1. $\left(\begin{array}{cccc|c} 1 & 1 & a & 1 & 1 \\ 1 & a & 1 & 1 & a \\ a & 1 & 1 & 1 & a^2 \end{array}\right)$

det $=$
$(a-1)+(-1)(1-a)$
$+a(1-a^2)$
$= 2a-2+a-a^3$
$= -a^3+3a-2$

$\sim\left(\begin{array}{ccc|c} 1 & 1 & a & 1 \\ 0 & a-1 & 1-a & a-1 \\ 0 & 1-a & 1-a^2 & a(a-1) \end{array}\right)$ $L2 = L2 - L_1$ $L3 = L_3 - aL_1$

a^2-a

$\sim\left(\begin{array}{ccc|c} 1 & 1 & a & 1 \\ 0 & a-1 & 1-a & a-1 \\ 0 & 0 & 2-a-a^2 & a^2 \cancel{-a} \cancel{+a} -1 \end{array}\right)$ $L3 = L3 - L2$

a^2-1

$\frac{1}{4}$

$\det(A) = (a-1)(2-a-a^2)$

$= 2a \cancel{-a^2} - a^3 - 2 + a \cancel{+a^2}$

$= -a^3 + 3a - 2$

(i) solução unica $\Longleftrightarrow \det(A) \neq 0$

$\Longleftrightarrow (a-1) \neq 0$ e $2 - a(1+a) \neq 0$

$\Longleftrightarrow a \neq 1$ e $a^2 + a - 2 \neq 0$

$(a+2)(a-1) \neq 0$

$\iff a \neq 1$ e $a \neq -2$ 1/4

(ii) Para $a = 1$, temos:

variáveis livres

$$\left(\begin{array}{ccc|c} 1 & 1 & a & 1 \\ 0 & a-1 & 1-a & a-1 \\ 0 & 0 & 2-a-a^2 & a^2-1 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$\therefore$ # infinito de soluções 1/4

(iii) Para $a = -2$ temos $2 - a - a^2$
$= 2 + 2 - 4 = 0$

$$\left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & -3 & 3 & -3 \\ 0 & 0 & 0 & 3 \end{array}\right)$$

$\therefore$ Nenhuma solução 1/4

(b) (ii) $S = \left\{ \begin{pmatrix} 1-y-z \\ y \\ z \end{pmatrix} : y, z \in \mathbb{R} \right\}$

$x + y + z = 1$
$x = 1 - y - z$

$= \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} : \alpha, \beta \in \mathbb{R} \right.$ 1/2

(i) $\left(\begin{array}{ccc|c} 1 & 1 & a & 1 \\ 0 & a-1 & 1-a & a-1 \\ 0 & 0 & 2-a(1+a) & a^2-1 \end{array}\right)$

$$z = \frac{a^2-1}{2-a(1+a)}$$

$$(a-1)y + \frac{\cancel{(1-a)}(1+a)(1-a)}{2-a(1+a)} = a-1$$

$$y - (\cancel{1+a})\ \cancel{(1-a)}$$

$\rightarrow \left(\begin{array}{ccc|c} 1 & 1 & a & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 2-a-a^2 & a^2-1 \end{array}\right)$

$\rightarrow \left(\begin{array}{ccc|c} 1 & 0 & a+1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 2-a-a^2 & a^2-1 \end{array}\right)$

$= (a+2)(a-1)$ $\quad 1-a^2 = (1-a)(1+a)$

$$y - z = 1 \Leftrightarrow y = 1 + z = 1 + \frac{a^2-1}{2-a(1+a)}$$

$$= 1 + \frac{(a+1)\cancel{(a-1)}}{-(a+2)\cancel{(a-1)}}$$

$x + a + 1\, z = 0$

$$\Leftrightarrow x = -(a-1)\, z = (1-a)\, z = -\frac{(a-1)^2(a+1)}{2-a(1+a)}$$

$$S = \left\{ \begin{pmatrix} -\frac{(a-1)^2(a+1)^2}{2-a(1+a)} \\ 1 + \frac{a^2-1}{2-a(1+a)} \\ \frac{a^2-1}{2-a(1+a)} \end{pmatrix} \right\} = \frac{+\cancel{(a-1)}^2(a+1)^2}{+(a+2)\cancel{(a-1)}} = \frac{(a+1)\cancel{(a-1)}}{-(a+2)(a-1)} = \left\{ \begin{pmatrix} \frac{(a-1)^2}{a+2} \\ 1 - \frac{a+1}{a+2} \\ -\frac{a+1}{a+2} \end{pmatrix} \right\}$$

$$S = \left\{ \begin{pmatrix} \frac{(a+1)^2}{a+2} \\ 1 - \frac{a+1}{a+2} \\ -\frac{a+1}{a+2} \end{pmatrix} \right\}$$

