

Implementation of Absorbing Boundary Conditions in an Adaptive Wavelet Code for Electromagnetic Wave Propagation in Inhomogeneous Media

Margarete O. Domingues*, Paulo J. S. G. Ferreira[†], Sônia M. Gomes[‡]
Anamaria Gomide[§], Andrielber S. Oliveira[¶], José R. Pereira^{||} and Pedro Pinho^{**}

Abstract

The main purpose of the present paper is to describe new developments to incorporate arbitrary inhomogeneous media and absorbing boundary conditions of Berenger Perfectly Matched Layer type for finite difference simulations of transient electromagnetic waves in adaptive grids. The principle is to represent the fields in the space domain by non uniform samples chosen from uniform meshes, which are dynamically updated as time evolves. For the organization of the discrete information, interpolating wavelets are used to indicate where a fine spatial resolution is needed, by a thresholding procedure that chooses those points where significant wavelet coefficients occur. The method leads to sparse grids that adapt to the local smoothness of the fields and track their evolution over time, with substantial gain in computational resources. illustrative numerical simulations show that the solutions produced by the updated adaptive code and by the finite difference scheme in the uniform grid are in good agreement.

1 Introduction

In this paper, we present bidimensional high order adaptive finite difference algorithms for the transient simulation of electromagnetic wave propagation. The techniques fall into the category

*Instituto Nacional de Pesquisas Espaciais, LAC/CTE. Caixa Postal 515. 12245-970 - São José dos Campos, SP, Brazil. mo.domingues@lac.inpe.br

[†]Signal Processing Lab, DET/IEETA, University of Aveiro, Portugal. pjf@ieeta.pt

[‡]Universidade Estadual de Campinas, IMECC. Caixa Postal 6065. 13083-970 Campinas SP, Brazil. soniag@ime.unicamp.br

[§]Universidade Estadual de Campinas, IC. Caixa Postal 6176, 130874-971 - Campinas, SP, Brazil. anamaria@ic.unicamp.br

[¶]Universidade Estadual de Campinas, IMECC. Caixa Postal 6065. 13083-970 Campinas SP, Brazil. andriel@ime.unicamp.br

^{||}Instituto de Telecomunicações/Universidade de Aveiro, Portugal. jrp@det.ua.pt

^{**}Instituto Superior de Engenharia de Lisboa DEETC/Instituto de Telecomunicações. Rua Conselheiro Emidio Navarro, 1, 1950-062 - Lisboa, Portugal. ppinho@deetc.isel.ipl.pt

of multiresolution methods, which have been successfully applied in the definition of adaptive strategies for different forms of discretizations of partial differential equations. Precisely, the focus is on the Sparse Point Representation (SPR) method [1], which is employed here for adaptive simulations of the TE_z (transverse electric) mode Maxwell's equations.

The SPR method is an adaptive finite difference strategy for numerical solution of evolution partial differential equations. The principle is to represent the fields in the space domain by non uniform samples chosen from uniform meshes, which are dynamically updated as time evolves. For the organization of the discrete information, interpolating wavelets are used to indicate where a fine spatial resolution is needed, by a thresholding procedure that chooses those points where significant wavelet coefficients occur. The computation of partial derivatives of a function from the information of its SPR representation is performed in two steps. The first one is a refinement procedure to extend the SPR representation by the inclusion of new interpolated point values in a security zone. Then, for points in the refined grid, such derivatives are approximated by uniform finite differences, using step size proportional to each point local scale. Eventually, some required neighboring stencils are not present in the grid. In such case, the corresponding missing point values are approximated from coarser scales. As indicated by earlier simulation results [2, 3], the method leads to sparse grids that adapt to the local smoothness of the fields and track their evolution over time, with substantial gain in computational resources. Since the SPR grids are dynamically adapted to the fields, there is no reduction of the stability limit and, depending on the choice of threshold parameters, the level of accuracy of the finite difference scheme of the regular grid at the finest level of resolution can be preserved.

The previous version of our 2D SPR code [2] is meant for electromagnetic wave propagation in homogeneous medium, and treats the fields at the outer boundary of the computational domain by only imposing PEC conditions. The main purpose of the present paper is to describe new developments to incorporate arbitrary inhomogeneous media and absorbing boundary conditions of Berenger Perfectly Matched Layer (PML) type [4] for SPR simulations of the TE mode Maxwell's equations.

Unlike other wavelet based methods, as the W-MRTD method [5], nonlinear operations (as products or squares, for example) are easily implemented in the SPR method, since the data are samples of the fields. For the same reasons, enforcing boundary conditions in the SPR scheme is a simpler matter. The concern that we could have in the incorporation of inhomogeneous media and PML condition in the SPR code is with respect to the use of very irregular grids and the computation of SPR derivatives, using local step size, in the vicinity of sharp interfaces. However, as shown by some illustrative numerical simulations, the solutions produced by the current fourth order scheme and a standard FDTD [4] algorithm are in good agreement. As expected, the adaptive simulations introduce some perturbation in the presence of inhomogeneities in the medium, but they are proportional to the magnitude of the threshold parameter.

The sections of the paper are organized as follows. Section 2 gives a brief overview of the SPR method. In Section 3, we present the PML set of equations. The finite difference models for these equations, which are used in the operational part of the SPR method, are described in Section 4. In Section 5, we discuss the results of some numerical simulations in terms of accuracy and CPU performance. Our concluding remarks are included in Section 6.

2 A brief overview of the SPR Method

The method combines two sets of numerical tools. For the discretization of differential operators, the background context is of an accurate and stable finite difference scheme in uniform grids, which we refer as the reference scheme. The other context is of interpolating wavelet analysis where the data is organized by levels of resolution $0 \leq \ell \leq j$, j being the finest scale level. In such multilevel context, the key role is played by interpolating subdivision schemes, and wavelet coefficients are precisely the interpolation errors at the new points occurring when going from one level of discretization to the next finer one. Grid adaptation naturally occurs by choosing selecting the points where wavelet coefficients have absolute values greater than a given threshold.

