Cálculo Numérico (MS211) Prova 2 (26/11/2013)

GABARITO

1.

(a)

$$Y=\begin{pmatrix} r \\ f \end{pmatrix} \quad , \quad Y'=\begin{pmatrix} r' \\ f' \end{pmatrix}=\begin{pmatrix} ar-brf \\ crf-df \end{pmatrix}=\begin{pmatrix} 2r-1.2rf \\ 0.9rf-f \end{pmatrix} \quad , \quad Y(0)=\begin{pmatrix} r(0) \\ f(0) \end{pmatrix}=\begin{pmatrix} 0.8 \\ 0.4 \end{pmatrix} \quad .$$

(b)

Seja $Y=\begin{pmatrix} r \\ f \end{pmatrix}$, $h=0.1$, $Y'=\begin{pmatrix} r' \\ f' \end{pmatrix}$, $Y(0)=\begin{pmatrix} 0.8 \\ 0.4 \end{pmatrix}$

x_k	$Y_k=\begin{pmatrix} r_k \\ f_k \end{pmatrix}$	$Y'_k=\begin{pmatrix} r'_k \\ f'_k \end{pmatrix}=\begin{pmatrix} 2r_k-1.2r_k f_k \\ 0.9 r_k f_k - f_k \end{pmatrix}$	$\bar{Y}_{k+1}=\begin{pmatrix} \bar{r}_{k+1} \\ \bar{f}_{k+1} \end{pmatrix}=Y_k+Y'_k\cdot h$	$\bar{Y}'_{k+1}=\begin{pmatrix} 2\bar{r}_{k+1}-1.2\bar{r}_{k+1}\bar{f}_{k+1} \\ 0.9\bar{r}_{k+1}\bar{f}_{k+1}-\bar{f}_{k+1} \end{pmatrix}$	$\Delta Y_k \approx \frac{Y'_k+\bar{Y}_{k+1}}{2}\cdot h$
0	$\begin{pmatrix} 0.8 \\ 0.4 \end{pmatrix}$	$\begin{pmatrix} 1.2160 \\ -0.1120 \end{pmatrix}$	$\begin{pmatrix} 0.9216 \\ 0.3888 \end{pmatrix}$	$\begin{pmatrix} 1.4132 \\ -0.0663 \end{pmatrix}$	$\begin{pmatrix} 0.1315 \\ -0.0089 \end{pmatrix}$
0.1	$\begin{pmatrix} 0.9315 \\ 0.3911 \end{pmatrix}$	$\begin{pmatrix} 1.4258 \\ -0.0632 \end{pmatrix}$	$\begin{pmatrix} 1.0740 \\ 0.3848 \end{pmatrix}$	$\begin{pmatrix} 1.6522 \\ -0.0128 \end{pmatrix}$	$\begin{pmatrix} 0.1539 \\ -0.0038 \end{pmatrix}$
0.2	$\begin{pmatrix} 1.0854 \\ 0.3873 \end{pmatrix}$				

$Y(0.2)=\begin{pmatrix} r(0.2) \\ f(0.2) \end{pmatrix}\approx\begin{pmatrix} 1.0854 \\ 0.3873 \end{pmatrix}$, então população das prezas é aproximadamente 1.0854 e dos predadores 0.3873.

2.

(a)

As variáveis Y_{k-1} , Y_k , Y_{k+1} ocorrem de forma linear em Y' e Y'' . $\cos x_k$ é constante.

(b)

$$h = 0.25 = \frac{1}{4} \rightarrow \frac{1}{h^2} = 16$$

$$16\left(y_{k+1} - 2\,y_k + y_{k-1}\right) = 2\left(y_{k+1} - y_{k-1}\right) + \cos x_k$$

$$y_4 = y'(2) \approx \frac{y_5 - y_3}{0.5} = 2\left(y_5 - y_3\right) \quad \rightarrow \quad 2\,y_5 - 2\mathrm{y}_3 = y_4 \rightarrow y_5 = \frac{y_4}{2} - y_3$$

Para k = 1, 2, 3, 4, temos:

$$16\,y_{k+1} - 32\mathrm{y}_k + 16\mathrm{y}_{k-1} = 2\mathrm{y}_{k+1} - 2\mathrm{y}_{k-1} + \cos x_k$$
$$14\mathrm{y}_{k+1} - 32\mathrm{y}_k + 18\mathrm{y}_{k-1} = \cos x_k$$

k = 2 : $14\mathrm{y}_3 - 32\mathrm{y}_2 + 18\mathrm{y}_1 = \cos(1.5) = 0.0707$

k = 3 : $14\mathrm{y}_4 - 32\mathrm{y}_3 + 18\mathrm{y}_2 = \cos(1.75) = -0.1782$

