1. (a) FALSO:

$$\nexists \bigvee [0.3, 0.4) \in \underbrace{[0.3, 0.4] \cup (0.6, 0.7)}^{\mathcal{P}}$$

Suponha $s = \bigvee [0.3, 0.4) \in \mathcal{P}$

$\Rightarrow a \leq s \quad \forall a \in [0.3, 0.4)$

$\Rightarrow s \notin [0.3, 0.4)$

$\Rightarrow s \in (0.6, 0.7]$

n tal que $s \leq r \quad \forall r \in (0.6, 0.7]$

$\nexists$ Suponha que $\exists\, s$ com esta prop

Visto que $\exists\, \varepsilon$ com $B_\varepsilon(s) \subseteq (0.6, .7)$

$\exists\, \varepsilon$ com $s - \varepsilon \in (0.6, 0.7]$

$\Rightarrow$ e obviamente $s - \varepsilon < s$ $\lightning$

(b) FALSO:

Considere $[0.1, 0.2) \cap [0.3, 0.4] = \emptyset \notin$

$\cancel{\emptyset \notin \Pi}$

1. (Pós) Vamos mostrar que $\forall I$ e $\forall x_i, i \in I$

(a) $\varphi\left(\bigvee_{i\in I} x_i\right) = \bigvee_{i\in I} \varphi(x_i)$

(b) $\varphi\left(\bigwedge_{i\in I} x_i\right) = \bigwedge_{i\in I} \varphi(x_i)$

Demo de (a): Sejam $I$, $x_i$, $i \in I$ arbitrários

(i) Note que $\bigvee_{i\in I} x_i \in \mathbb{L}$ porque $\mathbb{L}$ é um reticulado compl

e $\bigvee_{i\in I} x_i \geq x_i \quad \forall i \in I$

$\Rightarrow \varphi\left(\bigvee_{i\in I} x_i\right) \geq \varphi(x_i) \quad \forall i \in I$

$\Rightarrow \varphi\left(\bigvee_{i\in I} x_i\right) \geq \bigvee_{i\in I} \varphi(x_i) \in \mathbb{M}$

(ii) Para concluir a demo de (a), é suficiente de mostrar que $\bigvee_{i\in I} \varphi(x_i) \geq \varphi\left(\bigvee_{i\in I} x_i\right)$

Visto que ~~$\varphi$~~ $\varphi$ é bijetivo, $\exists(!)\, y \in \mathbb{L}$ t.q. $\varphi(y) = \bigvee_{i\in I} \varphi(x_i) \in \mathbb{M}$

Tem-se que $\varphi(y) = \bigvee_{i\in I} \varphi(x_i) \geq \varphi(x_i) \quad \forall i \in I$

$\overset{\varphi(b)\geq\varphi(a) \Rightarrow b\geq a}{\Longrightarrow} \quad y \geq x_i \quad \forall i \in I$

$\Longrightarrow \quad y \geq \bigvee_{i\in I} x_i \in \mathbb{L}$

$\overset{b\geq a \Rightarrow \varphi(b)\geq\varphi(a)}{\Longrightarrow} \quad \varphi(y) \geq \varphi\left(\bigvee_{i\in I} x_i\right)$

$\Longleftrightarrow \quad \bigvee_{i\in I} \varphi(x_i) \geq \varphi\left(\bigvee_{i\in I} x_i\right)$ q.e.d

(b): a prova é similar.

2. ~~(a)~~ e 2(a) Pós

Seja $A = (a; u; b)$ onde $a \leq u \leq b$ $\Rightarrow supp(A) = [A]^{0+} = (a, b)$

Seja $\alpha \in (0, 1]$:

$[A]^\alpha = \{x \in \mathbb{R} \mid A(x) \geq \alpha\}$

$= \{x \in [a, u) \mid \frac{x-a}{u-a} \geq \alpha\} \cup \{u\} \cup \{x \in (u, b] \mid \frac{x-b}{u-b}$

$= \{x \in [a, b] \setminus \{u\} \mid \frac{x-a}{u-a}, \frac{x-b}{u-b} \geq \alpha\} \cup \{u\}$

