1. (a) Mostre que $\mathcal{R}$ e $\mathcal{L}$ são reversas

(i) $\mathcal{R} \cap \mathcal{L} = \emptyset$:

$\mathcal{R} = \{\alpha \overrightarrow{AB} + \overrightarrow{OA}\}_{\alpha \in \mathbb{R}}$, $\mathcal{L} = \{\beta \overrightarrow{CD} + \overrightarrow{OC} \mid \beta \in \mathbb{R}\}$

Suponha que $\mathcal{R} \cap \mathcal{L} \neq \emptyset$

$\Leftrightarrow \exists\, \alpha, \beta \in \mathbb{R}$ tais que
$$\alpha \overrightarrow{AB} + \overrightarrow{OA} = \beta \overrightarrow{CD} + \overrightarrow{OC}$$

$\Leftrightarrow \exists\, \alpha, \beta \in \mathbb{R}$ tais que
$$\alpha \overrightarrow{AB} = \beta \overrightarrow{CD} + \overrightarrow{AC}$$

$\Leftrightarrow \exists\, \alpha, \beta \in \mathbb{R}$ tais que
$$\alpha \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} = \beta \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

$\Leftrightarrow \exists\, \alpha, \beta \in \mathbb{R}$ t.q.
$$\alpha = 0, \quad \beta = 1 \text{ e } \beta = 2 \quad \lightning \quad \therefore\ \mathcal{R} \cap \mathcal{L} = \emptyset$$
(então)

(ii) $\mathcal{R} \nparallel \mathcal{L}$:

Suponha que $\mathcal{R} \parallel \mathcal{L}$

$\Leftrightarrow \exists\, \alpha \neq 0$ tal que $\overrightarrow{AB} = \alpha \overrightarrow{CD}$

$\Leftrightarrow \exists\, \alpha \neq 0$ tal que $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} = \alpha \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix}$

$\lightning$

$\therefore\ \mathcal{R} \nparallel \mathcal{L}$
(então)

(i) & (ii): $\mathcal{R}$ e $\mathcal{L}$ são reversas

1(b) Seja $V_R = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ e $W = \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix}$

$V$ é vetor diretor de $R$ e $W$ é vetor diretor de $L$

Seja $N = V \times W$.

$$N = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} = \det \begin{pmatrix} E_1 & E_2 & E_3 \\ -1 & 1 & 0 \\ 0 & -1 & -1 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$$

$N$ é vetor normal de $P$ e de $Q$, então

$$P = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \mid -x - y + z + d = 0, \text{ onde } d \text{ é t.q. } -1 - 0 + 0 + d = 0 \right\}$$

$$= \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \mid -x - y + z + 1 = \right\}$$

$$Q = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \mid -x - y + z + d = 0, \text{ onde } d \text{ é t.q. } -1 - 0 + 1 + d = 0 \right\}$$

$$= \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \mid -x - y + z = 0 \right\}$$

(c) Temos $A \in P$ e $C \in Q$. Seja $P_0 = A$ e $P_1 = C$.

$$\overrightarrow{P_0P_1} = \overrightarrow{AC} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \Rightarrow \operatorname{proj}_{V \times W} (\overrightarrow{P_0P_1})$$

$$= \operatorname{proj}_N (\overrightarrow{AC}) = \langle U, \overrightarrow{AC} \rangle U, \text{ onde } U = \frac{N}{\|N\|}$$

$$= \frac{\langle N, \overrightarrow{AC} \rangle \cdot N}{\|N\|^2} = \frac{\left[ (-1, -1, 1) \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \right] \cdot \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}}{3}$$

$$= \frac{1}{3} \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \Rightarrow \| \operatorname{proj}_{V \times W} (\overrightarrow{AC}) \| = \sqrt{\left(\frac{1}{3}\right)^2 [(-1)^2 + (-1)^2 + 1^2]}$$

$$= \frac{1}{3} \cdot \sqrt{3} \left( = \frac{1}{\sqrt{3}} \right)$$

1(d) Encontre $P \in \mathcal{R}$ e $Q \in \mathcal{L}$ t.q. $\vec{PQ} \perp \mathcal{R}, \mathcal{L}$

$\vec{PQ} \perp \mathcal{R}, \mathcal{L} \iff \vec{PQ} \perp V, W$

$\iff \langle \vec{PQ}, V \rangle = 0 = \langle \vec{PQ}, W \rangle$

Note que $\vec{PQ} = (\beta \vec{CD} + \vec{OC}) - (\alpha \vec{AB} + \vec{OA})$
para alguns $\alpha, \beta \in \mathbb{R}$

Então $\vec{PQ} = \begin{pmatrix} \alpha \\ -\beta - \alpha + 1 \\ -\beta + 2 \end{pmatrix}$ para alguns $\alpha, \beta \in \mathbb{R}$

