1. I.(a)

$$(x\ y)\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}+(10\ \ -6)\begin{pmatrix}x\\ y\end{pmatrix}+25=0$$

(b) Autovalores de A:

$$0=\det(A-\lambda I)=\det\begin{pmatrix}1-\lambda & -1\\ -1 & 1-\lambda\end{pmatrix}=(\lambda-1)^2-1=\lambda(\lambda-2)$$

$$\therefore \lambda_1=0,\ \lambda_2=2$$

(c) Autovetores de A:

$\lambda_1=0$: $\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}\sim\begin{pmatrix}1 & -1\\ 0 & 0\end{pmatrix}$ $x=y$ $S=\left\{\alpha\begin{pmatrix}1\\ 1\end{pmatrix}:\alpha\in\mathbb{R}\right\}$

$\lambda_2=2$: $\begin{pmatrix}-1 & -1\\ -1 & -1\end{pmatrix}\sim\begin{pmatrix}1 & 1\\ 0 & 0\end{pmatrix}$ $x=-y$ $S=\left\{\alpha\begin{pmatrix}-1\\ 1\end{pmatrix}:\alpha\in\mathbb{R}\right\}$

Para $V_1=\begin{pmatrix}1\\ 1\end{pmatrix}$ e $V_2=\begin{pmatrix}-1\\ 1\end{pmatrix}$, temos $\det\left(\begin{pmatrix}V_1 & V_2\end{pmatrix}\right)=2>0$

(d) $\bar{V}_1=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix}$, $\bar{V}_2=\frac{1}{\sqrt{2}}\begin{pmatrix}-1\\ 1\end{pmatrix}\Rightarrow R=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}$

é matriz de rotação e $R=I_{\mathcal{B}}^{\bar{\mathcal{B}}}$, onde $\bar{\mathcal{B}}=\{\bar{V}_1,\bar{V}_2\}$ e $\mathcal{B}=\{E_1,E_2\}$

(e)

$$(\bar{x}\ \bar{y})\begin{pmatrix}0 & 0\\ 0 & 2\end{pmatrix}\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}+(10\ \ -6)R\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}+25=0$$

$$\Leftrightarrow 2\bar{y}^2+\frac{1}{\sqrt{2}}(4\ \ -16)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}+25=0$$

$$\Leftrightarrow 2\bar{y}^2-8\sqrt{2}\,\bar{y}+2\sqrt{2}\,\bar{x}+25=0$$

II. $2\bar{y}^2 - 8\sqrt{2}\,\bar{y} = 2(\bar{y}^2 - 4\sqrt{2}\,\bar{y})$

$$\bar{y}^2 - 4\sqrt{2}\,\bar{y} = (\bar{y} - 2\sqrt{2})^2 - 8$$

Obtemos

$$2[(\bar{y} - 2\sqrt{2})^2 - 8] + 2\sqrt{2}\,\bar{x} + 25 = 0$$

$$\Leftrightarrow 2(\bar{y} - 2\sqrt{2})^2 + 2\sqrt{2}\,\bar{x} + 9 = 0$$

$$\Leftrightarrow (\bar{y} - 2\sqrt{2})^2 + \sqrt{2}\,\bar{x} + \frac{9}{2} = 0$$

$$\Leftrightarrow (\bar{y} - 2\sqrt{2})^2 = -\sqrt{2}\left(\bar{x} + \frac{9}{\sqrt{2}\cdot 2}\right)$$

$$\Leftrightarrow \underbrace{(\bar{y} - 2\sqrt{2})}_{\hat{y}}{}^2 = -\sqrt{2}\underbrace{\left(\bar{x} + \frac{9\sqrt{2}}{4}\right)}_{\hat{x}}$$

