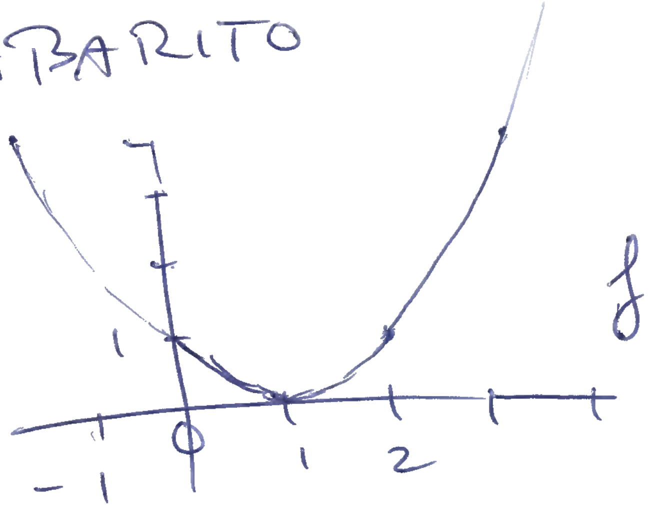
GABARITO

1.



(a) $f(x) = x^2 - 2x + 1 = (x-1)^2$ $\quad \frac{1}{2}$

única raiz $\xi = 1$

(b) $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} = x_k - \frac{(x_k-1)^2}{2x_k - 2}$

$\frac{1}{4}$ $\quad = x_k - \frac{1}{2}\frac{(x_k-1)^2}{(\cancel{x_k-1})} = x_k - \frac{1}{2}(x_k - 1)$

$= x_k - \frac{1}{2}x_k + \frac{1}{2}$ $\quad \frac{1}{4}$

$\xi = 1$ e $x_k \neq 1 \ \forall k$ $\quad = \frac{1}{2}x_k + \frac{1}{2} = \frac{1}{2}(x_k + 1)$

$e_{k+1} = x_{k+1} - 1 = \frac{1}{2}x_k + \frac{1}{2} - 1 = \frac{1}{2}x_k - \frac{1}{2}$

$= \frac{1}{2}(x_k - 1) = \frac{1}{2}e_k$ $\quad \frac{1}{4}$

$\Rightarrow \lim_{k\to\infty} \frac{e_{k+1}}{e_k} = \frac{1}{2}$

$\Rightarrow \lim_{k\to\infty} \left|\frac{e_{k+1}}{e_k}\right| = \frac{1}{2}$ $\quad \frac{1}{4}$ $\quad \therefore$ convergência linear

(c) Se $f'(\xi) \neq 0$ $\quad \frac{1}{4}$

(d) $x_1 = \frac{1}{2}(x_0+1) = \frac{1}{2} \cdot \cancel{2.2} \; 2.1 = 1.05$

$x_2 = \frac{1}{2}(x_1+1) = \frac{1}{2}(2.05) = 1.02$

2.

(a) $PA = LU$

$\Rightarrow \det \cancel{(PA)} = \det(P) \cdot \det(A) = \det(PA)$

$= \det(LU) = \det(L) \cdot \det(U)$

$\Rightarrow$ ~~det~~ Temos $\det(P) = -1$

$\Rightarrow \det(A) = \dfrac{1 \cdot (-4) \cdot (-9)}{-1} = -36$

(b) $Ax = b$

Pb

2. (a) $PA = LU$

$\Rightarrow \det(P)\cdot\det(A) = \det(L)\cdot\det U$

$\Rightarrow (-1)\cdot\det(A) = 1\cdot(-4)\cdot(-9) = 36$

$\rightarrow \det(A) = -36$

$\frac{1}{2}$

(b) $Ax = b \Leftrightarrow PAx = Pb$

$\Leftrightarrow L(\underbrace{Ux}_{y}) = Pb = \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ $\frac{1}{2}$

1. Resolve $Ly = Pb$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -1 \\ 2 & 1 & 0 & 0 & 1 \\ -3 & 0 & 1 & 0 & 0 \\ 0 & -1 & -2 & 1 & 0 \end{array}\right) \Rightarrow y = \begin{pmatrix} -1 \\ 3 \\ -3 \\ -3 \end{pmatrix}$$

$3 + y_3 = 0$

$-3 + 6 + y_4 = 0$

$\frac{1}{2}$

2. Resolve $Ux = y$

$$\left(\begin{array}{cccc|c} -2 & -2 & -1 & 0 & -1 \\ 0 & 2 & 1 & -1 & 3 \\ 0 & 0 & 3 & 0 & -3 \\ 0 & 0 & 0 & -3 & -3 \end{array}\right)$$