For the solution of transient partial differential equations by the SPR method one must adapt the multilevel organization from one time step to the next one following some basic stages. For more details of the procedures we refer to [1, 2, 3]

1. At a time instance t^n , the numerical solution is represented in an adapted sparse grid Γ^n . Γ^n is expected to be coarse where the solution is smooth, and refined in regions of sharp variations.
2. An extended grid Γ^{n+} is formed by the inclusion of new points in a security zone, where the solution is interpolated from coarser scales by means of the interpolating subdivision scheme. This refinement procedure prevents possible movement or steepness of the solution between time steps.
3. At each point of Γ^{n+} , the reference finite difference scheme is applied to compute the solution at the next time step t^{n+1} . However, the spatial step sizes in each direction are proportional to each point local scales in the corresponding direction using step size that can be made proportional to the point local scale. Here the interpolating subdivision scheme is used again, when some required stencil points are not present in the extended SPR representation of the numerical solution at t^n .
4. Finally, a thresholding operation (i.e., coarsening) is applied to form Γ^{n+1} by eliminating from Γ^{n+} those points that are unnecessary for an accurate representation of the solution at t^{n+1} .

3 PML Model for TE Mode Maxwell's Equations

We consider the electrical and magnetic fields $\mathbf{E} = (E^x, E^y, 0)$ and $\mathbf{H} = (0, 0, H^z)$ in the (x, y) plane, without variation in the z direction. In a medium with electric conductivity σ , magnetic conductivity σ^* , and without source term, the TE_z mode system reads [4]

$$\begin{aligned}\mu \frac{\partial H^z}{\partial t} + \sigma^* H^z &= \frac{\partial E^x}{\partial y} - \frac{\partial E^y}{\partial x}, \\ \epsilon \frac{\partial E^x}{\partial t} + \sigma E^x &= \frac{\partial H^z}{\partial y}, \\ \epsilon \frac{\partial E^y}{\partial t} + \sigma E^y &= -\frac{\partial H^z}{\partial x},\end{aligned}$$

where ϵ is the electrical permittivity and μ is the magnetic permeability. In order to avoid wave reflection in vacuum-medium interfaces, the next condition must be satisfied.

$$\frac{\sigma}{\epsilon} = \frac{\sigma^*}{\mu} \quad (1)$$

For the truncation of the infinite domain, we restrict the problem to the computational region $R = [0, 1] \times [0, 1]$. We assume PEC boundary conditions, meaning that tangential components of the electrical field vanish at the boundary. This means that $E^y(0, y, t) = E^y(1, y, t) = 0$ and $E^x(x, 0, t) = E^x(x, 1, t) = 0$ for all $0 \leq x, y \leq 1$ and $t > 0$.

3.1 PML Medium

To reduce outer boundary reflections, the computational domain is surrounded by PML zones, as shown in Figure 1. The Berenger PML model is an hypothetical medium based on a mathematical formulation. The idea is to split magnetic field into two components $H^{z,x}$ and $H^{z,y}$ and the new set of equations read

$$\begin{aligned}\mu \frac{\partial}{\partial t}(H^{z,x}) + \sigma^{x*} H^{z,x} &= -\frac{\partial E^y}{\partial x} \\ \mu \frac{\partial}{\partial t}(H^{z,y}) + \sigma^{y*} H^{z,y} &= \frac{\partial E^x}{\partial y} \\ \epsilon \frac{\partial(E^x)}{\partial t} + \sigma^x E^x &= \frac{\partial(H^{z,x} + H^{z,y})}{\partial y}, \\ \epsilon \frac{\partial(E^y)}{\partial t} + \sigma^y E^y &= -\frac{\partial(H^{z,x} + H^{z,y})}{\partial x}\end{aligned}$$

Here, for $r \in \{x, y\}$, σ^r and σ^{r*} denote magnetic conductivities. Outside the PML region,

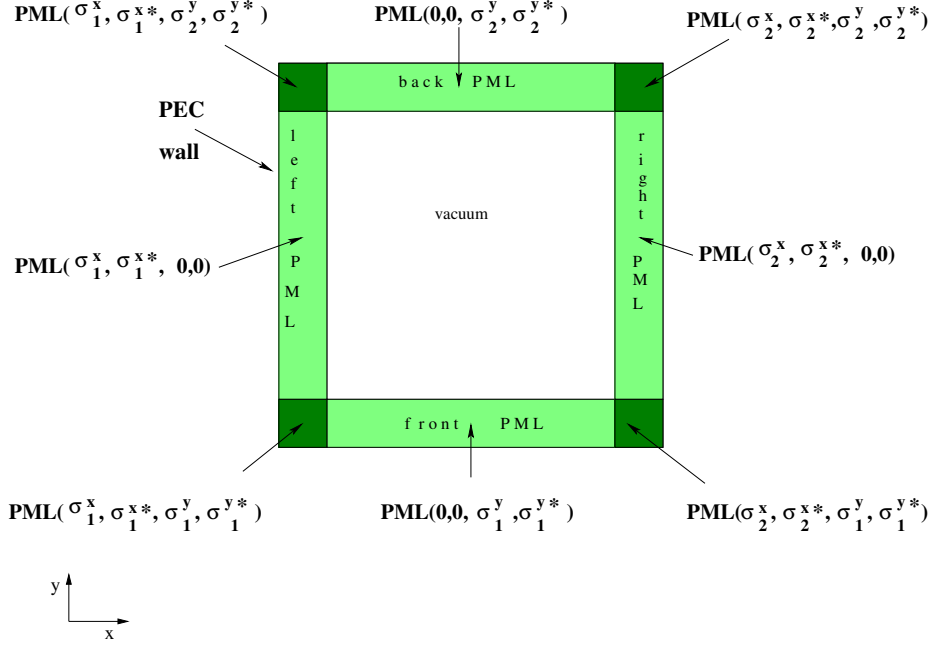


Figure 1: Berenger PML absorbing layer

$\sigma^{x*} = \sigma^{y*} = \sigma^*$, and $\sigma^x = \sigma^y = \sigma$. Consequently the first two equations can be merged and the system reduces to the usual TE_z mode equations for E^x , E^y and $H^z = H^{z,x} + H^{z,y}$.