$= \{x \in [a, b] \setminus \{u\} \mid x - a \geq \alpha(u-a)$ e $x - b \leq \alpha(u-b)\} \cup \{u$

$= \{x \in [a, b] \setminus \{u\} \mid \alpha(u-a) + a \leq x \leq \alpha(u-b) + b\} \cup \{u$

$= \left( [\alpha(u-a) + a, \alpha(u-b) + b] \setminus \{u\} \right) \cup \{u\}$

$= [\alpha(u-a) + a, \alpha(u-b) + b]$

Visto que $A(x)$ é contínua da ~~es~~ direita em $(-\infty, a]$ e " esquerda em $[b, \infty)$ temos que $[A]^{0+} = \lim_{\alpha \to 0} [A]^\alpha = (a, b)$ mas isto já sabemos pq $A = (a; u; b)$

2.(b) Vamos mostrar que

$(a;u;b)+(c;v;d)=(a+c;u+v;b+d)$

Seja $A=(a;u;b)$, $B=(c;v;d)$ e $C=(a+c;u+v;b+d)$

$\forall \alpha\in(0,1]$ temos

$[A+B]^\alpha=[A]^\alpha+[B]^\alpha=$

$=[\alpha(u-a)+a,\alpha(u-b)+b]+[\alpha(v-c)+c,\alpha(v-d)+$

$=[\alpha(u-a+v-c)+a+c,\alpha(u-b+v-d)+b+d]$

$=[\alpha(u+v-(a+c))+a+c,\alpha(u+v-(b+d))+b+d$

$=[$

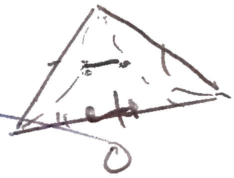
Para $\alpha=0$: $[A+B]^0=\mathbb{R}([A]^0+[B]^0)$

$=[C]^0$

Portanto, $[A+B]^\alpha=[C]^\alpha\ \forall\alpha\in[0,1]$

$\Rightarrow A+B=C=(a+c;u+v;b+d)$

(c) Seja $A=(a;u;b)=(-1;0;1)$



$\Rightarrow [A]^{\frac12}=(-\frac12;0;\frac12)$

$[A\cdot A]^{\frac12}=[A]^{\frac12}\cdot[A]^{\frac12}=[-\frac12,\frac12]\cdot[-\frac12,\frac12]$

$=[-\frac14\wedge\frac14,-\frac14\vee\frac14]=[-\frac14,\frac14]$ e $[A\cdot A]^1=[0,0]\cdot[$

mas supp$(A\cdot A)=(-1,1)\cdot(-1,1)=(-1,1)$ $=\{0\}$

S... $A\cdot A$ tem ... ~~Portanto~~ $A^0\cdot A=(-1,0,1)$ mas $(-1,0,1)\neq$

4. Seja $X = \{x_1, x_2 \dots, x_n\}$, onde $n \in \mathbb{N} \setminus \{0\}$, e seja $|F| = \sum_{i=1}^{n} F(x_i)\ \forall F \in \mathcal{F}(X)$. Considere a relação fuzzy $\mathcal{S} \in \mathcal{F}(\mathcal{F}(X) \times \mathcal{F}(X))$ definida por:

$$\mathcal{S}(A,B) = \begin{cases} \frac{|A \cap B|}{|A \cup B|} & \text{se } A \cup B \neq \emptyset, \\ 1 & \text{caso contrário}. \end{cases} \quad \forall A, B \in \mathcal{F}(X).$$

Quais das condições seguintes são satisfeitas por $\mathcal{S}$?

(a) Transitividade; [0,5 pt]

(b) Se $A \subseteq B \subseteq C$ então $\mathcal{S}(A,C) \leq \mathcal{S}(A,B) \wedge \mathcal{S}(B,C)$. [1 pt]

5. Seja $X = Y = \mathbb{R}$, $f(x) = \sin(x)\ \forall x \in X$ e $A = (0; \frac{\pi}{2}; \pi)$.

(a) Determine $f(A)$ e interprete o resultado geometricamente (como feito na aula). [1,5 pt]

(b) Determine $f^{-1}(B)$, onde $B = f(A)$, e interprete o resultado geometricamente. [1 pt]

**Boa sorte!**

2.(c) (Pós)

