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Fuzzy Numbers in the Credit Rating of Enterprise Financial Condition

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Abstract. Most of the parameters used to describe the credit rating are in linguistic terms, which are vague and difficult to put into precise numerical values. Fuzzy set theory, which was developed to handle this kind of vagueness, is used to represent and to aggregate the various linguistic data usually used in commercial banks. To illustrate the approach, numerical examples are solved and compared with existing approaches.

Key words: credit rating, fuzzy sets, linguistic representation

JEL Classification: C10, P41

1. Introduction

Many factors, which are usually vague, difficult to define, and even conflicting, need to be considered in determining the credit rating of an enterprise. To make the problem even more complicated, the relative importance between the different factors must also be decided in order to obtain the overall rating. Thus, the final credit rating obtained is not accurate and tremendous amount of human judgement is needed in order to use the results. To overcome this approximate and unreliable nature of credit rating, the so-called rule-of-thumb is frequently used to screen loan applicants.

Fuzzy set theory was developed to handle this kind of vague and linguistic situation, and thus is ideally suited for improving the accuracy of credit rating. However, the use of fuzzy concept in credit rating is fairly new. It appears that Su and Chen (1980) are the earliest investigators to study this problem using fuzzy sets. Within the last two years, several researchers have investigated this problem by the use of neural-fuzzy approaches. Based on the actual data used by the banks in Taiwan, Su and Chen (1980) proposed the use of fuzzy numbers to represent the various linguistic factors. As will be shown in this paper, their approaches frequently give unreasonable results. Malhotra and Malhotra (1999)

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proposed the use of artificial intelligence, expert system, neural network, and fuzzy logic to reduce the complexity and to improve the accuracy in credit approval. These authors didn't give any algorithms or any detailed approaches. Rast (1997) studied price forecasting and customer credit rating by the use of fuzzy neural network with emphasis based on time series analysis. The approach is useful only if a large amount of data is available and the emphasis is on the trend and on the future predictions. Piramuthu (1999) proposed the use of neural network and neuro-fuzzy system to improve the credit evaluation decisions. They argued that neuro-fuzzy with fuzzy logic rules can be used for credit rating.

Except the studies of Su and Chen, all the other approaches mentioned above emphasize neural learning, which is time consuming and needs a lot of data. Another problem is that the approaches are either too simplified or too theoretical to be of practical use. In this research, we wish to propose a fairly simple and practical approach based on fuzzy linguistic representation and fuzzy multi-attribute decision making. By using actual credit rating data, we illustrate the basic concept of the approach and compare with the existing literature.

Based on the general practices of the banks of Taiwan, the current rating of credit approach is first summarized. Then, fuzzy numbers and fuzzy linguistic representation are presented. Finally, to illustrate and to show the advantages of the proposed approach, some numerical results are obtained and compared with the literature.

2. Financial credit rating

Many factors influence the credit rating of an enterprise. According to the Committee of the Banks of the City of Taipei [4], the important factors can be approximately classified into the following three categories:

1. Financial conditions of the enterprise
   (a) ability to pay debt
   (b) financial structure
   (c) earning ability
   (d) management ability
2. Management
   (a) the rating of the management
   (b) the legal form of the enterprise
   (c) the efficiency of the organization
   (d) the cooperation between production and sales
   (e) the influence of additional capital
   (f) past history on credit liability
3. The particular characteristics of the main products, competitions and expectations
   (a) competitive position in the particular product field
   (b) product demand and suitability
   (c) development expectations
   (d) the coming one year's market condition

The first two categories are more influenced by the management and the last one is more intrinsic to the company and its environment. Thus, category three cannot be controlled or improved easily by management.

Not only the above linguistic terms are difficult to represent numerically they are even difficult to define. For example, terms like *management ability, efficiency, competitive position, market conditions,* and etc. are difficult to describe and, in fact, they are composite terms. As a general practice, these composite terms are expressed linguistically based on experience, past history, and other factors.

In addition to the problem of representing the above terms numerically, a more difficult problem is the relative importance between the different terms and between the different categories. According to the general practice of the banks in Taiwan [4], the following different weights are used to express the relative importance of the different categories:

$$w_1 = 0.5, \qquad w_2 = 0.3, \qquad w_3 = 0.2$$

where $\sum_{i=1}^{3} w_i = 1$ and $i = 1, 2, 3 =$ Category 1, Category 2, and Category 3. In actual practice, instead of the fractional weights used above, weights in percentages were used.

Obviously, these weights or percentages are very arbitrary. For example, if the product of an enterprise is in the new high technology area and is fast growing, category three certainly should dominate the weighting.