Typically, in the left, right and corner layers, σ^x and σ^{x*} are chosen as functions of x only, and they are equal to zero in the front and back layers. Similarly, in the front, back and corner layers σ^y and σ^{y*} are function of y only, and they equal to zero in the left and right layers. The conductivities should be functions increasing from zero at the vacuum-layer interface to a maximum value σ_{max} at the outer side of the layer. For instance, we may consider $\sigma_1^x(x) = \sigma(\delta - x)$, $\sigma_2^x(x) = \sigma(-1 + \delta + x)$, $\sigma_1^y(y) = \sigma(\delta - y)$ and $\sigma_2^y(y) = \sigma(-1 + \delta + y)$ in terms of the basic function [4]

$$\sigma(\rho) = \sigma_{max} \left(\frac{\rho}{\delta} \right)^q,$$

where δ is the layer width, and q is an integer that controls the conductivity profile, hereinafter referred as conductivity power. Usually, q is taken equal to 2.

4 Finite Difference Expressions for PML Equations

For the definition of the finite difference model of reference to be used in the SPR scheme, we consider finite difference models where the fields are sampled at the same uniform grid

$$\begin{aligned}
H_{k,l}^{zx,n} &= H^{zx}(k\Delta x, l\Delta y, \Delta t), \\
H_{k,l}^{zy,n} &= H^{zy}(k\Delta x, l\Delta y, n\Delta t), \\
E_{k,l}^{x,n} &= E^x(k\Delta x, l\Delta y, n\Delta t), \\
E_{k,l}^{y,n} &= E^y(k\Delta x, l\Delta y, n\Delta t).
\end{aligned}$$

If on R we consider a grid with uniform spacing in all directions $\Delta x = \Delta y = 1/N$, the discrete values $E_{k,l}^{x,n}$, $E_{k,l}^{y,n}$, $H_{k,l}^{zx,n}$ and $H_{k,l}^{zy,n}$ have index range in $0 \leq k, l \leq N$. The PEC boundary condition implies that $E_{0,l}^{y,n} = E_{N,l}^{y,n} = E_{k,0}^{x,n} = E_{k,N}^{x,n} = 0$. Since some extra virtual finite difference stencils outside the boundary are required, we use the symmetry properties $E_{-k,l}^{y,n} = -E_{k,l}^{y,n}$, $E_{N+k,l}^{y,n} = -E_{N-k,l}^{y,n}$, $E_{k,-l}^{x,n} = -E_{k,l}^{x,n}$ and $E_{k,N+l}^{x,n} = -E_{k,N-l}^{x,n}$ for the electrical field, and $H_{-k,l}^{zr,n} = H_{k,l}^{zr,n}$, $H_{N+k,l}^{zr,n} = H_{N-k,l}^{zr,n}$, $H_{k,-l}^{zr,n} = H_{k,l}^{zr,n}$, and $H_{k,N+l}^{zr,n} = H_{k,N-l}^{zr,n}$, $r \in \{x, y\}$ for the magnetic field.

For partial derivatives in space we introduce the finite difference operators

$$(\partial^x v)_{k,l} = \frac{1}{\Delta x} \sum_{k'} v_{k',l} \beta(k - k') \quad (\partial^y u)_{k,l} = \frac{1}{\Delta y} \sum_{l'} u_{k,l'} \beta(l - l'),$$

and we consider a central the fourth order scheme where the non vanishing values for $\beta(k)$, $k > 0$ are $\beta(1) = -\frac{2}{3}$, $\beta(2) = \frac{1}{2}$, $\beta(-k) = -\beta(k)$.

4.1 Finite Difference Expression for PML Equations

Applying the finite difference scheme in space to the set of equations of the PML medium, we obtain a ODE system of the form

$$\frac{d}{dt} V_{k,l} + \Sigma_{k,l} V_{k,l} = [GV]_{k,l},$$

where $V(t) = [V_{k,l}(t)]$ denote the fields samples at time t , $\Sigma_{k,l}$ represent the point wise values of the electric or magnetic conductivities multiplying the fields components, and G is the matrix representing the discretization of the partial differential operators. Integrating in time from t^n to t^{n+1} , and using the notation $V^n = V(t^n)$, we obtain

$$\begin{aligned}
V_{k,l}^n &= e^{-\Sigma_{k,l}\Delta t} V_{k,l}^n + e^{-\Sigma_{k,l}t^{n+1}} \left[\int_{t^n}^{t^{n+1}} [GV(\tau)]_{k,l} e^{\Sigma_{k,l}\tau} d\tau \right] \\
&\approx e^{-\Sigma_{k,l}\Delta t} V_{k,l}^n + [GV(\bar{t})]_{k,l} \frac{1 - e^{-\Sigma_{k,l}\Delta t}}{\Sigma_{k,l}}
\end{aligned}$$

When $\Sigma_{k,l} = 0$, we assume in this expression that $\frac{1-e^{-\Sigma_{k,l}\Delta t}}{\Sigma_{k,l}} := \Delta t$. If we choose $\bar{t} = t^n$ for the electric fields and $\bar{t} = t^{n+1}$ for the components of the magnetic field, we get the set of equations

$$H_{k,l}^{zx,n+1} = Dahx H_{k,l}^{zx,n} - Dbhx_{k,l} \left[\frac{1}{\Delta x} (\partial^x E^{y,n})_{k,l} \right], \quad (2)$$

$$H_{k,l}^{zy,n+1} = Dahy_{k,l} H_{k,l}^{zy,n} + Dbhy_{k,l} \left[\frac{1}{\Delta y} (\partial^y E^{x,n})_{k,l} \right], \quad (3)$$

$$E_{k,l}^{x,n+1} = Caex_{k,l} E_{k,l}^{x,n} + Cbex_{k,l} \left[\frac{1}{\Delta y} (\partial^y H^{zx,n+1} + \partial^y H^{zy,n+1})_{k,l} \right], \quad (4)$$

$$E_{k,l}^{y,n+1} = Caey_{k,l} E_{k,l}^{y,n} - Cbey_{k,l} \left[\frac{1}{\Delta x} (\partial^x H^{zx,n+1} + \partial^x H^{zy,n+1})_{k,l} \right], \quad (5)$$

where, for $r \in \{x, y\}$, the coefficients are given by the formulae

$$\begin{aligned} Caer_{k,l} &= e^{-\frac{\sigma_{k,l}^r \Delta t}{\epsilon}} & Cber_{k,l} &= \frac{1 - e^{-\frac{\sigma_{k,l}^r \Delta t}{\epsilon}}}{\sigma_{k,l}^r} \\ Dahr_{k,l} &= e^{-\frac{\sigma_{k,l}^{r*} \Delta t}{\mu}} & Dbhr_{k,l} &= \frac{1 - e^{-\frac{\sigma_{k,l}^{r*} \Delta t}{\mu}}}{\sigma_{k,l}^{r*}} \end{aligned}$$

In the vacuum medium $\sigma^r = \sigma^{r*} = 0$, and without PML treatment, the model (2-5) reduces to the one analyzed in [3]. In such case, when the grid has the same spacing in both directions $\Delta = \Delta x = \Delta y$, the Von Neumann stability analysis gives the stability limit $CFL = \frac{\Delta t}{\sqrt{\mu\epsilon}\Delta} \leq 1.0306$.