Seja $A = (-4; 0; 4)$ e $B = A \cdot A$

Daí, $[B]^{\frac{1}{2}} = [A \cdot A]^{\frac{1}{2}} = [A]^{\frac{1}{2}} \cdot [A]^{\frac{1}{2}} = [-2,2] \cdot [-2,2]$

$= [(-4) \wedge 4, (-4) \vee 4] = [-4, 4]$ (*)

$[A \cdot A]^{\frac{1}{4}} = [A]^{\frac{1}{4}} \cdot [A]^{\frac{1}{4}} = [-3,3] \cdot [-3,3] = [-9,9]$ (**)

e $[B]^1 = [A]^1 \cdot [A]^1 = [0,0] \cdot [0,0] = \{0\}$ (***)

Suponha que $B$ é um # fuzzy triangular, i.e., $B$ é da forma $(a; u; b)$ com $a \leq u \leq b$. (***) implica que $u = 0$.

(*) implica que $4 = \frac{1}{2}(0-b) + b = \frac{b}{2} \Rightarrow b = 8$

(**) " " $9 = \frac{1}{4}(0-b) + b = \frac{3b}{4} \Rightarrow b = 12$ ↯

Portanto, $B \notin \{$#'s fuzzy triangulares$\}$

3. (a) $S = T_1 + T_2 + T_3$, onde

$T_1 = (20;30;40)$, $T_2 = (16;20;24)$, $T_3 = (4;5;6)$

$\Rightarrow S = (\underbrace{40}_{a}; \underbrace{55}_{u}; \underbrace{70}_{b})$ em minutos

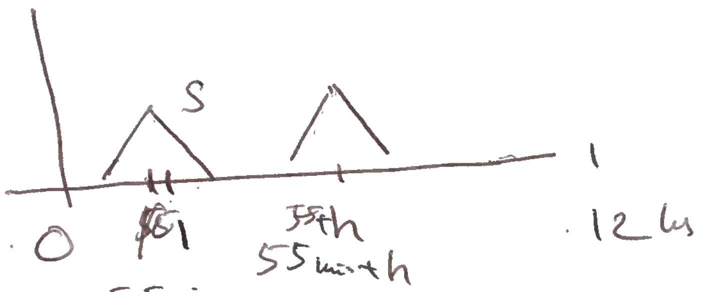
(b) $C_h = h + S = (\underbrace{\cancel{4}40+h}_{a}; \underbrace{55+h}_{u}, \underbrace{70+h}_{a}) \in \mathcal{F}$

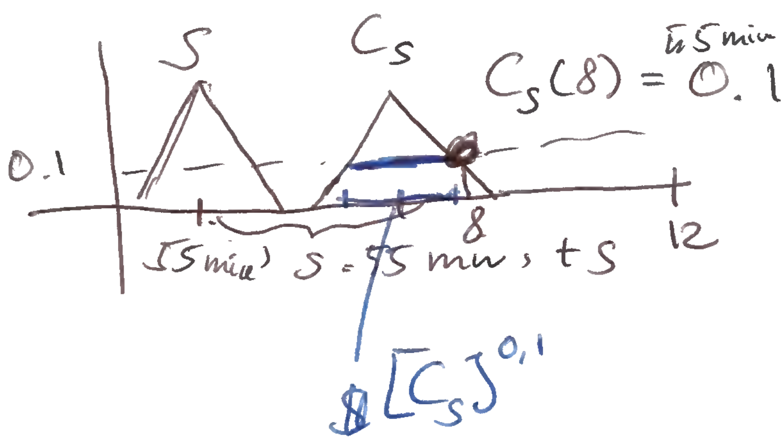
(Note que aqui $C_h, h, S$ são restritos a $X$.)

$$[C_h]^{0.1} = \{x \in X \mid C_h(x) \geq 0.1\}$$

$$\stackrel{2.}{=} [0.1 \cdot 15 + 40 + h, 0.1(-15) + 70 + h]$$

$$= [\overset{41}{\cancel{38}},5 + h \,, 68,5 + h]$$ (em minutos)