2. (a) Seja $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

$$\Rightarrow A^2 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = A \Rightarrow A^3 = A^2 \cdot A =$$

$$= A \cdot A = A^2 = A$$

mas $A \neq I$ e $A \neq 0$ $\boxed{\text{Falso}}$

(b) Seja $A \in \mathbb{R}^{m \times n}$ com $m < n$

$$m \left(\quad \right) \xrightarrow[\text{Gauß-Jordan}]{\text{op's el's por linha}} \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \quad \text{pivôs}$$

$r = \#$ de pivôs $\leq m < n$

$\Rightarrow$ temos $n - r > 0$ variáveis livres e $m - r$ linhas nulas

$\Rightarrow \#$ infinito de soluções

$\boxed{\text{VERDADEIRO}}$

(c) FALSO: Seja $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2$

$B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $C = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow B + C = I_2$

e $A(B+C) = I_2$

Então $1 = \det(A(B+C)) = 1 \neq 0 = 0 + 0$

$= \det(B) + \det(C)$

$= \det(AB) + \det(AC)$

(d) Suponha que $A = (B \mid C) \in \mathbb{R}^{n \times n}$

e $\exists X$ t.q. $BX = C$

$(B \mid C) \underset{\text{op's el}}{\overset{GJ}{\rightsquigarrow}} \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix} = I_n$

pq $\det(A) \neq 0 \Leftrightarrow$ a forma escalonada reduzida de A tem um pivo p/ linha

$\left(\begin{array}{ccc|c} 1 & & 0 & 0 \\ & \ddots & & \vdots \\ 0 & & 1 & 0 \\ \hline 0 & 0 & \cdots 0 & 1 \end{array}\right)$ não é soluvel porque $\nexists \begin{pmatrix} x_1 \\ x_{n-1} \end{pmatrix}$ t.q. $0x_1 + \dots + 0x_{n-1} = 1$

$\Rightarrow \nexists X$ t.q. $BX = C$ ↯

~~FALSO~~ $\boxed{\text{VERDADEIRO}}$

$$\left|\begin{array}{cccc|cccc} 2 & 3 & 5 & -2 & 0 & 1 & 0 & 0 \\ 4 & -2 & -1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 & 0 & 0 & 1 \end{array}\right|$$

$L2 = L2 - 2L1$

$L3 = L3 - 4L1$

$$\left|\begin{array}{cccc|cccc} 1 & -1 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 5 & 7 & -4 & -2 & 1 & 0 & 0 \\ 0 & 2 & 3 & -3 & -4 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 & 0 & 0 & 1 \end{array}\right|$$

$\boxed{L2 \leftrightarrow L4}$

$(\det = \det \cdot (-1))$

$$\left|\begin{array}{cccc|cccc} 1 & -1 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & -1 & 0 & 0 & 0 & \\ 0 & 2 & 3 & -3 & -4 & 0 & 1 & 0 \\ 0 & 5 & 7 & -4 & -2 & 1 & 0 & 0 \end{array}\right|$$

$L3 = L3 - 2L2$

$L4 = L4 - 5L2$

$$\left|\begin{array}{cccc|cccc} 1 & -1 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & -1 & -4 & 0 & 1 & -2 \\ 0 & 0 & -3 & 1 & -2 & 1 & 0 & -5 \end{array}\right|$$

$\boxed{L3 = (-1)L3}$

$(\det = \det \cdot (-1))$

$L4 = L4 + 3L3$

$$\left|\begin{array}{cccc|cccc} 1 & -1 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 4 & 0 & -1 & 2 \\ 0 & 0 & 0 & 4 & 10 & 1 & -3 & 1 \end{array}\right|$$

$\therefore \det(A) = 1 \cdot 1 \cdot 1 \cdot 4 = 4$

$L1 = L1 + L2 \rightsquigarrow$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & \cancel{4}1 & 4 & 0 & -1 & 2 \\ 0 & 0 & 0 & 4 & 10 & 1 & -3 & 1 \end{array}\right)$$

$L2 = L2 + L3 \rightsquigarrow$

$L4 = \frac{1}{4} L4$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 3 & 0 & 4 & 0 & -1 & 3 \\ 0 & 0 & 1 & 1 & 4 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & \frac{5}{2} & \frac{1}{4} & -\frac{3}{4} & \frac{1}{4} \end{array}\right)$$

