1. $A = \begin{pmatrix} 1 & 2 & 0 \\ -1 & 0 & 4 \\ 3 & 2 & 2 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 10 \\ -10 \end{pmatrix}$

(a) $\tilde{A} = \begin{pmatrix} 3 & 2 & 2 \\ -1 & 0 & 4 \\ 1 & 2 & 0 \end{pmatrix}, \quad p = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

$$R = \begin{pmatrix} 3 & 2 & 2 \\ -\frac{1}{3} & \frac{2}{3} & \frac{14}{3} \\ \frac{1}{3} & \frac{4}{3} & -\frac{2}{3} \end{pmatrix} \rightsquigarrow \begin{pmatrix} 3 & 2 & 2 \\ \frac{1}{3} & \frac{4}{3} & -\frac{2}{3} \\ -\frac{1}{3} & \frac{2}{3} & \frac{14}{3} \end{pmatrix}, \quad p = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$$

$$\rightsquigarrow \begin{pmatrix} 3 & 2 & 2 \\ \frac{1}{3} & \frac{4}{3} & -\frac{2}{3} \\ -\frac{1}{3} & \frac{1}{2} & 5 \end{pmatrix}$$

$$\therefore L = \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{3} & 1 & 0 \\ -\frac{1}{3} & \frac{1}{2} & 1 \end{pmatrix}, \quad U = \begin{pmatrix} 3 & 2 & 2 \\ 0 & \frac{4}{3} & -\frac{2}{3} \\ 0 & 0 & 5 \end{pmatrix}$$

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

(b) $Ax = b \Leftrightarrow \underbrace{PA}_{LU} x = Pb$

(i) $Ly = Pb$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -10 \\ \frac{1}{3} & 1 & 0 & 0 \\ -\frac{1}{3} & \frac{1}{2} & 1 & 10 \end{array}\right)$$

$y_1 = -10$

$-\frac{10}{3} + y_2 = 0$

$\Rightarrow y_2 = \frac{10}{3}$

$\frac{10}{3} + \frac{5}{3} + y_3 = 10$

$\Rightarrow y_3 = 10 - 5 = 5$

$$\therefore y = \begin{pmatrix} -10 \\ \frac{10}{3} \\ 5 \end{pmatrix}$$

(ii) $Ux = y$

$$\left(\begin{array}{ccc|c} 3 & 2 & 2 & -10 \\ 0 & \frac{4}{3} & -\frac{2}{3} & \frac{10}{3} \\ 0 & 0 & 5 & 5 \end{array}\right)$$

$3x_1 + 6 + 2 = -10 \Rightarrow x_1 =$

$\frac{4}{3} x_2 = \frac{10}{3} + \frac{2}{3} = 4 \Rightarrow x_2 =$

$x_3 = 1$

$$\therefore x = \begin{pmatrix} -6 \\ 3 \\ 1 \end{pmatrix}$$

2.(a)

$$A = \begin{pmatrix} -128.01 & 64 & & & & & & \\ 64 & -128.01 & 64 & & & & 0 & \\ & 64 & -128.01 & 64 & & & & \\ & & 64 & -128.01 & 64 & & & \\ & & & 64 & -128.01 & 64 & & \\ & & & & 64 & -128.01 & 64 & \\ & 0 & & & & 64 & -128.01 & 64 \\ & & & & & & 64 & -128.0 \end{pmatrix}$$

$b = (-2560\ -0.2\ -0.2\ -0.2\ -0.2\ -0.2\ -12800$

Note que $\frac{64}{128.01} \approx 0.49996$ e usando 4 dígitos de e arredondamento obtemos 0.

(b) $|C| = C = \begin{pmatrix} 0 & 0.5 & 0 & 0.000 \\ 0.5 & 0 & 0.5 & 0.000 \\ & \vdots & & \vdots \end{pmatrix}$ $\alpha_2 = 1 \not< 1$ Portanto, o critério das linhas não é satisfeito

