4.(a) $W \overset{t_L}{\vee} A = B$

$$\iff \underbrace{A^T}_{S} \overset{t_L}{\vee} \underbrace{W^T}_{R} = \underbrace{B^T}_{T}$$

$\therefore \exists W^*$ com $W^* \overset{t_L}{\vee} A = B$

$\iff \exists R^*$ com $S \overset{t_L}{\vee} R^* = T$

$\iff \bar{R} = S^T \overset{\wedge}{\Rightarrow}_L T$ satisfaz $S \overset{t_L}{\vee} \bar{R} = T$

$\iff \bar{W} = \left(S^T \overset{\wedge}{\Rightarrow}_L T\right)^T = \left(A \overset{\wedge}{\Rightarrow}_L B^T\right)^T$ satizfaz

$\bar{W} \overset{t_L}{\vee} A = B$

$$\bar{W}^T = A \overset{\wedge}{\Rightarrow}_L B^T = \begin{pmatrix} 0.7 & 0.7 \\ 0.8 & 0.4 \\ 0 & 0.8 \end{pmatrix} \overset{\wedge}{\Rightarrow}_L \begin{pmatrix} 0.5 & 0.2 & 0.7 & 0.3 \\ 0.3 & 0.6 & 0.3 & 0 \end{pmatrix} = \begin{pmatrix} 0.6 & 0.5 & 0.6 & 0.3 \\ 0.7 & 0.4 & 0.9 & 0.5 \\ 0.5 & 0.8 & 0.5 & 0.2 \end{pmatrix}$$

$$\bar{W} \overset{t_L}{\vee} A = \begin{pmatrix} 0.6 & 0.7 & 0.5 \\ 0.5 & 0.4 & 0.8 \\ 0.\cancel{8}6 & 0.9 & 0.5 \\ 0.3 & 0.5 & 0.2 \end{pmatrix} \overset{t_L}{\vee} \begin{pmatrix} 0.7 & 0.7 \\ 0.8 & 0.4 \\ 0 & 0.8 \end{pmatrix} = \begin{pmatrix} 0.5 & 0.3 \\ 0.2 & 0.6 \\ 0.7 & 0.3 \\ 0.3 & 0 \end{pmatrix} \text{ o.k.!}$$

Então $\bar{W}$ tem a propriedade desejada

(b) Sabemos que $\bar{W}$ é a maior solução de $W \overset{t_L}{\vee} A = B$.
Eventualmente tem outra solução W^* com $W^* \leq W$?

$$\text{Seja } W^* = \begin{pmatrix} 0.5 & 0.7 & 0.5 \\ 0.5 & 0.4 & 0.8 \\ 0.6 & 0.9 & 0.5 \\ 0.3 & 0.5 & 0.2 \end{pmatrix} \neq \bar{W}$$

Verificamos que $W^* \overset{t_L}{\vee} A = B$ então $\exists$ outra matriz que satisfaz $W \overset{t_L}{\vee} A = B$.