Seja $s$ tal que ~~$[C_s]^0$~~ $C_s(8) = 0.1$

$\Rightarrow$ 68,5 mins $+ s = 8$ hs

$\Rightarrow s = 7$ hs $- 8,5$ mins $=$ ~~7 6:6~~

$= \underline{6:51:30}$ hs

Note que para $h < s$

68,5 min's $+ h < 8$ hs

$\Rightarrow 8 \notin [C_h]^{0.1}$

Para $h > s$:

68,5 min's $+ h > 8$ hs $\Rightarrow C_h(8) > 0.1$

$\therefore$ Este $s = \bigvee \{ h \in X \mid C_h(X) \leq 0.1 \}$

(c) $[C_h]^{0+} = (40 + h, 70 + h)$

$\Rightarrow t = \bigvee \{ h \in X \mid [C_h]^{0+} \cap [8, 12] = \emptyset \}$

$= \bigvee \{ h \in X \mid 70 + h < 8 \text{ hs} \}$

$= 6:50$ hs

(d) $s = 6:51:30$hs é tal que a possibilidade que o horario de chegada do M. seja as 8hs o de 12... é $\leq 0.1$ se M. seja as 6:51:30hs

4. (Pos) $X = \{x_1, \dots, x_n\}, n \in \mathbb{N} = \{1, 2, 3, \dots\}$

a) Vamos mostrar que $\exists A, B, C$ tal que

$$S(A,B) \wedge S(B,C) \not\leq S(A,C)$$

Seja $A = \frac{1}{4} 1_{\mathcal{F}(X)}$ e $B = \frac{1}{2} 1_{\mathcal{F}(X)}$ e $C = 1_{\mathcal{F}(X)}$,

quer dizer $C(x) = 1 \quad \forall x \in X$

Tem-se $S(A,B) = \dfrac{|A|}{|B|} = \dfrac{\sum_{i=1}^{n} A(x_i)}{\sum_{i=1}^{n} B(x_i)} = \dfrac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$,

$$S(B,C) = \frac{|B|}{|C|} = \frac{\frac{1}{2} \cdot n}{n} = \frac{1}{2},$$

$$S(A,C) = \frac{|A|}{|C|} = \frac{\frac{1}{4} \cdot n}{n} = \frac{1}{4}$$

Portanto

$$S(A,B) \wedge S(B,C) = \frac{1}{2} > \frac{1}{4} = S(A,C)$$

4(b) Pós

Vamos mostrar que $A \subseteq B \subseteq C \Rightarrow$ ~~S(A,C) ≤~~ implica

que $S(A,C) \leq S(A,B) \wedge S(B,C)$

Caso 1: $C = \emptyset$.

Neste caso, $A = B = \emptyset \Rightarrow S(A,B) \wedge S(B,C) = 1 = S(A,C)$

Caso 2: $B = \emptyset$ e $C \neq 0$:

Tem-se $S(A,B) \wedge S(B,C) = 1 \wedge S(B,C)$

$$= S(B,C) = \frac{|B|}{|C|} = \frac{\sum_{i=1}^{n} B(x_i)}{|C|} \geq \frac{\sum_{i=1}^{n} A(x_i)}{|C|} = S(A,C)$$

Caso 3: $B, C \neq \emptyset$:

Temos $S(A,B) = \frac{|A|}{|B|} = \frac{|A|}{\sum_{i=1}^{n} B(x_i)} \geq \frac{|A|}{\sum_{i=1}^{n} C(x_i)}$

porque $B(x_i) \leq C(x_i) \quad \forall i \in \{1, \dots, n\}$

$\Rightarrow S(A,B) \geq S(A,C) \quad (*)$

Além disso,

$$S(B,C) = \frac{|B|}{|C|} = \frac{\sum_{i=1}^{n} B(x_i)}{|C|} \geq \frac{\sum A(x_i)}{|C|}$$