(c)

$$|C| = \begin{pmatrix} 0 & 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0.5\cdot\gamma_1 & 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0\cdot\gamma_1 & 0.5\cdot\gamma_2 & 0 & 0.5 & 0 & 0 & 0 \\ 0\cdot\gamma_1 & 0\cdot\gamma_2 & 0.5\gamma_3 & 0 & 0.5 & 0 & 0 \\ 0\cdot\gamma_1 & 0\cdot\gamma_2 & 0\cdot\gamma_3 & 0.5\gamma_4 & 0 & 0.5 & 0 \\ 0\cdot\gamma_1 & 0\gamma_2 & 0\cdot\gamma_3 & 0\gamma_4 & 0.5\gamma_5 & 0 & 0.5 \\ 0\cdot\gamma_1 & 0\gamma_2 & 0\cdot\gamma_3 & 0\gamma_4 & 0\gamma_5 & 0.5\gamma_6 & 0 \end{pmatrix}$$

$\gamma_1 = 0.5 < 1$

$\gamma_2 = 0.75 < 1$

$\gamma_3 = 0.875 < 1$

$\gamma_4 = 0.9375 < 1$

$\gamma_5 = 0.9688 < 1$

$\gamma_6 = 0.9844 < 1$

$\gamma_7 = 0.4922 < 1$

VIRE PÁGINA!

... satisfeito ~~posso ver~~ justo! que ... é a única vez que ...

(c) $\therefore$ O critério de Sassenfeld é satisfeito

$\Rightarrow$ O método de Gauss-Seidel converge para a solução de $At=b$ para qualquer chute inicial $t^{(0)}$.

$$t^{(0)} = (60, 80, 100, 120, 140, 160, 180)^T$$

(d) $t_1^{(1)} = (0\ 0.5\ 0\ 0\ 0\ 0\ 0) \cdot t^{(0)} + \beta_1$, onde

$$\beta_1 = -\frac{1}{128{,}01} \cdot b_1 = 19{,}9984$$

Então $t_1^{(1)} = 0.5 \cdot 80 + 19.9984 = 59{,}9998$

$\rightsquigarrow$ vetor intermediário $(59{,}9998, 80, 100, \ldots, 180)$

$$t_2^{(1)} = (0.5\ 0\ 0.5\ 0\ 0\ 0\ 0) \begin{pmatrix} 59.9998 \\ 80 \\ 100 \\ \vdots \\ 180 \end{pmatrix} + \beta_2$$

$$= 0.5 \cdot 59{,}9998 + 50 + \frac{0{,}2}{128{,}01} = 79{,}9999 + 0{,}00$$

$$= 80{,}0015$$

$\rightsquigarrow$ vetor intermediário $(59.9998, 80.0015, 100, \ldots$

$$t_3^{(1)} = (0\ 0.5\ 0\ 0.5\ 0\ 0\ 0) \begin{pmatrix} 59{,}9998 \\ 80.0015 \\ 100 \\ 120 \\ \vdots \\ 180 \end{pmatrix} + \beta_3 =$$

$$= 40.0008 + 60 + 0.0016 = 100{,}0024$$

$$t_3^{(1)} = 100{,}0$$

5.(a) $f(x) = x^{4/3} \Rightarrow f'(x) = \frac{4}{3} x^{\frac{1}{3}}$

$x_{k+1} = x_k - \frac{f(x_k)}{f'(x)} = x_k - \frac{x_k \cdot \cancel{x_k^{1/3}}}{\frac{4}{3} \cancel{x_k^{1/3}}} = x_k - \frac{3}{4} x_k$

$= \frac{1}{4} x$

Se x_{k+}

k	x_k	$f(x_k)$	$\lvert x_k - x_{k-1} \rvert$
0	1	1	—
1	$\frac{1}{4} = 0.25$	0.1575	0.25
2	0.0625	0.0248	

Note que $\lvert f(x_2) \rvert = 0.0248 < \varepsilon$

Para

Resultado 0.0625

(b) Note que $\left(\varphi(x_k)\right)_{k\in\mathbb{N}\cup\{0\}}$

onde $\varphi(x_k) = \underbrace{x_k - \dfrac{f(x_k)}{f'(x_k)}}_{\frac{1}{4}x_k \text{ se } x_0 \neq 0} \to 0$

Note também que $f(0)=0$

Seja $e_k = \varphi(x_k) - \lim\limits_{k\to\infty} \varphi(x_k)$

$= \varphi(x_k) - 0 = \frac{1}{4}x_k$

Para $x_0 \neq 0$ temos $e_{k+1} = \frac{1}{4}e_k \quad \forall k = 0,1,\ldots$

$\Rightarrow \lim\limits_{k\to\infty} \dfrac{e_{k+1}}{e_k} = \dfrac{1}{4} > 0$

$\therefore$ A ordem de convergência de $\left(\varphi(x_k)\right)_{k=0,1,\ldots}$ é linear.