$-2x_1 = 3$

$-2x_1 - 5 + 1 = -1 \Rightarrow \cancel{2x_1}$

$2x_2 - 1 - 1 = 3 \Rightarrow x_2 = 2{,}5$

$x_3 = -1$

$x_4 = 1$

$$x = \begin{pmatrix} -1{,}5 \\ 2{,}5 \\ -1 \\ 1 \end{pmatrix}$$

$\frac{1}{2}$

(c) $d = c + b = Az + Ax$

$= A(z+x)$ $\frac{1}{4}$

$\Rightarrow t = z + x = \begin{pmatrix} -7 \\ 6 \\ 3 \\ 8 \end{pmatrix} + \begin{pmatrix} -1{,}5 \\ 2{,}5 \\ -1 \\ 1 \end{pmatrix}$

$\frac{1}{4} \quad = \begin{pmatrix} -8{,}5 \\ 8{,}5 \\ 2 \\ 9 \end{pmatrix}$ $= -\frac{15}{2}$ satisfaz $At = d$

3. (a)

$A = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 3 & -1 & 0 \\ 0 & 1 & 5 & -1 \\ 0 & 0 & 1 & 7 \end{pmatrix}$ $\quad a_{12} = 1 - 2 = -1$

$a_{21} = 2 - 1 = 1$

$b = \begin{pmatrix} 1 \\ 3 \\ 5 \\ 7 \end{pmatrix}$ $\quad \frac{1}{2}$

(b) $\cancel{x}$ b =

$C = \begin{pmatrix} 0 & +1 & 0 & 0 \\ -\frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & -\frac{1}{5} & 0 & \frac{1}{5} \\ 0 & 0 & -\frac{1}{7} & 0 \end{pmatrix}$

$\Rightarrow |C| = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{5} & 0 & \frac{1}{5} \\ 0 & 0 & \frac{1}{7} & 0 \end{pmatrix}$ $\quad \alpha_1 = \gamma_1 = |\cancel{1}|$

Nem o critério das linhas nem o critério de Sassenfeld está satisfeito. $\frac{1}{2}$

Isto implica que, usando o critério das linhas ~~e o~~ critério de Sassenfeld, não podemos afirmar nada, i.e., não sabemos (usando estes critérios) se GJ e/ou GS vão convergir ou não. $\frac{1}{2}$

(c) $\varepsilon = 0,2$, $x^{(0)} = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 1 \end{pmatrix}$, $\beta = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$

$$\begin{pmatrix} 0 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \beta_1 = +1 + 1 = \underline{\underline{2}}$$

$$\rightsquigarrow \begin{pmatrix} 2 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} -\frac{1}{3} & 0 & \frac{1}{3} & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \\ 1 \end{pmatrix} + 1 = -\frac{1}{3} + 1 = \frac{2}{3} \rightsquigarrow \begin{pmatrix} 2 \\ \frac{2}{3} \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -\frac{1}{5} & 0 & \frac{1}{5} \end{pmatrix} \begin{pmatrix} 2 \\ \frac{2}{3} \\ 1 \\ 1 \end{pmatrix} + 1 = \cancel{\frac{7}{15}} \cancel{\rightsquigarrow \begin{pmatrix} 2 \\ \frac{2}{3} \\ \frac{7}{15} \\ 1 \end{pmatrix}} = \frac{16}{15} \approx 1,0667$$

$$\rightsquigarrow \begin{pmatrix} 2 \\ 0,6667 \\ 1,0667 \\ 1 \end{pmatrix}$$

$$(0\ 0\ -\tfrac{1}{7}\ 0)\begin{pmatrix}2\\ 2/3\\ 16/15\\ 1\end{pmatrix}+1 \approx \underline{\underline{0,8476}}$$

$$d_r(x^{(1)},x^{(0)}) = \frac{\|x^{(1)}-x^{(0)}\|_\infty}{\|x^{(1)}\|_\infty} \approx \frac{\cancel{6,3333}\ 0,6667}{2} \approx 0,166 < 0,2$$

k	$x^{(k)}$	$d_r(x^{(k)},x^{(k+1)})$ Pare
0	$\begin{pmatrix}2\\1\\1\\1\end{pmatrix}$	—
1	$\begin{pmatrix}2\\0,6667\\1,0667\\0,8476\end{pmatrix}$	$0,1667 < 0,2 = \varepsilon$ PARE

1

1/2

Resultado:

Solução $x^* \approx \begin{pmatrix}2\\0,6667\\1,0667\\0,8476\end{pmatrix}$

4.(a) $70e^{\beta} + 20e^{\omega} = 27.5702$ $\frac{1}{2}$

$$70e^{2\beta} + 20e^{2\omega} = 17.6567$$

(b)
$$F(\beta,\omega) = \begin{pmatrix} 70e^{\beta} + 20e^{\omega} - 27.5702 \\ 70e^{2\beta} + 20e^{2\omega} - 17.6567 \end{pmatrix}$$

$$J(\beta,\omega) = \begin{pmatrix} 70e^{\beta} & 20e^{\omega} \\ 140e^{2\beta} & 40e^{2\omega} \end{pmatrix} \quad \frac{1}{4}$$

$$x^{(0)} = \begin{pmatrix} \beta_0 \\ \omega_0 \end{pmatrix} = \begin{pmatrix} -1{,}9 \\ -0{,}11 \end{pmatrix}$$

k	$x^{(0)}$	$F(x^{(0)})$	$\lVert F(x^{(0)})\rVert_\infty$	$\lVert s^{(k-1)}\rVert_\infty$	s
0	$\begin{pmatrix} -1{,}9 \\ -0.11 \end{pmatrix}$	$\begin{pmatrix} 0{,}8163 \\ -0{,}0404 \end{pmatrix}$	$0{,}8163$ $\frac{1}{4}$	–	

$$F(x^{(0)}) = \begin{pmatrix} 70e^{-1{,}9} + 20e^{-0{,}11} - 27{,}5702 \\ 140\cdot e^{-3.8} + 40e^{-0.22} - 17{,}6567 \end{pmatrix} = \begin{pmatrix} 0{,}8163 \\ -0{,}040 \end{pmatrix}$$

Para encontrar $s^{(0)}$:

$$J(x^{(0)}) \cdot s^{(0)} = -F(x^{(0)}) = \begin{pmatrix} -0,8163 \\ 0,0404 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} 70e^{-1,9} & 20e^{-0.11} & -0,8163 \\ 140e^{-3,8} & 40e^{-0.22} & 0,0404 \end{array}\right)$$

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$$\rightsquigarrow \left(\begin{array}{cc|c} 10.4698 & 17.9167 & -0,8163 \\ 3.1319 & 32.1008 & 0,0404 \end{array}\right)$$

$m = \frac{3.1319}{10.4698}$

$$\rightsquigarrow \left(\begin{array}{cc|c} 10,4698 & 17.9167 & -0.8163 \\ 0 & 26.7412 & 0.2846 \end{array}\right)$$

$$\rightsquigarrow \left(\begin{array}{cc|c} 10,4698 & 17.9167 & -0,8163 \\ 0 & 1 & 0.0106 \end{array}\right)$$

$$\rightsquigarrow \left(\begin{array}{cc|c} 10.4698 & 0 & -1.0069 \\ 0 & 1 & 0,0106 \end{array}\right)$$

$$\rightsquigarrow s^{(0)} = \begin{pmatrix} -0,0962 \\ 0,0106 \end{pmatrix}$$

1/4

Obtemos,

$$x^{(1)} = x^{(0)} + s^{(0)} = \begin{pmatrix} -1,9 \\ -0,11 \end{pmatrix} + \begin{pmatrix} -0,0962 \\ 0,0106 \end{pmatrix}$$

$$= \begin{pmatrix} -1,9962 \\ -0,0994 \end{pmatrix}$$

k	$x^{(k)}$	F(
0		

$$F(x^{(1)}) = \begin{pmatrix} 70e^{-1,9962} + 20e^{-0,0994} - 27,5202 \\ 70e^{-2\cdot 1,9962} + 20\cdot e^{2\cdot(-0,0994)} - 14,656 \end{pmatrix}$$

k	$x^{(k)}$	$F(x^{(k)})$	$\lVert F(x^{(k)})\rVert_\infty$	$\lVert s^{(k-1)}\rVert_\infty$	$s^{(k)}$
0	$\begin{pmatrix} -1,9 \\ -0,11 \end{pmatrix}$	$\begin{pmatrix} 0,8163 \\ -0,0404 \end{pmatrix}$	0,8163	–	$\begin{pmatrix} -1,996 \\ -0,099 \end{pmatrix}$
1	$\begin{pmatrix} -1,9962 \\ -0,0994 \end{pmatrix}$	$\begin{pmatrix} 0,0479 \\ 0,0309 \end{pmatrix}$	$0,0479 < 0,1 = \varepsilon$ PARE	$\frac{1}{4}$	

Resultado $x^{(1)} = \begin{pmatrix} -1,9962 \\ -0,0994 \end{pmatrix}$ $\frac{1}{4}$