$\langle \vec{PQ}, V \rangle = 0$

$\iff (\alpha, -\alpha - \beta + 1, -\beta + 2) \cdot \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} = 0$

$\iff -2\alpha - \beta + 1 = 0 \iff 2\alpha + \beta = 1$

$\langle \vec{PQ}, W \rangle = 0$

$\iff (\alpha, -\alpha - \beta + 1, -\beta + 2) \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} = 0$

$\iff \alpha + \beta - 1 + \beta - 2 = 0 \iff \alpha + 2\beta = 3$

Temos

$$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

Resolvendo $\left( \begin{array}{cc|c} 2 & 1 & 1 \\ 1 & 2 & 3 \end{array} \right) \sim \left( \begin{array}{cc|c} 2 & 1 & 1 \\ -3 & 0 & 1 \end{array} \right)$

$\sim \left( \begin{array}{cc|c} 2 & 1 & 1 \\ 1 & 0 & -\frac{1}{3} \end{array} \right) \sim \left( \begin{array}{cc|c} 0 & 1 & \frac{5}{3} \\ 1 & 0 & -\frac{1}{3} \end{array} \right)$

$\therefore \alpha = -\frac{1}{3}, \quad \beta = \frac{5}{3}$

$\Rightarrow P = \alpha \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = -\frac{1}{3} \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{4}{3} \\ -\frac{1}{3} \\ 0 \end{pmatrix}$,

$Q = \beta \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \frac{5}{3} \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ -\frac{2}{3} \\ \frac{1}{3} \end{pmatrix}$.

2.(a) $\langle u, v\rangle = 0$ e $u \times v = 0 \in \mathbb{R}^3$

$\iff \langle u, v\rangle = 0$ e $\|u \times v\| = 0$

$\iff \|u\| \cdot \|v\| \cdot \cos\measuredangle(u,v) = 0$ e

$\|u\| \cdot \|v\| \cdot \operatorname{sen}\measuredangle(u,v) = 0$

$\iff u = 0$ ou $v = 0$ ou $\underbrace{\cos\measuredangle(u,v) = 0 = \operatorname{sen}\measuredangle(u,v)}_{\text{somente acontece se } u = 0 \text{ ou } v = 0}$

$\iff u = 0$ ou $v = 0$

VERDADEIRO

(b) $u \times (v \times w) = (u \times v) \times w \quad \forall u, v, w \in \mathbb{R}^3$?

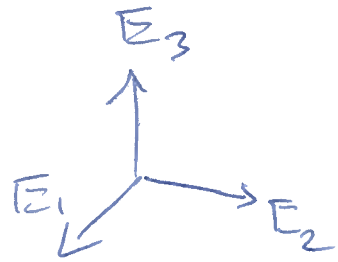
Seja $u = v = E_1$ e $w = E_2$

$u \times (v \times w) = E_1 \times (E_1 \times E_2)$

$= E_1 \times E_3 = -E_2 \neq 0$

$= 0 \times E_2 = (E_1 \times E_1) \times E_2$

$= (u \times v) \times w$




Afirmação FALSA

(c) Se $P$ é um plano então $\exists \tilde{v}, \tilde{w}, u \in \mathbb{R}^3$ t.q. com $\tilde{v}, \tilde{w} \neq 0$

$P = \{\tilde{\alpha}\tilde{v} + \tilde{\beta}\tilde{w} + u \mid \tilde{\alpha}, \tilde{\beta} \in \mathbb{R}\}$

$= \{\tilde{\alpha}\|\tilde{v}\| \frac{\tilde{v}}{\|\tilde{v}\|} + \tilde{\beta} \cdot \|\tilde{w}\| \cdot \frac{\tilde{w}}{\|\tilde{w}\|} + u \mid \tilde{\alpha}, \tilde{\beta} \in \mathbb{R}\}$

$= \{\alpha \frac{\tilde{v}}{\|\tilde{v}\|} + \beta \frac{\tilde{w}}{\|\tilde{w}\|} + u \mid \alpha, \beta \in \mathbb{R}\}$

Então $\exists v, w$ unitários e $u \in \mathbb{R}^3$ t.q.