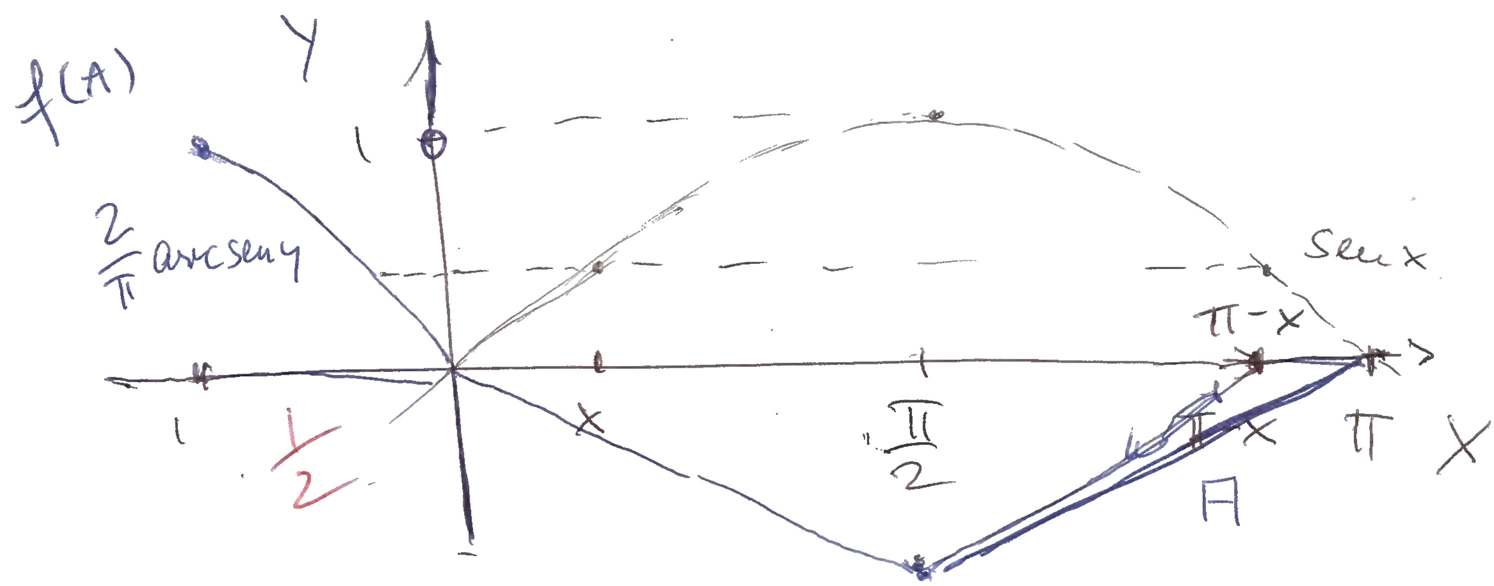
porque $B(x_i) \geq A(x_i) \quad \forall i = \{1, \dots, n\}$

$\Rightarrow S(B,C) \geq S(A,C) \quad (**)$

$(*) \& (**)$: $S(A,B) \wedge S(B,C) \geq S(A,C)$

q.e.d.

5. (a) (Pós), 4. (Geral)




Para $y \notin [-1,1]$ temos

$$f(A)(y) = \bigvee_{x \mid f(x)=y} A(x) = \bigvee_{x \mid \operatorname{sen} x = y} A(x) = \bigvee \emptyset = 0 \qquad \frac{1}{4}$$

Para $y \in [-1,0)$: $f(A)(y) = \bigvee_{x \mid \operatorname{sen} x = y} A(x) = 0 \qquad \frac{1}{4}$

Para $y \in [0,1]$:

$$f(A)(y) = \bigvee_{x \mid \operatorname{sen} x = y} A(x) = \bigvee_{x \in [0,\frac{\pi}{2}] \mid \operatorname{sen} x = y} A(x) \vee \bigvee_{x \in [\frac{\pi}{2},\pi] \mid \operatorname{sen} x = y} A(x) \qquad \frac{1}{4}$$

$$= \bigvee_{x \in [0,\frac{\pi}{2}] \mid \operatorname{sen} x = y} A(x) \vee \bigvee_{x \in [0,\frac{\pi}{2}] \mid \operatorname{sen}(\pi - x) = y} A(\pi - x) \qquad \frac{1}{4}$$

$$= \bigvee_{x \in [0,\frac{\pi}{2}] \mid \operatorname{sen} x = y} A(x) \vee A(\pi - x) = \bigvee_{x \in [0,\frac{\pi}{2}] \mid \operatorname{sen} x = y} A(x)$$

$$= A(\operatorname{arcsen} y) = \frac{2}{\pi} \operatorname{arcsen} y. \qquad \frac{1}{4}$$

Com a saida as $t = 6{:}50$ hs da cama, não há possibilidade (segundo o modelo empregado) que o horário de chegada do Mathias na estação de trem seja as 8hs ou depois.