In order to simplify and to illustrate the problem, let us consider only the first category which is *the financial conditions of the enterprise*. According to the Committee of the Banks of Taipei City [4], each of the composite terms such as *ability to pay, financial structure,* etc. was first separated into different sub-terms, which have different meanings. The degree of importance of the different sub-terms was represented by the use of ratios as that shown in Table 1. The ratios listed in Table 1 for each sub-term are defined as:

1. Ability to pay
   (a) quick ratio = (current assets investor)/(current liabilities)
   (b) current ratio = (current assets)/(current liabilities)
2. Financial structure
   (a) debt-equity ratio = (total debt)/(equity)
   (b) fixed long term turnover = (fixed assets + long term investment)/(equity + long term liabilities)
3. Earning ability
   (a) expense ratio = (expenses)/(sales)
   (b) profit margin = (net income)/(sales)
   (c) return on equity = (net income)/(equity)
4. Management ability
   (a) inventory turnover = (sales)/(inventory)
   (b) receivables turnover = (sales)/receivables)
   (c) total assets turnover = (sales)/(total assets)

The universe of the ratio for each sub-item in Table 1 is divided into five intervals, which represent the five different grades. For example, the five intervals for the five grades for

Table 1. The financial conditions of an enterprise

Sub-Term	Ratio	Grade: Interval
Ability to pay debt	Quick ratio	1: below 34%, 2: 35%–49%, 3: 50%–64% 4: 65%–79%, 5: above 80%
	Current ratio	1: below 59%, 2: 60%–89%, 3: 90%–119% 4: 120%–149%, 5: above 150%
Financial structure	Debt-equity ratio	1: above 401%, 2: 400%–301%, 3: 300%–201% 4: 200%–101%, 5: below 100%
	Fixed long long turnover	1: above 181%, 2: 180%–131%, 3: 130%–101% 4: 100%–81%, 5: below 80%
Earning ability	Expense ratio	1: above 6%, 2: 5.1%–6.0%, 3: 3.1%–5.0% 4: 2.1%–3.0%, 5: below 2%
	Profit margin	1: negative, 2: 0.0%–1.9%, 3: 2.0%–4.9% 4: 5.0%–7.9%, 5: above 8.0%
	Return on equity	1: negative, 2: 0.0%–5.0%, 3: 5.1%–9.9% 4: 10%–14.9%, 5: above 15%
Management ability	Inventory turnover	1: below 1.9%, 2: 2.0%–2.9%, 3: 3.0%–3.9% 4: 4.0%–5.9%, 5: above 6%
	Receivables turnover	1: below 1.9%, 2: 2.0%–2.9%, 3: 3.0%–3.9% 4: 4.0%–5.9%, 5: above 6.0%
	Total assets turnover	1: below 0.59, 2: 0.6%–0.79%, 3: 0.8%–0.99% 4: 1.0%–1.19%, 5: above 1.2%

profit margin ratio are:

- Grade 1 if the ratio is negative
- Grade 2 if the ratio is 0.0%–1.9%
- Grade 3 if the ratio is 2.0%–4.9%
- Grade 4 if the ratio is 5.0%–7.9%
- Grade 5 if the ratio is above 8.0%

In order to obtain a fairly reliable estimation based on the three categories, we must first represent these categories and then aggregate or combine them so that some reasonable results can be obtained. Although Table 1 presented a fairly reasonable representation, the separation of the different grades is very arbitrary and the aggregation of the different categories and sub-terms with the addition of weighting to represent the relative importance also need to be carefully considered.

3. Fuzzy numbers

Only the basic ideas and essential definitions will be introduced in this section. The interested reader can consult many of the excellent books in the literature (Kaleva and Seikkala, 1984; Kaufmann and Gupta, 1991; Zimmermann, 1991).

Let R denote the set of real numbers, A fuzzy number is defined as:

Definition 1. A fuzzy number is a fuzzy set $\mu : R \to [0, 1]$ with the following properties:

1. μ is normal, i.e., there exists a real number m such that $\mu(m) = 1$.
2. μ is fuzzy convex, i.e., for any pair $x, y \in [a, b]$, $\mu(\lambda x + (1-\lambda)y) \geq \min\{\mu(x), \mu(y)\}$ for all $\lambda \in [0, 1]$.
3. μ is upper semi-continuous, i.e., for each $\alpha \in (0, 1]$, the α-level set $\{x \in R \mid \mu(x) \geq \alpha\}$ is closed.