5 Numerical Results

In the present section we analyze the performance of the SPR method for the numerical solution of the TE_z mode system of Maxwell's equations. In the operational part, we shall use the finite difference scheme described in Section 4. Our purpose is to analyze the effects of the presence of the PML absorbing boundary condition and inhomogeneities in the medium.

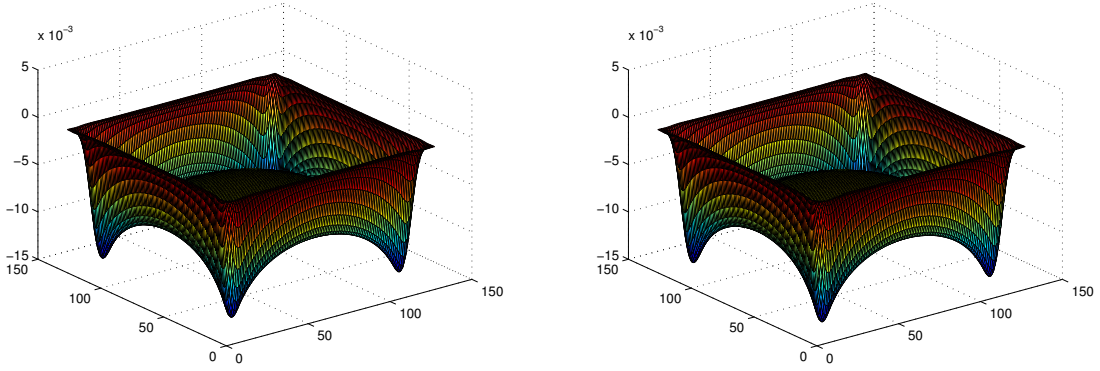


Figure 2: The magnetic fields $t = 2.74 \text{ ns}$ computed with the FDTD (left) and the fourth order scheme (right) in a uniform grid with $j = 7$

We restrict the problem to the computational domain $R = [0, 1] \times [0, 1]$ where the uniform grid X^j at the finest level of resolution has the same spacing in both directions $\Delta x = \Delta y = 2^{-j}$, for different values for j . Due to the stability limitation of the FDTD scheme, and for comparison purpose, in all simulations we use $CFL = 0.7$.

We assume PEC+PML boundary conditions, as described in Section 3. All simulations are for a initial condition where the fields at $t = 0$ are assumed to be $E^x(x, y) = E^y(x, y) = 0$ and the magnetic fields are the Gaussian pulse $H^{zx}(x, y) = H^{zy}(x, y) = \frac{1}{2}e^{-2500(x-\frac{1}{2})^2}e^{-2500(y-\frac{1}{2})^2}$.

In the SPR simulations, we use the cubic interpolating subdivision scheme for the multiresolution algorithms. If ε_H is the threshold for the wavelet coefficients of the magnetic field, then the threshold for the electrical fields is $\varepsilon_E = 100 \times \varepsilon_H$. Since the time evolution of the electrical fields requires the partial derivatives of the magnetic field, and vice versa, the time stepping is performed on a nonuniform grid formed by the union of the extended SPR grids of the three fields. In all simulations, the coarsest grid is X^3 , having 9×9 points.

5.1 Study of the Absorbing Boundary Layer Effect

To analyze the effect of the presence of a PML absorbing boundary condition, first we consider a vacuum medium surrounded by a PML layer of width $\delta = 2^{-3}$, corresponding to one cell of the coarsest level, with $\sigma_{max} = -(q + 1) \ln(10^{-7})/2\sqrt{\mu\epsilon}\delta$, and conductivity power $q = 2$. The fields at $t = 0$ are assumed to be $E^x(x, y) = E^y(x, y) = 0$ and $H^{zx}(x, y) = H^{zy}(x, y) = \frac{1}{2}e^{-2500(x-\frac{1}{2})^2}e^{-2500(y-\frac{1}{2})^2}$.

The magnetic fields at $t = 2.74 \text{ ns}$ computed with the FDTD method and the fourth order scheme in a uniform grid with $j = 7$ are shown in Figure 2. The plots in Figure 3 indicates the maximum absolute difference between the magnetic H^z and electrical fields E^x obtained with the FDTD scheme and the present higher order scheme in the uniform grid X^7 , for different

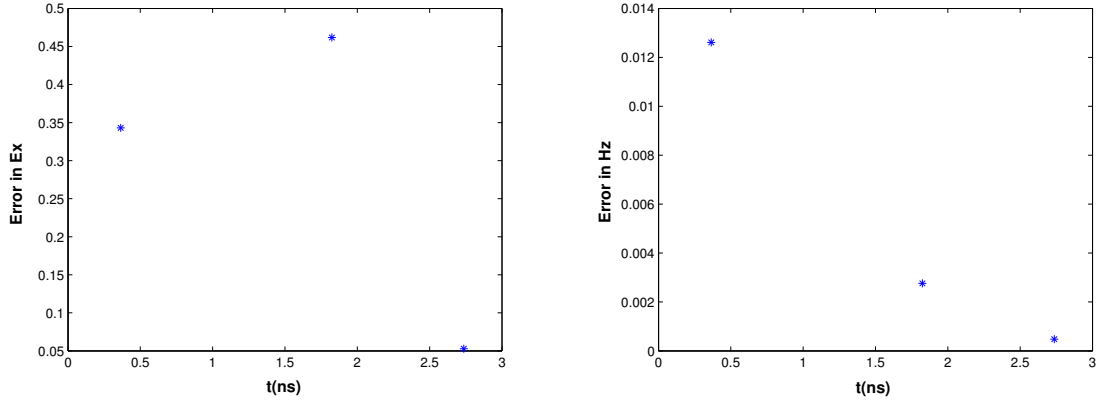


Figure 3: The maximum absolute difference between the fields E^x and $H^{zx} + H^{zy}$ obtained with the FDTD and the fourth order schemes in the uniform grid X^7 , and $CFL = 0.7$.

time instances. These graphics show that both finite difference schemes give similar results in a uniform grid.