(b) ℓ é uma parábola da forma

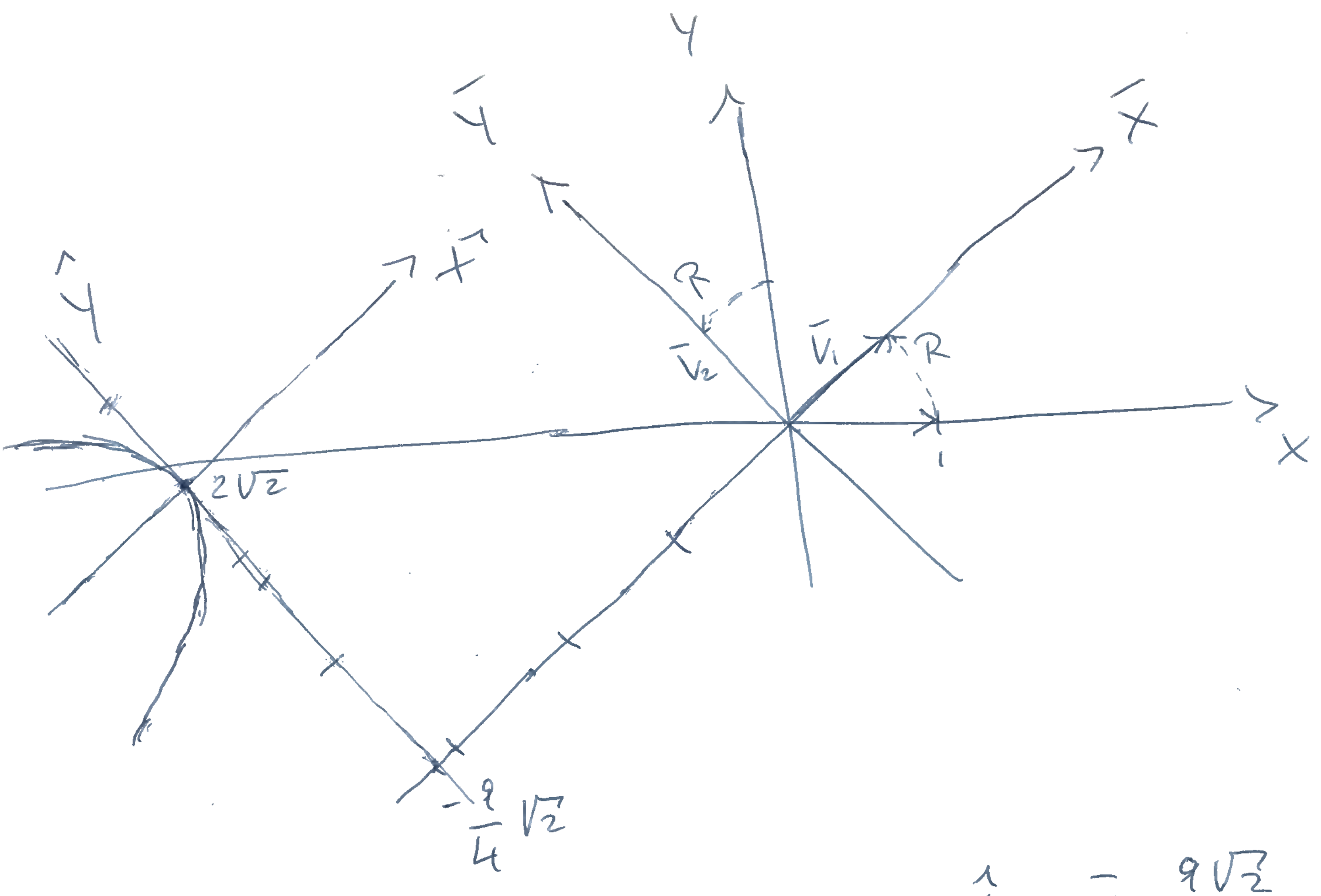
$$\hat{y}^2 = 4p\,x, \text{ onde } 4p = -\sqrt{2}$$

$$\Leftrightarrow p = -\frac{\sqrt{2}}{4}$$

Foco no sist. $\hat{x},\hat{y}$: $F = \begin{pmatrix} -\frac{\sqrt{2}}{4} \\ 0 \end{pmatrix}$; Vértice em $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ no sist $\hat{x},\hat{y}$

Excentricidade de ℓ é $e=1$ (parábola)

(c)



(d) $F = \begin{pmatrix} -\frac{\sqrt{2}}{4} \\ 0 \end{pmatrix}$ em $\hat{x}, \hat{y}$, onde $\hat{x} = \bar{x} + \frac{9\sqrt{2}}{4} \Leftrightarrow \bar{x} = \hat{x} - \frac{9\sqrt{2}}{4}$

$\hat{y} = \bar{y} - 2\sqrt{2} \Leftrightarrow \bar{y} = \hat{y} + 2\sqrt{2}$

$\Rightarrow F = \begin{pmatrix} -\frac{5\sqrt{2}}{2} \\ 2\sqrt{2} \end{pmatrix}$ em $\bar{x}, \bar{y}$, quer dizer $F = \begin{pmatrix} -\frac{5\sqrt{2}}{2} \\ 2\sqrt{2} \end{pmatrix}_{\bar{\mathcal{B}}}$