Each α-level set of a fuzzy number is a closed interval, which can be represented as: $[a(\alpha), b(\alpha)]$, where the limits $a(\alpha) = -\infty$ and $b(\alpha) = \infty$ are admissible. A fuzzy number is determined by the family of its α-level cuts.

A fuzzy number, A, is usually denoted as:

$$A = \{(x, \mu_A(x)) \mid x \in R\} \tag{1}$$

where $\mu_A(x)$ is the grade of membership of x in the fuzzy concept A. The arithmetic operations between two fuzzy sets, A and B, can be expressed as (Zadeh, 1965):

$$\mu_C(z) = \mu_{A*B}(z) = sup_{z=x*y} \min(\mu_A(x), \mu_B(y)) \tag{2}$$

where $*$ denotes one of the binary operations: $+$, $-$, $\times$, and $\div$, and where C is the new fuzzy concept or fuzzy number with membership function $\mu_C(x)$.

Definition 2. A trapezoidal fuzzy number, A, is a fuzzy number with membership function:

$$\mu_A(x) = \begin{cases} \frac{x-c}{a-c} & c \leq x \leq a \\ 1 & a \leq x \leq b \\ \frac{x-d}{b-d} & b \leq x \leq d \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

where $a, b, c, d, \in R$ and $c \leq a \leq b \leq d$.

A triangular fuzzy number can be considered as a special case of the trapezoidal fuzzy number with $a = b$.

4. Fuzzy financial grading system

Since the sub-terms are represented by linguistic ratios, the problem is how to represent these ratios and then how to aggregate them so that some meaningful results can be obtained. Although the ratio of each sub-term is divided into five intervals or five grades, the division between grades is too sharp or too "crisp." For example, consider the ratios in the profit margin, the distance between the ratios 1.9% and 2% is very near, but they belongs

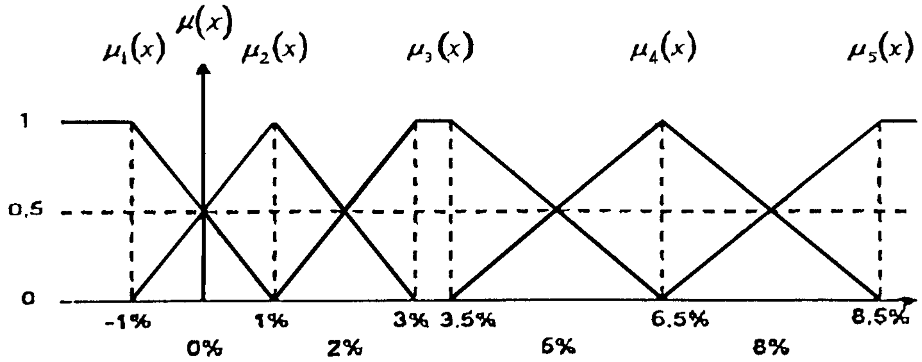

Figure 1. Proposed fuzzy representation.

to two different grades. A gradual transition from one grade to another would be more reasonable. Fuzzy set can provide this gradual transition. This is shown in Figure 1, where the different grades of fuzzy membership functions for the "profit margin ratio" is represented by trapezoidal or triangular fuzzy numbers. For example, instead of sudden transition from negative for Grade 1 to positive for Grade 2, the transition is gradual and the membership function for Grade 1 is gradually decreasing from 0.5 at zero ratio to zero at a ratio of one. At the same time, the membership function for Grade 2 is gradually increasing from zero at -1 ratio to 0.5 at a ratio of zero. This representation is clearly more reasonable than that shown in Table 1.

Let p_j, $j = 1, 2, 3, 4$, represent the horizontal coordinates in Figure 1, then we have:

$$p_1 = 0\%, \qquad p_2 = 2\%, \qquad p_3 = 5\%, \qquad p_4 = 8\%$$

The membership functions for the five grades can be represented by $\mu_j(x)$, $j = 1, 2, \ldots, 5$. These membership functions exhibit the following properties:

1. The membership functions for Grades 1 and 5, which can be represented by $\mu_1(x)$ and $\mu_5(x)$, respectively, are composed of half trapezoidal fuzzy numbers.
2. The membership functions for Grades 2 and 4 are symmetric triangular fuzzy numbers and for Grade 3 trapezoidal number.
3. For $j = 2, 3, 4$, the membership function, $\mu_j(x)$ reaches a maximum of one at $(p_{j-1} + p_j)/2$.
4. The membership functions.
5. For any given x, $\mu_1(x) + \mu_2(x) + \cdots \mu_5(x) = 1$. In other words, the sum of the membership functions of all the fuzzy numbers at any given point x is equal to 1.