$L3 = L3 - L4 \rightsquigarrow$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 3 & 0 & 4 & 0 & -1 & 3 \\ 0 & 0 & 1 & 0 & \frac{3}{2} & -\frac{1}{4} & -\frac{1}{4} & \frac{7}{4} \\ 0 & 0 & 0 & 1 & \frac{5}{2} & \frac{1}{4} & -\frac{3}{4} & \frac{1}{4} \end{array}\right)$$

$L1 = L1 - L3 \rightsquigarrow$

$L2 = L2 - 3L3$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{4} & \frac{1}{4} & -\frac{3}{4} \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & \frac{3}{4} & -\frac{1}{4} & -\frac{9}{4} \\ 0 & 0 & 1 & 0 & \frac{3}{2} & -\frac{1}{4} & -\frac{1}{4} & \frac{7}{4} \\ 0 & 0 & 0 & 1 & \frac{5}{2} & \frac{1}{4} & -\frac{3}{4} & \frac{1}{4} \end{array}\right)$$

$\underbrace{\qquad\qquad}_{n^{-1}}$

4.

(a) $AX = B$

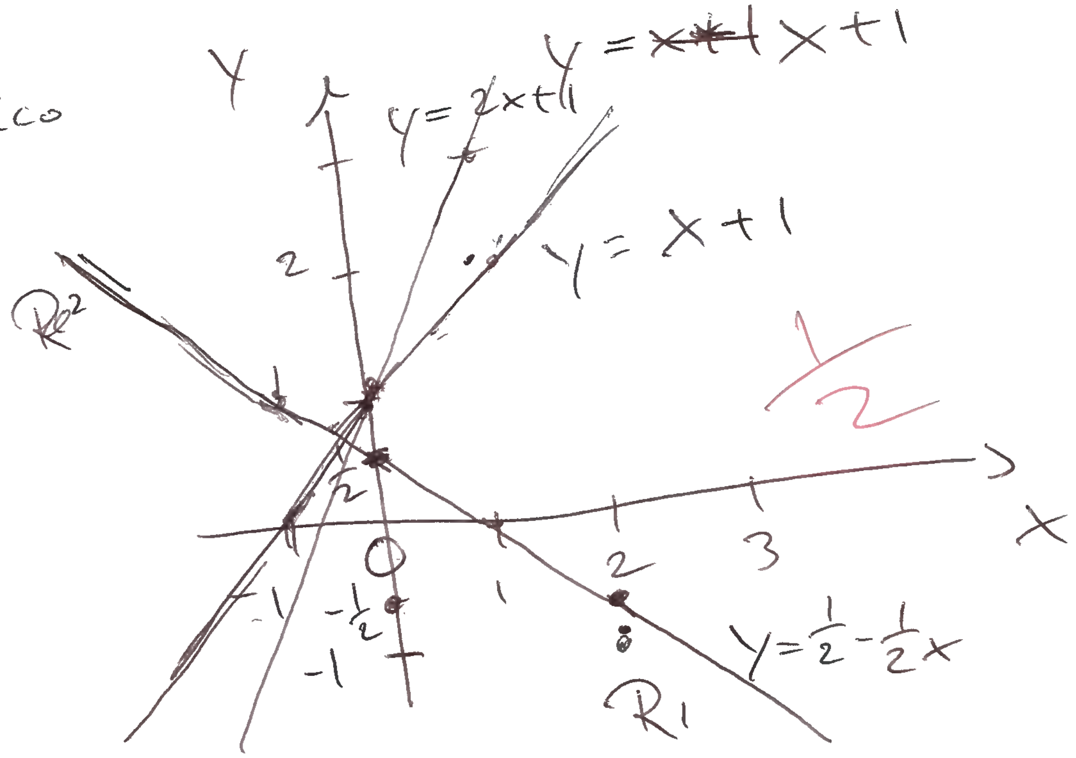
$\Leftrightarrow$ R1 $x + 2y = 1 \Leftrightarrow 2y = 1 - x \Leftrightarrow y = \frac{1}{2} - $

R2 $x - y = -1 \Leftrightarrow y = x + 1$

R3 $4x - 2y = -2 \Leftrightarrow 2x - y = -1$

$\Leftrightarrow y = 2x + 1$

Gráfico



$\frac{1}{2}$

$\begin{pmatrix} x \\ y \end{pmatrix} = X \in S_{A|B} \Leftrightarrow AX = B$

$\Leftrightarrow$ as 3 eq's são satisfeitas para $X = \begin{pmatrix} x \\ y \end{pmatrix}$