To analyse the effect of inclusion of PML boundary layers in combination with SPR adaptivity, Figure 4 present cut profiles at the line $y = 0.5$ of the magnetic fields at time $t = 0.5 \text{ ns}$ computed with the PML code in regular and SPR grids with $\varepsilon_E = 0.1$ and 0.01 (i.e., $\varepsilon_H = 0.001$ and 0.0001). As expected, the adaptive simulations introduce some perturbation, with respect to the solution computed in the finest uniform grid, all over the computational domain. These perturbations are proportional to the magnitude of the threshold parameter.

For a better analysis, the case the plots for $\varepsilon_E = 0.01$ ($\varepsilon_H = 0.0001$) are displayed in Figure 5 with the zoom in into the left PML layer.

5.2 Study of the Inhomogeneous Medium Effect

To illustrate the SPR algorithm in inhomogeneous media, we consider the scenario where a box is placed in the middle of the computational domain. The width of box walls is 2^{-5} , and we set the electrical conductivity $\sigma = 10^9$ there, as indicated in Figure 6.

Figure 7 shows the magnetic field (left side) at $t = 0.5 \text{ ns}$, for $j = 7$, corresponding to the finest uniform grid containing 129×129 points. The plots are for $\varepsilon_H = 0.0001$ (top), 0.001 (middle) and 0.01 (bottom). The corresponding SPR grids are displayed on the right side. Note that the SPR grids become sparser as ε_H increases, and they are concentrated inside the box, where the wave propagation is concentrated.

The curves in Figure 8 describe the time evolution of the number $\#N_{SPR}(\%)$ of points used in the SPR simulations (i.e. the total number of points in the union of the extended SPR grids of the electrical and magnetic fields) as a percentage of the total number of points in the uniform

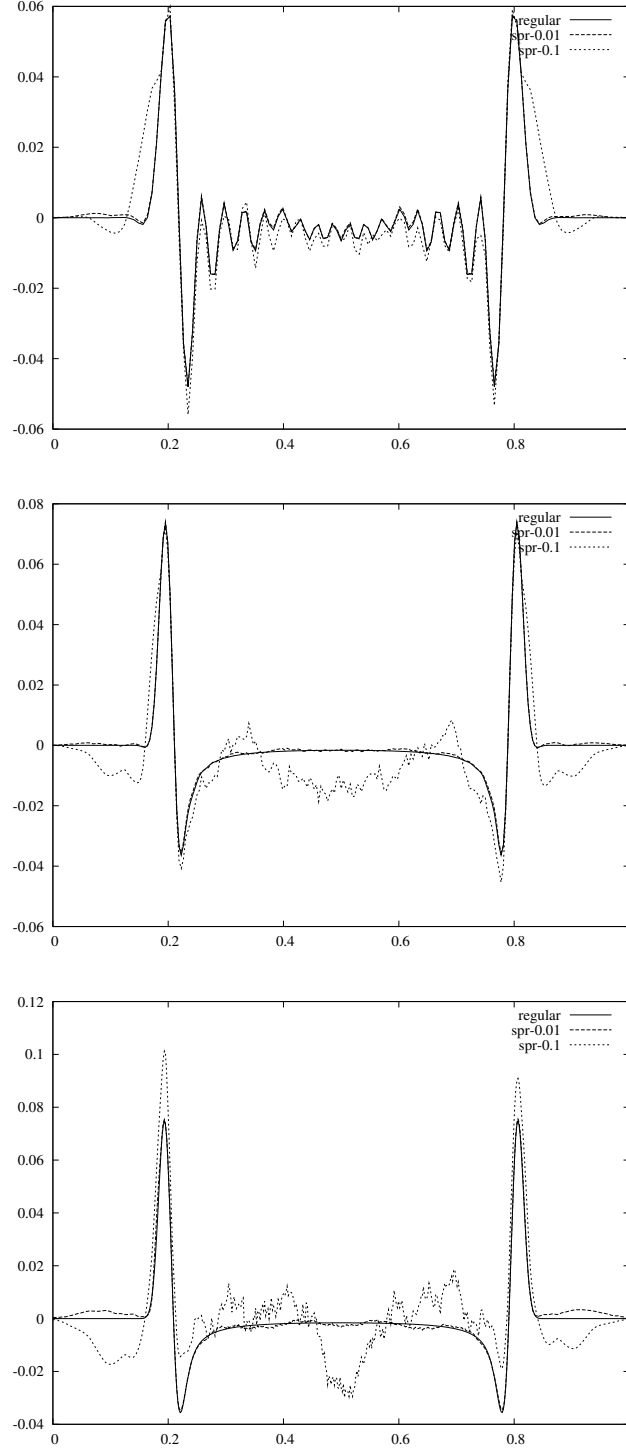


Figure 4: The effect of the thresholding parameter $\varepsilon_E = 0.01$ and $\varepsilon_E = 0.1$ (i.e. $\varepsilon_H = 0.0001$ and $\varepsilon_H = 0.001$): $j = 7$ (top), 8 (middle) and 9 (bottom).