Figure 1 is the proposed actual representation of the linguistic terms for the "profit margin" ratio. Nine similar figures to represent the remaining nine ratios listed in the second column

Table 2. Sample data from five enterprises

		Samples				
Sub-Term	Ratio	1	2	3	4	5
Ability to pay debt	Quick ratio	79	65	64	35	35
	Current ratio	170	150	149	60	60
Financial structure	Debt equity ratio	50	100	101	2500	400
	Fixed long term turnover	81	100	101	500	180
Earning ability	Expense ratio	2.1	3	3.1	16	6
	Profit margin	12	8	7.9	0.0	0.0
	Return on equity	9.9	5.1	5	0.0	0.0
Management ability	Inventory turnover	10	6	5.9	2	10
	Receivable turnover	5.9	4	3.9	2	10
	Total assets turnover	1.19	1	0.99	0.6	5

of Table 1 were also obtained. Based on these ten fuzzy linguistic representations and based on the actual data of a given enterprise, the credit rating of this enterprise can be obtained.

Suppose the ratings of an enterprise based on the ratios of Table 1 for each sub-term are given such as the examples listed in Table 2, then the horizontal coordinates for each ratio can be obtained from the ten figures. Let these horizontal coordinates be represented by x_i, with $i = 1, 2, \ldots, 10$. According to Figure 1, each horizontal coordinate gives a maximum of two different membership functions corresponding to two different grades. For generality, let these membership functions be represented by

$$\mu_j(x_i), \quad i = 1, 2, \ldots, 10, \quad j = 1, 2, \ldots, 5$$

where i and j represent the number of horizontal coordinates and the number of different grades, respectively. Thus, the rating of each ratio—or a given value of x_i—of a given enterprise can be obtained by:

$$\sum_{j=1}^{5}(x_i)\{\mu_j(x_i)\}, \quad i = 1, 2, \ldots, 10 \tag{4}$$

where, from Figure 1, we can see that only one or a maximum of two membership functions, $\mu_j(x_i)$, can be different from zero. The final aggregated rating for this given enterprise can now be obtained as:

$$\sum_{i=1}^{10}\sum_{j=1}^{5}(x_i)\{\mu_j(x_i)\} \tag{5}$$

Even though the above approach is very simplified, it is still a much better representation of the overall rating than the traditional approach.

5. Comparative numerical examples

Numerical samples from five different enterprises are listed in Table 2. Using equation (5), the overall credit ratings for these enterprises are obtained and listed in Table 3. For comparison purposes, the results obtained by the traditional approach are also listed in Table 3. From Table 2, we can see that the differences between Samples 2 and 3 are very small while the differences between Samples 1 and 2 are fairly large. But, the results obtained by the traditional approach show the opposite is true (see Table 3), where, Samples 1 and 2 have the same overall score of 43 while Sample 3 has a score of 33. As can be seen from Table 3, the proposed approach obtained a much more reasonable overall score.

For comparison purposes, the approach of Su and Chen (1980) are approximately summarized. Their approach is based on the method of ranking of fuzzy numbers proposed by Chen (1985), where the concept of maximizing and minimizing sets are used. Let $A_j = (c_j, a_j, d_j)$, $j = 1, 2, \ldots, n$, be n triangular fuzzy numbers, then, the utility of each fuzzy number is given by:

$$U_T(A_j) = \frac{1}{2}\left\{\frac{d_j - x_{min}}{x_{max} - x_{min} - a_j + d_j} + \frac{x_{max} - c_j}{x_{max} - x_{min} + a_j - c_j}\right\}, j = 1, 2, \ldots, n \quad (6)$$

where $x_{max} = \max(d_1, d_2, \ldots, d_n)$ and $x_{min} = \min(c_1, c_2, \ldots, c_n)$.

Instead of the five grades, Su and Chen defined the following four fuzzy triangular numbers, A_j, $j = 1, 2, 3, 4$ (see Figure 2). Let p_j, $j = 1, 2, 3, 4$ be the horizontal coordinates—

Table 3. Comparison between the proposed and traditional approaches

Samples	1	2	3	4	5
Proposed approach	45	38	37.5	13.5	25.5
Traditional approach	43	43	33	17	29