5(b) (Pós)

Seja $B = f(A)$.

$$f^{-1}(B)(x) = B(f(x)) = B(\operatorname{sen}(x))$$

Portanto, para $x \in [0, \frac{\pi}{2}] + 2\pi k$, onde $k \in \mathbb{Z}$ temos

$$f^{-1}(B)(x) = \frac{2}{\pi} \operatorname{arcsen}(\operatorname{sen} x) =$$

$$= \frac{2}{\pi}(x - 2\pi k).$$

$\frac{1}{4}$

Para $x \in (\frac{\pi}{2}, \pi] + 2k\pi$, onde $k \in \mathbb{Z}$:

$$f^{-1}(B)(x) = \frac{2}{\pi} \operatorname{arcsen}(\operatorname{sen} x) = \frac{2}{\pi} \operatorname{arcsen}(\underbrace{\operatorname{sen} x -}_{\in (\frac{\pi}{2}, \pi]}$$

$$= \frac{2}{\pi} \operatorname{arcsen}(\operatorname{sen}(\pi - x))$$

$$= \frac{2}{\pi}\left[\operatorname{arcsen}\left(\operatorname{sen}\left(\pi - (\underbrace{x - 2\pi k}_{\in (\frac{\pi}{2}, \pi]})\right)\right)\right]$$