(c) Note que $f''(x) = \begin{cases} \frac{4}{9}x^{-\frac{2}{3}} & \text{se } x \neq 0 \\ \nexists & \text{se } x = 0 \end{cases}$

Para $x \nearrow 0$, $\frac{4}{9}x^{-\frac{2}{3}} = \frac{4}{9}\cdot\frac{1}{x^{\frac{2}{3}}} \to -\infty$

e para $x \searrow 0$, $\frac{4}{9}x^{-\frac{2}{3}} = \frac{4}{9}\cdot\frac{1}{x^{\frac{2}{3}}} \to +\infty$

$\nexists\ f''(0)$ e portanto as condições ~~da~~ ~~teo~~ não são satisfeitas apesar do fato que 0 é a única raiz de f.

4.(a) $g_1(X) = -2x_1 e^{-(x_1^2+x_2^2)} - 2x_1 + 1$

$g_2(X) = -2x_2 e^{-(x_1^2+x_2^2)} - 2x_2 + 2$

$$G(x) = \begin{pmatrix} g_1(x) \\ g_2(x) \end{pmatrix}$$

(b) $$J(x) = \begin{pmatrix} \frac{\partial g_1}{\partial x_1}(x) & \frac{\partial g_1}{\partial x_2}(x) \\ \frac{\partial g_2}{\partial x_1}(x) & \frac{\partial g_2}{\partial g x_2}(x) \end{pmatrix}$$

$$\frac{\partial}{\partial x_1} g_1(x) = -2e^{-(x_1^2+x_2^2)} + 4x_1^2 e^{-(x_1^2+x_2^2)} - 2$$
$$= 2(2x_1^2-1)e^{-(x_1^2+x_2^2)} - 2$$

$$\frac{\partial g_1}{\partial x_2}(x) = 4x_1x_2 e^{-(x_1^2+x_2^2)} = \frac{\partial g_2}{\partial x_1}(x)$$

$$\frac{\partial g_2}{\partial x_2}(x) = -2e^{-(x_1^2+x_2^2)} + 4x_2^2 e^{-(x_1+x_2)} - 2$$
$$= 2(2x_2^2-1)e^{-(x_1^2+x_2^2)} - 2$$

(c) $x^{(0)} = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix}$, $G(x^{(0)}) = \begin{pmatrix} 1 \\ -e^{-0.25}+1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0.221\ldots \end{pmatrix}$

$$\|G(x^{(0)})\|_\infty = 1 > 0.1 = \varepsilon$$

$$J(x^{(0)}) = \begin{pmatrix} +2(-1)e^{-0.25} - 2 & 0 \\ 0 & 2(0.5-1)e^{-0.25} - 2 \end{pmatrix} = \begin{pmatrix} -3.5576 & \\ 0 & -2.77\ldots \end{pmatrix}$$

$$J(x^{(0)}) \cdot s^{(0)} = -G(x^{(0)})$$

$$\left(\begin{array}{cc|c} -3.5576 & 0 & -1 \\ 0 & -2.7788 & -0.2212 \end{array}\right)$$

$$s^{(0)} = \begin{pmatrix} +0.2811 \\ +6.0796 \end{pmatrix}$$

$$x^{(1)} = x^{(0)} + s^{(0)} = \begin{pmatrix} 0.2811 \\ 0.5796 \end{pmatrix}$$

K	$x^{(k)}$	$G(x^{(k)})$	$\|G(x^{(k)})\|_\infty$	$\|s^{(k-1)}\|_\infty$	$s^{(\ldots}$
0	$\begin{pmatrix} 0 \\ 0.5 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0.2212 \end{pmatrix}$	$1 > \varepsilon$	$-$	$\begin{pmatrix} 0.28\ldots \\ 0.07\ldots \end{pmatrix}$
1	$\begin{pmatrix} 0.2811 \\ 0.5796 \end{pmatrix}$	$\begin{pmatrix} 0.0665 \\ 0.0753 \end{pmatrix}$	$0.0753 < \varepsilon$ Pare!		

$x^{(1)} \approx x^*$ onde x^* é a solução de $G(x) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ e também o máximo de f. Portanto $x^{(1)}$ ... de f ... o valor máximo ...