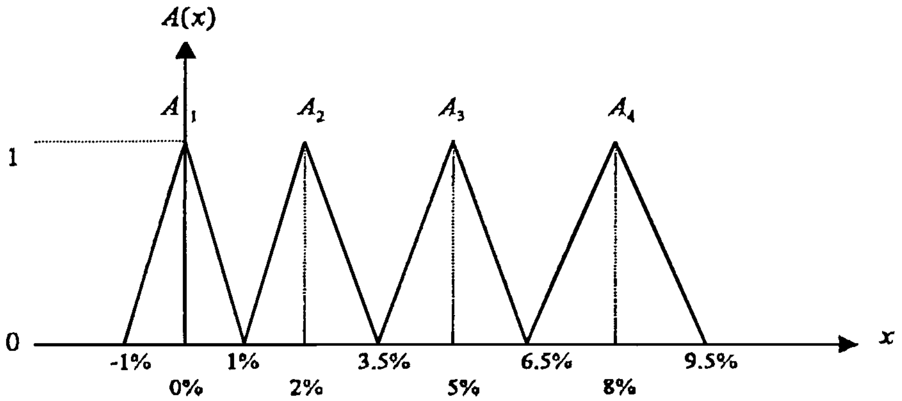

Figure 2. Fuzzy representation of Su and Chen.

Table 4. Comparison with the approach of Su and Chen

Samples	4	5
Proposed approach	13.5	25.5
Su and Chen	44.82	41.87

or the boundaries between the five grades listed in Table 1, then we have:

$$p_1 = 0\%, \qquad p_2 = 2\%, \qquad p_3 = 5\%, \qquad p_4 = 8\%$$

Thus, the four fuzzy numbers can be represented as:

$$A_j = (c_j, p_j, d_j), \quad j = 1, 2, 3, 4$$

where $d_j = c_{j+1} = (p_j + p_{j+1})/2$ for $j = 1, 2, 3$, $c_1 = 2p_1 - d_1$, and $d_4 = 2p_4 - c_4$.

With a given set of actual ratio values as those shown in Table 2, we can obtain the four fuzzy evaluation values, C_j, $j = 1, 2, 3, 4$, as:

$$C_j = \sum_{i=1}^{10} A_{ij} = \left(\sum_{i=1}^{10} c_{ij}, \sum_{i=1}^{10} p_{ij}, \sum_{i=1}^{10} d_{ij} \right), \quad j = 1, 2, 3, 4 \tag{7}$$

where j and i represent the four fuzzy numbers and the ten sampled data, respectively. Based on the four fuzzy evaluation numbers and the utility function, Su and Chen (1980) obtained the total utility. The final score was obtained by multiplying the total utility by the factor (0.5×100).

The results obtained by using the approach of Su and Chen are compared with the proposed approach in Table 4. From Table 2, we know that all the data in Sample 5 are better than those in Sample 4 except for the first two data points, which are the same. But, a better score was obtained for Sample 4 by the approach of Su and Chen (see Table 4). The proposed approach obtained a more reasonable result.

It appears that the approach of Su and Chen cannot distinguish between cost and profit data. For example, debt-to-equity ratio is cost, where the lower the value the more desirable the result.

6. Discussions

At least two aspects need to be further considered, the aggregation of the different sub-terms and different categories and the relative weights or relative importance of these sub-terms and categories. Since fuzzy number is a set, it is not simple to aggregate the different sets, which are usually partial order or even conflicting. Various approaches have been proposed to rank and to aggregate fuzzy numbers (Chang and Lee, 1994; Zhu and Lee, 1992). The approaches can be classified into compensatory and noncompensatory aggregations (Lee and Li, 1993). Theoretically, the noncompensatory max-min approach is most reasonable according to the possibility theory. However, in practice, noncompensatory approach frequently ignores certain important aspects. Another consideration is the computation involved in ranking or aggregation. Some approaches resulting in nonlinear operations, which are much more difficult to solve. In general, linear operation is preferred.

There are other well-known approaches for credit rating such as the use of statistical analysis, data envelopment analysis (Charnes and Cooper, 1985), etc. All these approaches are based on the traditional or "crisp" concept, which does not have the flexibility for treating linguistic expressions. It should be emphasized that linguistic expressions are not only vague but also subjective. For example, the credit rating of an enterprise is frequently described as *very good, good, average, bad* and *very bad*. It would be impossible to judge the credit of an enterprise as *good* or other grades completely objectively. It is true that if enough data are available, some kind of objective results can be obtained. However, the different grades based on these objective results must constantly change with time and with the economic conditions. In fact, it even changes with the current state of the mind. For example, during difficult economic times, a fairly bad credit rating may be considered good because the bank is anxious to lend money out. Another advantage is that fuzzy set represents the vague or linguistic system as it is, it is neither overly accurate nor overly simplified representation.

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