$\Leftrightarrow X \in R1 \cap R2 \cap R3$

Gráficamente vemos que $S_{A|B} = \emptyset$

$1\frac{1}{2}$

$$\left(\begin{array}{cc|c|c} 1 & 2 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 4 & -2 & & -2 \end{array}\right)$$

$$\sim \left(\begin{array}{cc|c} 1 & 2 & 1 \quad 1 \\ 1 & -1 & -1 \\ 2 & -1 & -1 \end{array}\right)$$

$$\sim \left(\begin{array}{cc|c} 1 & 2 & 1 \quad 1 \\ -1 & 1 & 1 \\ 2 & -1 & -1 \end{array}\right) \sim \left(\begin{array}{cc|c} 0 & 3 & 2 \\ -1 & 1 & 1 \\ 0 & 1 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{cc|c} 0 & 3 & 2 \\ -1 & 0 & 0 \\ 0 & 1 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{cc|c} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{array}\right) \quad 0x + 0y = -1$$

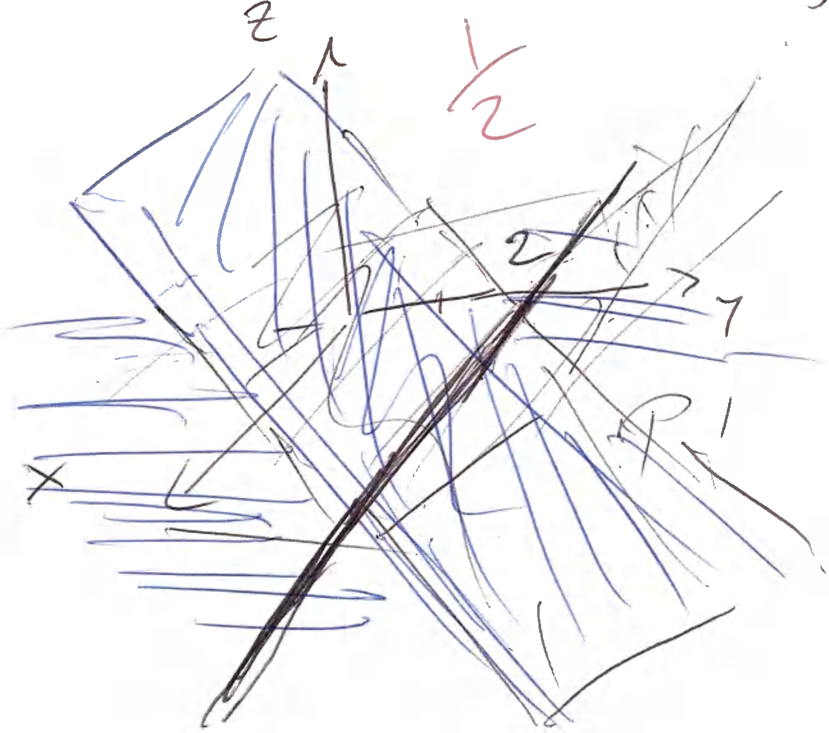
$$\therefore S_{A|B} = \emptyset$$

$\frac{1}{2}$

(b) $P1: \; z = 0$

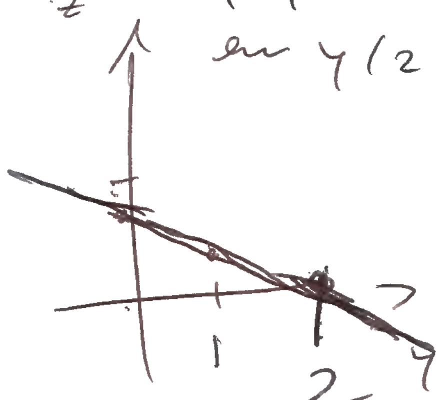
$P2: \; 2y + 5z = 4 \iff y = 2 - \frac{5}{2}z$

$\iff z = \frac{4}{5} - \frac{2}{5}y$



$\frac{1}{2}$

Projeen auf y/z



$\frac{1}{2}$

$$\left(\begin{array}{cccc|c} 0 & 0 & 1 & & 0 \\ 0 & 2 & 5 & & 4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 4 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \end{array}\right) \qquad \begin{array}{l} z = 0 \\ y = 2 \end{array}$$

$$S_{A|B} = \left\{ \begin{pmatrix} x \\ 2 \\ 0 \end{pmatrix} \middle| \; x \in \mathbb{R} \right\}$$

$\frac{1}{2}$

$$= \left\{ \alpha \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} \middle| \; \alpha \in \mathbb{R} \right\}$$