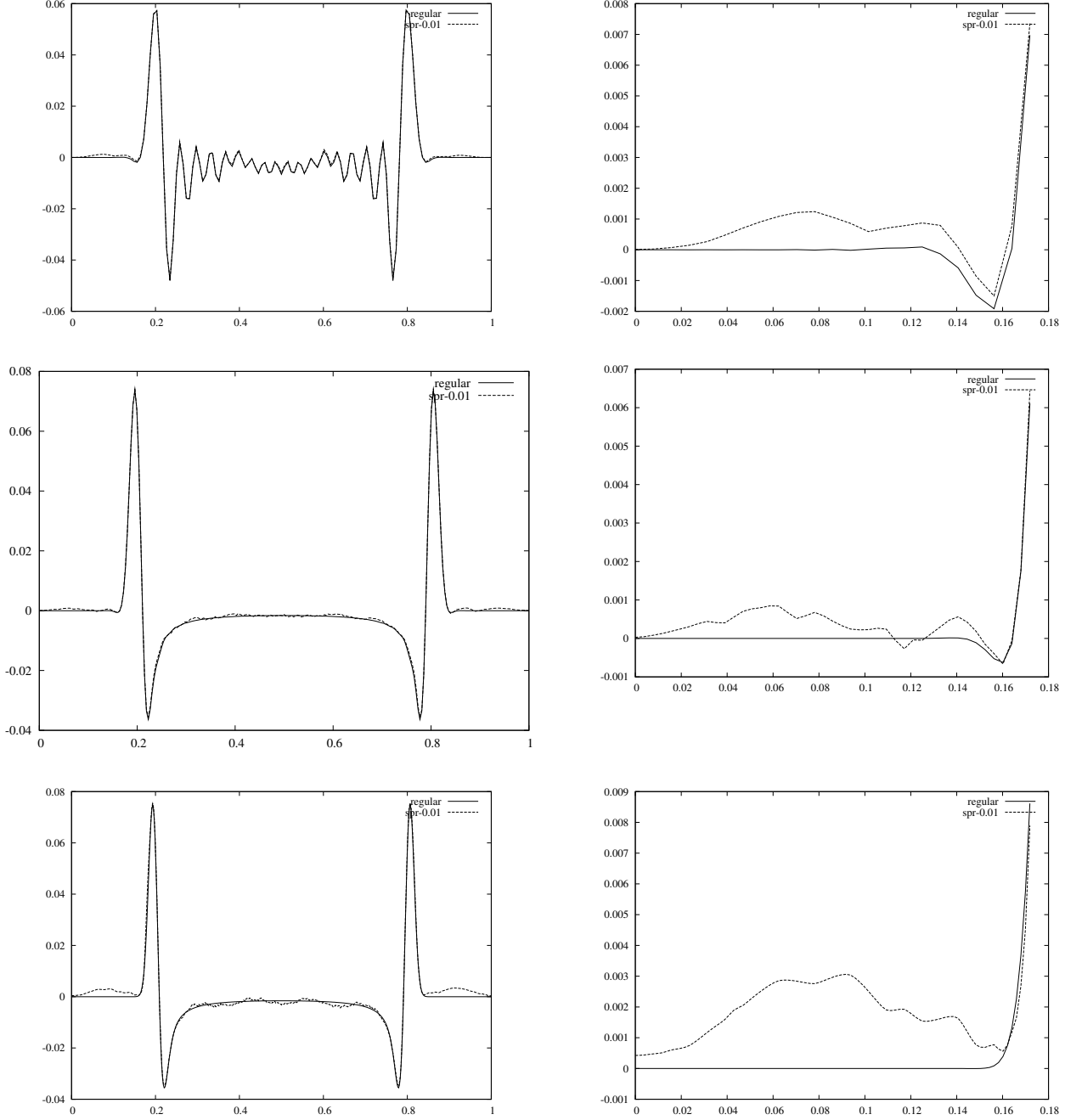


Figure 5: Comparison of the solutions for the PML model in regular and SPR grids: $j = 7$ (top), $j = 8$ (middle) and $j = 9$ (bottom), using $\varepsilon_E = 0.01$ (i.e., $\varepsilon_H = 0.0001$).

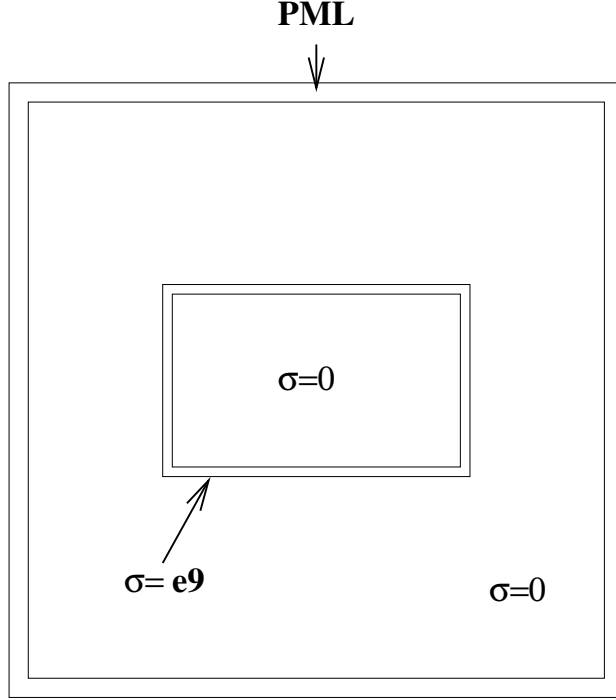


Figure 6: Box geometry

finest grids X^j , $j = 7, 8$, and 9 (corresponding to 129×129 , 257×257 , and 513×513 points, respectively). For this example, using the threshold $\varepsilon_H = 10^{-2}$, (top side) the percentage of points in the SPR grids, at the final time $t = 0.5 \text{ ns}$, ranges from 12% ($j = 7$) down to less than 4.5% ($j = 9$). As expected, the effect of decreasing the threshold parameter is more denser grids. For instance, with $\varepsilon_H = 10^{-3}$ (middle) the percentage of points in the SPR grids at $t = 0.5 \text{ ns}$ reaches 20% with $j = 7$, and it is kept in the range 8% to 10% with $j = 8$ and $j = 9$. We can also observe that for $\varepsilon_H = 10^{-4}$ the discrepancy in the percentage of points in the SPR grids is less sensitive to the number of scale levels, since for such small threshold parameter the SPR grids at $t = 0.5 \text{ ns}$ almost coincide with the coarser uniform grid outside the box and the finest uniform grid inside the box, and some extra points used around the box walls. This behavior is illustrated for the case of $j = 7$ in Figure 7 (top-right side).

The plots in Figure 9 correspond to cuts at the line $y = 0.5$ of the magnetic fields at the final time $t = 0.5 \text{ ns}$ computed with the PML code in regular and SPR grids with $j = 7, 8$ and 9 . These plots show a good agreement between both computations, which is further confirmed by the zoom in into the central region of the computational domain (Figure 9 (right)).