$$= \frac{2}{\pi}\underbrace{[\pi - (x - 2\pi k)]}_{\in [0, \frac{\pi}{2})}$$

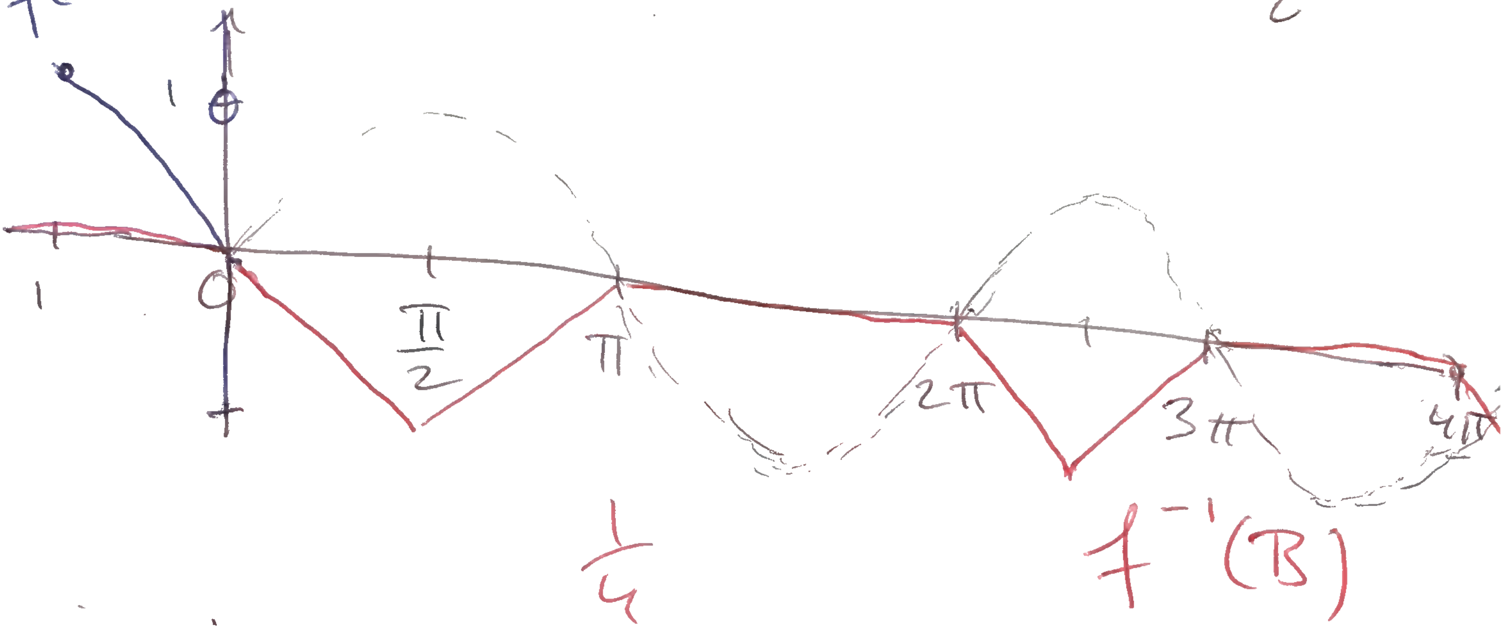
$\frac{1}{4}$

Para $x \notin [0, \pi] + 2\pi k$ para todos $k \in \mathbb{Z}$, quer dizer para $x \in (\pi, 2\pi) + 2\pi k$ para algum $k \in \mathbb{Z}$ temos

$$\operatorname{sen} x \in [-1, 0] \Rightarrow f^{-1}(B)(x) = B(\operatorname{sen}(x)) = 0$$

$\frac{1}{4}$

$f(A)=B$




$f^{-1}(B)$

5. Seja $X = \{1, 2 \ldots, 5\}$. Considere a relação fuzzy $\mathcal{R} \in \mathcal{F}(X \times X)$ definida por:

$$\mathcal{R}(x,y) = \frac{1}{xy} \quad \forall x, y \in X.$$

(a) Calcule $\mathcal{R} \circ \mathcal{R}$. [1 pt]

(b) Quais das condições seguintes são satisfeitas por $\mathcal{R}$?

i. Reflexividade; [0,5 pt]

ii. Simetria; [0,5 pt]

iii. Anti-Simetria; [0,5 pt]

iv. Transitividade. [0,5 pt]

5. Graduação. 4

**Boa sorte!**

(a) $R \circ R = R \overset{\wedge}{\vee} R =$

$$= \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{6} & \frac{1}{8} & \frac{1}{10} \\ \frac{1}{3} & \frac{1}{6} & \frac{1}{9} & \frac{1}{12} & \frac{1}{15} \\ \frac{1}{4} & \frac{1}{8} & \frac{1}{12} & \frac{1}{16} & \frac{1}{20} \\ \frac{1}{5} & \frac{1}{10} & \frac{1}{15} & \frac{1}{20} & \frac{1}{25} \end{pmatrix} \overset{\wedge}{\vee} \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{6} & \frac{1}{8} & \frac{1}{10} \\ \frac{1}{3} & \frac{1}{6} & \frac{1}{9} & \frac{1}{12} & \frac{1}{15} \\ \frac{1}{4} & \frac{1}{8} & \frac{1}{12} & \frac{1}{16} & \frac{1}{20} \\ \frac{1}{5} & \frac{1}{10} & \frac{1}{15} & \frac{1}{20} & \frac{1}{25} \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{pmatrix}$$

(b) (i) Reflexivo? NÃO! $R \not\geq I_{5\times5}$ porque $R(2,2) = \frac{1}{4} \neq$

(ii) Simétrico? SIM! $R = R^T$

Note que $R(x,y) = \frac{1}{xy} = \frac{1}{yx} = R(y,x) \quad \forall x,y \in X$

(iii) Anti-Simetrico? $R(x,y) \wedge R(y,x) > 0 \Rightarrow x = y$?

$R \wedge R^T = R \wedge R^{-1} \leq I_{5\times5}$? NÂO!

$(R \wedge R^T)(2,1) = \frac{1}{2} \neq 0$. Alternativamente, $R(1,2) \wedge R(2,1)$

$R(1,2) \wedge R(2,1) = \frac{1}{2} > 0$

(iv) Transitivo? $R(x,y) \wedge R(y,z) \leq R(x,z) \quad \forall x,y,z$

NÃO! $R(2,1) \wedge R(1,2) = \frac{1}{2} \not\leq \frac{1}{4} = R(2$

Note que $R \circ R \not\leq R$ porque $(R \circ R)(2,2) = \frac{1}{2} \not\leq \frac{1}{4} = R(2$