Para k = 1, $y_{k-1} = y_0 = -1$:

$$14\mathrm{y}_2 - 32\mathrm{y}_1 = \cos(1.25) + 18 = 18.3153$$

Para k = 4, temos, $y_5 = \frac{y_4}{2} - y_3$:

$$14\mathrm{y}_5 - 32\mathrm{y}_4 + 18\,y_3 = \cos(2)$$
$$-25\mathrm{y}_4 + 4\mathrm{y}_3 = -0.4161$$

Se $A \cdot y = b$, onde

$$\begin{pmatrix} -32 & 14 & 0 & -0 \\ 18 & -32 & 14 & 0 \\ 0 & 18 & -32 & 14 \\ 0 & 0 & 4 & 25 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} 18.3153 \\ 0.0707 \\ -0.1782 \\ -0.4161 \end{pmatrix}$$

3.

$$y \approx 2\sin(\omega t + \Phi)$$

(a)

$$\frac{y}{2} \approx \sin(\omega t + \Phi)$$

$$z = \arcsin\left(\frac{y}{2}\right) \approx \omega t + \Phi \quad , \quad \begin{matrix} g_1 = t \\ g_2 = 1 \end{matrix}$$

t	1	1.5	2	2.5	3
z	1.4307	0.8353	0.1055	-0.5174	-1.0646

$$A\begin{pmatrix} \omega \\ \Phi \end{pmatrix} \approx z \Leftrightarrow A^T A \begin{pmatrix} \omega \\ \Phi \end{pmatrix} = A^T z$$

$$A = \begin{pmatrix} \vdots & \vdots \\ g_1 & g_2 \\ \vdots & \vdots \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1.5 & 1 \\ 2 & 1 \\ 2.5 & 1 \\ 3 & 1 \end{pmatrix} \qquad A^T A = \begin{pmatrix} 22.5 & 10 \\ 10 & 5 \end{pmatrix} \qquad A^T z = \begin{pmatrix} -1.5926 \\ 0.7895 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} 22.5 & 10 & -1.5926 \\ 10 & 5 & 0.7895 \end{array}\right) \rightarrow \begin{matrix} \omega \approx -1.2687 \\ \Phi \approx 2.6952 \end{matrix} \qquad y \approx 2\sin(-1.2687\, t + 2.6952)$$

(b)

O resíduo do ajuste é

$$\|y-2\sin(-1.2687\,t+2.6952)\|_2 \rightarrow$$

$$\left\|\begin{pmatrix}1.9804\\1.483\\0.2106\\-0.9892\\-1.7492\end{pmatrix}-\begin{pmatrix}1.9792\\1.4237\\0.3143\\-0.9174\\-1.7922\end{pmatrix}\right\|_2=\left\|\begin{pmatrix}0.0012\\0.0593\\-0.1037\\-0.0718\\0.0430\end{pmatrix}\right\|_2=0.1458$$

4.

Seja $C=y$ taxa de CO_2 e $t=ano-1973$:

C=y	332.07	357.09	369.28
t	0	20	40

		Ordem	
C	0	1	2
332.07	0		
		0.7994	
357.09	20		-0.0045
		0.5103	
396.28	40		

$$f_0 = 0$$
$$f_1 = 0.7994$$
$$f_3 = -0.0045$$

$$p_2(y) = 0.7994\,(y - 332.07) - 0.0045\,(y - 332.07)(y - 357.09)$$

$$p_2(350) \approx 14.9053 \approx 15$$
$$ano = 1973 + 15 = 1988$$

Portanto a estimativa para o ano qual a taxa de CO_2 superou 350 ppm é 1988.

$$p_2(420) \approx 45.3987 \approx 45$$
$$ano = 1973 + 45 = 2018$$

Assim a estimativa para o ano qual a taxa de CO_2 ultrapassará 420 ppm é 2018.

5.

$$\int_{\varphi(-1)=2.5}^{\varphi(1)=3} \cos(e^x)\,dx$$
$$\varphi(t) = \frac{1}{2}[5.5 + t \cdot 0.5] = \frac{1}{4}[11 + t]$$
$$\varphi'(t) = \frac{1}{4} \qquad d\varphi(t) = \frac{1}{4}dt$$

$$\int_{-1}^{1} \cos\left(e^{\frac{11+t}{4}}\right)\frac{1}{4}dt \;=\; \frac{1}{4}\int_{-1}^{1} f(t)\,dt \;\approx f\left(\frac{-\sqrt{3}}{3}\right) + f\left(\frac{\sqrt{3}}{3}\right)$$
$$\frac{1}{4}[0.5621 + 0.7123] = \frac{1}{4} \cdot 1.2744 = 0.3186$$