$R = I^{\bar{\mathcal{B}}}_{\mathcal{B}}$, então $F = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -\frac{5\sqrt{2}}{2} \\ 2\sqrt{2} \end{pmatrix} = \begin{pmatrix} -\frac{9}{2} \\ -\frac{1}{2} \end{pmatrix}$

(em x, y)

O vértice $V = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ em $\hat{x}, \hat{y}$

$\Rightarrow V_{\bar{\mathcal{B}}} = \begin{pmatrix} -\frac{9\sqrt{2}}{4} \\ 2\sqrt{2} \end{pmatrix}_{\bar{\mathcal{B}}}$

$\Rightarrow V = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -\frac{9\sqrt{2}}{4} \\ 2\sqrt{2} \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -\frac{9}{4} \\ 2 \end{pmatrix} = \begin{pmatrix} - \\ \frac{1}{4} \end{pmatrix}$

2. (a) Seja $\bar{\mathcal{B}} = \{\bar{V}_1, \bar{V}_2\} = \left\{ \begin{pmatrix} \cos\theta \\ \operatorname{sen}\theta \end{pmatrix}, \begin{pmatrix} -\operatorname{sen}\theta \\ \cos\theta \end{pmatrix} \right\}$

Tem-se $V_{\bar{\mathcal{B}}} = \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \bar{x}\,\bar{V}_1 + \bar{y}\,\bar{V}_2$

$= R \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$, onde $R = \begin{pmatrix} \cos\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \cos\theta \end{pmatrix}$

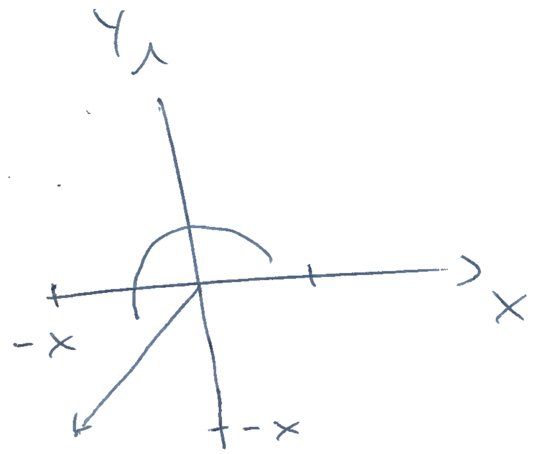
Alternativamente, observe que

$V = V_{\mathcal{B}}$, onde $\mathcal{B} = \{E_1, E_2\}$, e $R = I_{\mathcal{B}}^{\bar{\mathcal{B}}}$

Então $V = V_{\mathcal{B}} = I_{\mathcal{B}}^{\bar{\mathcal{B}}} \cdot V_{\bar{\mathcal{B}}} = R \cdot \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$

VERDADEIRO

(b)



Para $\begin{pmatrix} -x \\ -x \end{pmatrix}$, temos $r = \sqrt{(-x)^2 + (-x)^2} = \sqrt{2}\, x > 0$