$P = \{\alpha v + \beta w + u \mid \alpha, \beta \in \mathbb{R}\}$

VERDADEIRO

2.(d) $R: \dfrac{x-x_0}{v_1} = \dfrac{y-y_0}{v_2} = \dfrac{z-z_0}{v_3}$

$\Leftrightarrow R = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \middle| \exists\, \alpha \in \mathbb{R} \text{ t.q. } \alpha = \dfrac{x-x_0}{v_1} = \dfrac{y-y_0}{v_2} = \dfrac{z-z_0}{v_3} \right\}$

$\Leftrightarrow R = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \middle| x - x_0 = \alpha v_1, y - y_0 = \alpha v_2, z - z_0 = \alpha v_3 \text{ para algum } \alpha \in \mathbb{R} \right\}$

$\Leftrightarrow R = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \middle| x = \alpha v_1 + x_0, y = \alpha v_2 + y_0, z = \alpha v_3 + z_0 \text{ para algum } \alpha \in \mathbb{R} \right\}$

$\Leftrightarrow R = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \middle| \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \alpha \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} + \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} \text{ para algum } \alpha \in \mathbb{R} \right\}$

$\Leftrightarrow R = \{ \alpha V + U \mid \alpha \in \mathbb{R} \}$

onde $V = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ e $U = \overrightarrow{OP_0}$ VERDADEIRO

(e) Todo plano $P$ é da forma

$P = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \middle| ax + by + cz + d = 0 \right\}$

para algum vetor normal $N = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \neq 0 \in \mathbb{R}^3$ e $d \in \mathbb{R}$

Seja $N \neq 0$

$0 \neq N$ é vetor normal de $P \Leftrightarrow$

$N \perp \overrightarrow{PQ} \quad \forall P, Q \in P$

$\Leftrightarrow \langle N, \overrightarrow{PQ} \rangle = 0$ — " —

$\Leftrightarrow \alpha \langle N, \overrightarrow{PQ} \rangle = 0$ — " — $\forall \alpha \neq 0$

$\Leftrightarrow \langle \alpha N, \overrightarrow{PQ} \rangle = 0$ " $\forall \alpha \neq 0$

$\Leftrightarrow \alpha N$ é vetor normal de $P$ $\forall \alpha \neq 0$

VERDADEIRO

(f) $proj_{V\times W}(U) = \langle U, V\times W\rangle \cdot \frac{V\times W}{\|V\times W\|^2}$

$\Rightarrow \|proj_{V\times W}(U)\| = \frac{|\langle U, V\times W\rangle|}{\|V\times W\|}$

$\Rightarrow \|proj_{V\times W}(U)\| \cdot \|V\times W\| = |\langle U, V\times W\rangle|$

= volume do paralelepípedo determinado por U, V e W

3.(a) $\mathcal{P}: 5x - y + z + 2 = 0$.

Um vetor normal de $Q$ é dado por

$$V\times W = \det\begin{pmatrix} E_1 & E_2 & E_3 \\ 2 & 0 & 3 \\ 1 & 3 & -1 \end{pmatrix} = \begin{pmatrix} -9 \\ 5 \\ 6 \end{pmatrix}$$

$$(0,0,3)^T \in Q \Rightarrow Q = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \middle| -9x + 5y + 6z - 18 = 0 \right\}$$

Seja $R = \mathcal{P} \cap Q$ (Obs.: $\mathcal{P} \not\parallel Q$)

Um vetor diretor $V_R$ de $R$ é dado por

$$V_R = \begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} = \det\begin{pmatrix} E_1 & E_2 & E_3 \\ 5 & -1 & 1 \\ -9 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -11 \\ -39 \\ 16 \end{pmatrix}$$

Vamos encontrar $P = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathcal{P} \cap Q$ com $z = 0$

$$\left(\begin{array}{cc|c} 5 & -1 & -2 \\ -9 & 5 & 18 \end{array}\right) \rightsquigarrow \left(\begin{array}{cc|c} 5 & -1 & -2 \\ 16 & 0 & 8 \end{array}\right) \rightsquigarrow \left(\begin{array}{cc|c} 5 & -1 & -2 \\ 1 & 0 & \frac{1}{2} \end{array}\right)$$

$$\rightsquigarrow \left(\begin{array}{cc|c} 0 & -1 & -\frac{9}{2} \\ 1 & 0 & \frac{1}{2} \end{array}\right) \rightsquigarrow \left(\begin{array}{cc|c} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{9}{2} \end{array}\right)$$

$x = \frac{1}{2}, y = \frac{9}{2}$

$$R: \begin{cases} x = -11\alpha + \frac{1}{2} \\ y = -39\alpha + \frac{9}{2} \\ z = 16\alpha \end{cases}$$

Nome do Emissor: Peter Sussner

Parecer registrado eletronicamente através da internet junto ao CNPq, mediante uso de senha pessoal em 04/06/2015 17:37

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3.(b): Volume do paralelepípedo determinado por N, V e U

$$= |\langle N, V \times U \rangle|$$

$$= \left| \det \begin{pmatrix} 5 & -1 & 1 \\ 2 & 0 & 3 \\ 0 & -4 & -5 \end{pmatrix} \right|$$

$$= |5 \cdot 12 + (-10) - 8|$$

$$= |60 - 10 - 8|$$

$$= 42$$