2(b) É suficiente de encontrar um conjunto $\mathcal{B}$ L.I de 4 matrizes (2×2) tal que este conjunto $\mathcal{B} \supseteq \mathcal{C}$.

porque $\dim(\operatorname{Span}(\mathcal{B})) = 4 = \dim(\mathbb{R}^{2\times 2})$

e daí $\operatorname{Span}(\mathcal{B}) = \mathbb{R}^{2\times 2}$

o que implica que $\mathcal{B}$ é base de $\mathbb{R}^{2\times 2}$.

Considere

$$\alpha_1 \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix} + \alpha_2 \begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix} + \alpha_3 \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} + \alpha_4 \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = O$$

onde $O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$$\Leftrightarrow \begin{pmatrix} 1 & 1 & 0 & 0 \\ 2 & -2 & 0 & 0 \\ 3 & 3 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

matriz aumentada

$$\left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 2 & -2 & 0 & 0 & 0 \\ 3 & 3 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right) \rightsquigarrow \left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 2 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right)$$

$$\rightsquigarrow \left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right)$$

então

$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_3 = 0$

$\mathcal{B} = \mathcal{C} \cup \left\{ \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$ é base de $\mathbb{R}^{2\times 2}$

q.e.d