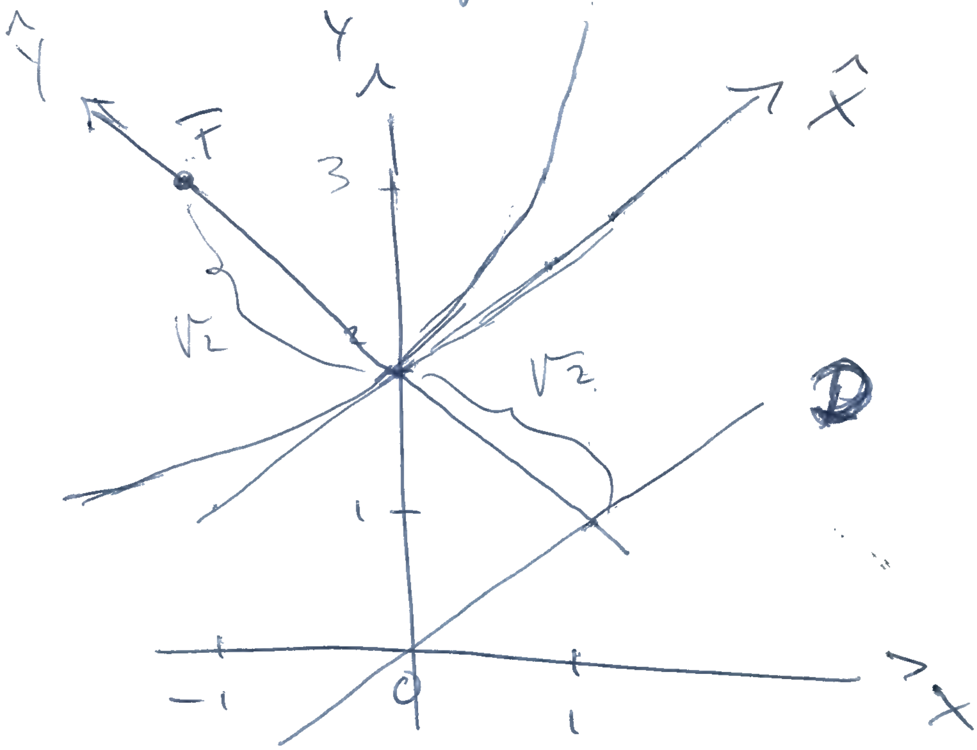
onde $x > 0$

mas $\theta = \measuredangle \left(\text{eixo } x, \begin{pmatrix} -x \\ -x \end{pmatrix} \right) = -\frac{5\pi}{4} \; (+2\pi k)$

$\neq \frac{\pi}{4} \; (+2\pi k)$

FALSO

3. $\mathcal{C}$ representa uma parábola



Coordenadas de F em $\hat{x}, \hat{y}$: $\begin{pmatrix} 0 \\ \sqrt{2} \end{pmatrix}$ porque

$$\mathrm{dist}(F, \mathcal{D}) = 2\,\mathrm{dist}(F, V) = 2\,\mathrm{dist}(\mathcal{D}, V) = 2\sqrt{2} = 2p$$

onde V é vértice de $\mathcal{C}$

$\mathcal{C}$ é da forma $\cancel{y}$ $\hat{x}^2 = 4p\hat{y}$, onde $p = \sqrt{2}$

Por exemplo, para $\hat{x} = \pm 1$, obtemos

$$\hat{y} = \frac{1}{4\sqrt{2}} = \frac{\sqrt{2}}{8} \approx 0{,}18$$

4. a) π_R tem $N_R = \begin{pmatrix} 2 \\ 1 \\ 6 \end{pmatrix}$ como vetor normal

π_S tem $N_S = \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix}$ como vetor normal

Observe que $\pi_R : 4x + 2y + 12z - \ldots$

$\pi_S : 4x + 10y + 0z - \ldots$

$$\measuredangle(\pi_R, \pi_S) = \measuredangle(N_R, N_S)$$

$$\cos \measuredangle(N_R, N_S) = \frac{\langle N_R, N_S \rangle}{\|N_R\| \, \|N_S\|} = \frac{9}{\sqrt{41}\sqrt{29}}$$

$$\Rightarrow \measuredangle(N_R, N_S) = \arccos\left(\frac{9}{\sqrt{41}\sqrt{29}}\right) = \arccos\left(\frac{9}{\sqrt{1189}}\right)$$

(b) Determinar Q t.q. $\pi_Q \parallel \mathcal{P}$ e $Q \in \mathcal{E}$ e $q_1, q_2, q_3 > 0$

$$\pi_Q \parallel \mathcal{P} \iff \begin{pmatrix} q_1 \\ 2q_2 \\ 3q_3 \end{pmatrix} \parallel \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \iff \begin{pmatrix} q_1 \\ 2q_2 \\ 3q_3 \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

para algum $\alpha \neq 0$

$$\iff q_2 = \frac{1}{2} q_1, \; q_3 = \frac{1}{3} q_1 \text{ e } q_1 \neq 0$$

Além disso $Q \in \mathcal{E}$ e daí

$$q_1^2 + 2\left(\frac{q_1}{2}\right)^2 + 3\left(\frac{q_1}{3}\right)^2 = 66$$

$$\iff q_1^2\left(\frac{11}{6}\right) = 66 \iff q_1^2 = 36 \iff q_1 = \pm 6$$

Já que $q_i > 0$ para $i = 1, 2, 3$, obtemos $Q = (6, 3, 2)^T$.

(c) $\mathrm{dist}(\mathcal{E}, \mathcal{P}) = \mathrm{dist}(Q, \mathcal{P}) = \|\mathrm{proj}_{N_\mathcal{P}}(\overrightarrow{QP})\|$

onde $N_\mathcal{P} = (1, 1, 1)^T$

e ~~P =~~ { $P \in \mathcal{P}$, por ex. $P = \begin{pmatrix} 10 \\ 10 \\ 10 \end{pmatrix}$

$$= \frac{|\langle \overrightarrow{QP}, N_\mathcal{P} \rangle|}{\|N_\mathcal{P}\|} = \frac{|(4,7,8)\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}|}{\sqrt{3}} = \frac{19}{\sqrt{3}}$$