6 Concluding Remarks

In the present paper, we apply the SPR method for the solution of the bidimensional TE_z mode Maxwell's equations. A fourth order finite difference model in non-staggered grids is used

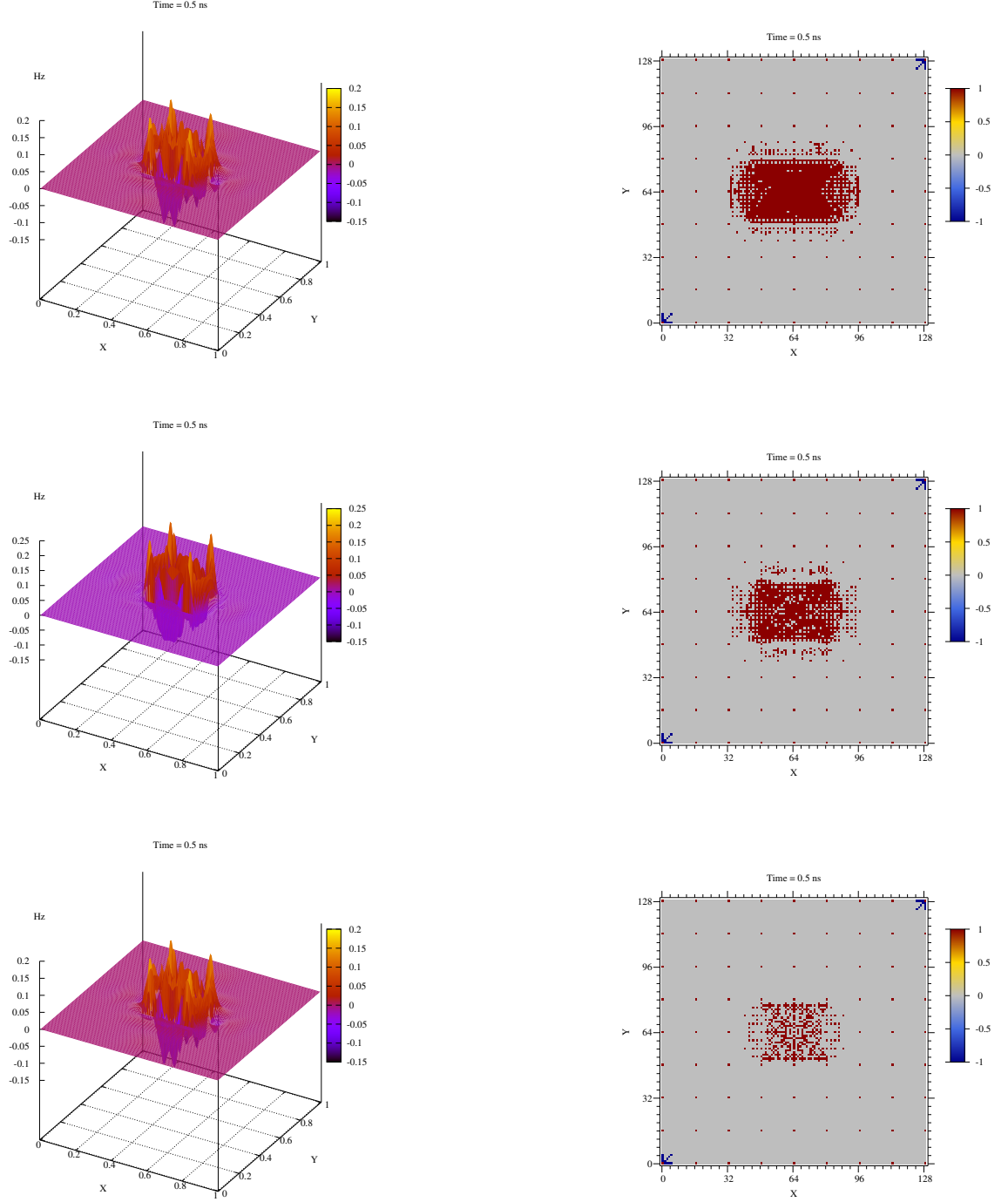


Figure 7: SPR magnetic fields (left) at $t = 0.5 \text{ ns}$, obtained with $j = 7$, and $\epsilon_H = 0.0001$ (top), 0.001(middle) and 0.01(bottom); and the corresponding SPR grids (right).

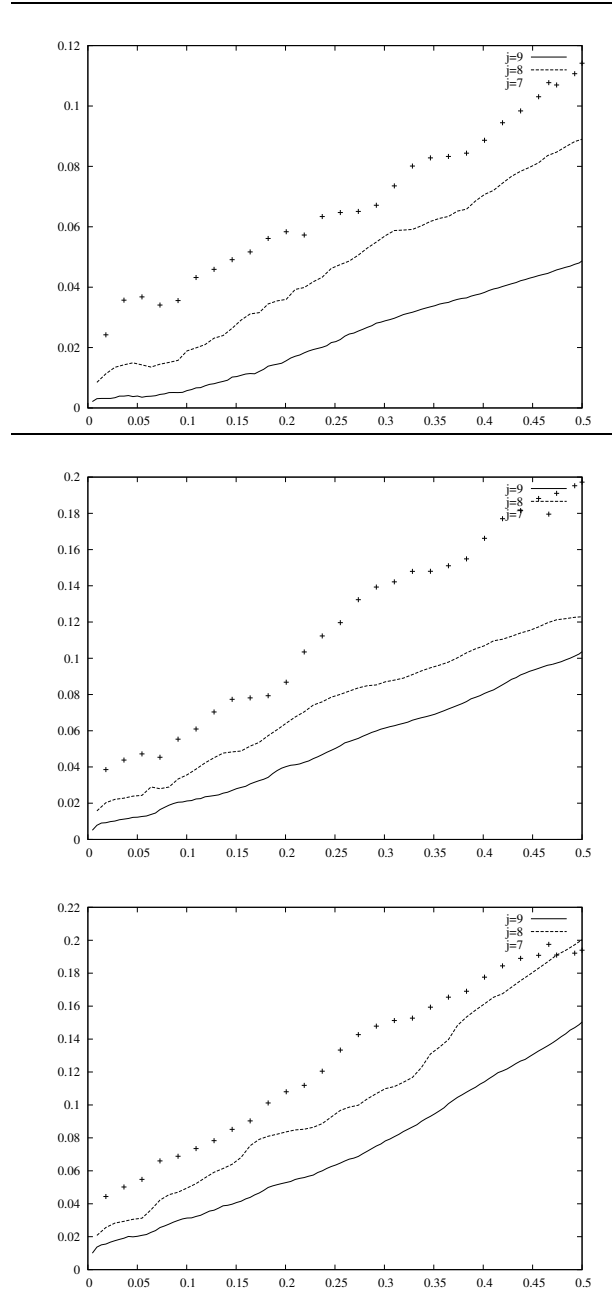


Figure 8: Percentage of grid points used in the SPR simulations; the thresholding parameter effect: $\varepsilon_H = 10^{-2}$ (top), 10^{-3} (middle) and 10^{-4} (bottom).

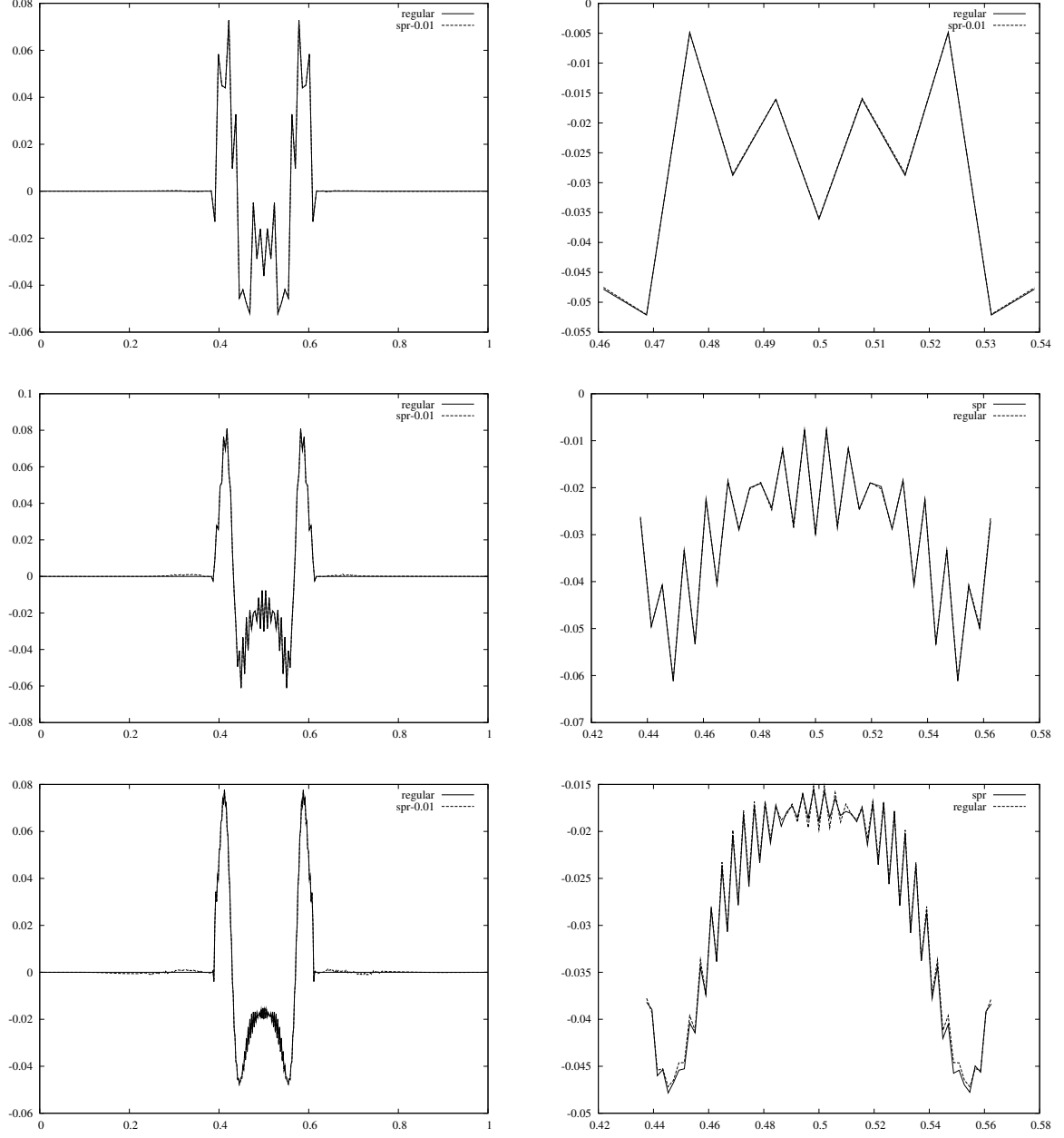


Figure 9: Comparison of the solutions for the PML model in regular and SPR grids: $j = 7$ (top), $j = 8$ (middle) and $j = 9$ (bottom), using $\varepsilon_E = 0.01$.

for the SPR discretization of spatial derivatives. We highlight the incorporation of arbitrary inhomogeneous media and absorbing PML boundary conditions. As expected, the adaptive simulations introduce some perturbation in the presence of inhomogeneities in the medium, but they are proportional to the magnitude of the threshold parameter.

Acknowledgments: The research of this paper is part of a scientific international cooperation between GRICES (Portugal) and CAPES (Brazil) (contract 142/05). This research has also been partially supported by the following Brazilian agencies: CNPq (contract 308632/20063-0) and FAPESP (contract 06/02277-6).

References

- [1] M. HOLMSTRÖM, *Solving hyperbolic PDEs using interpolating wavelets*, SIAM J. Scientific Computing 21 (1999), pp. 405–420.
- [2] P. PINHO, M. O. DOMINGUES, P. J. S. G. FERREIRA, S. M. GOMES, A. GOMIDE AND J. R. PEREIRA, *Interpolating wavelets and adaptive finite difference schemes solving Maxwell's equations: the effects of gridding*. IEEE Transactions in Magnetism 43 (2007), pp. 1013-1022.
- [3] M. O. DOMINGUES, P. J. S. G. FERREIRA, S. M. GOMES, A. GOMIDE, J. R. PEREIRA, AND P. PINHO, *High Order Finite-Difference Schemes for Maxwell's Equations with Adaptive Wavelet Grid Refinement*. Preprint 2007.
- [4] A. TAFLOVE AND S. C. HAGNESS, *Computational Electrodynamics: The Finite-Difference Time-Domain Method*, Norwood, MA, 2005.
- [5] M. KRUMPHOLTZ AND I. B. KATEHI, *MRTD: new time-domain schemes based in multiresolution analysis*. IEEE Transactions on Microwave, Theory and Techniques 44(4): 55-571, 1996.