# MA-111 Cálculo I- 6a Lista (Integração)

6. Calcule a área do conjunto dado.

a) $A = \{(x, y) \in \mathbb{R}^2 \mid 1 \leqslant x \leqslant 2 \text{ e } 0 \leqslant y \leqslant \sqrt{x-1}\}$

b) $A = \{(x, y) \in \mathbb{R}^2 \mid 0 \leqslant x \leqslant 2 \text{ e } 0 \leqslant y \leqslant \dfrac{x}{1+x^2}\}$

c) $A$ é o conjunto do plano limitado pela reta $x = 1$ e pelos gráficos de $y = e^{-2x}$ e $y = e^{-x}$, com $x \geqslant 0$

7. Calcule.

a) $\displaystyle\int_0^1 x\sqrt{x^2 + 3} \cdot dx$

b) $\displaystyle\int_0^1 x(x^2 + 3)^5\, dx$

c) $\displaystyle\int_1^2 x(x^2 - 1)^5\, dx$

d) $\displaystyle\int_0^1 x\sqrt{1 - x^2}\, dx$

e) $\displaystyle\int_{-1}^0 x^2 e^{x^3}\, dx$

f) $\displaystyle\int_0^1 x\sqrt{1 + 2x^2}\, dx$

g) $\displaystyle\int_1^2 \dfrac{3s}{1 + s^2}\, ds$

h) $\displaystyle\int_0^1 \dfrac{1}{1 + 4s}\, ds$

i) $\displaystyle\int_0^3 \dfrac{x}{\sqrt{x + 1}}\, dx$

j) $\displaystyle\int_0^1 \dfrac{s}{\sqrt{s^2 + 1}}\, ds$

l) $\displaystyle\int_0^3 \dfrac{x^2}{\sqrt{x + 1}}\, dx$

m) $\displaystyle\int_0^1 \dfrac{x^2}{(x + 1)^2}\, dx$

n) $\displaystyle\int_{-1}^1 x^3(x^2 + 3)^{10}\, dx$

o) $\displaystyle\int_0^{\sqrt{3}} x^3\sqrt{x^2 + 1}\, dx$

p) $\displaystyle\int_0^{\frac{\pi}{3}} \operatorname{sen} x \cos^2 x\, dx$

q) $\displaystyle\int_0^{\frac{\pi}{6}} \cos x \operatorname{sen}^5 x\, dx$

r) $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \operatorname{sen} x (1 - \cos^2 x)\, dx$

s) $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \operatorname{sen} x \operatorname{sen}^2 x\, dx$

t) $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \operatorname{sen}^3 x\, dx$

u) $\displaystyle\int_0^{\frac{\pi}{6}} \cos^3 x\, dx$

1.

1

10. Calcule

a) $\int \text{tg } x \, dx$

b) $\int \sec^2 x \, dx$

c) $\int \text{tg}^2 x \, dx$

d) $\int \sec x \, dx$

e) $\int \text{tg } 2x \, dx$

f) $\int \sec 3x \, dx$

g) $\int 3^x dx$

h) $\int \dfrac{5}{\sqrt{1-x^2}} \, dx$

i) $\int (5^x + e^{-x}) \, dx$

j) $\int (x + \sec^2 3x) \, dx$

l) $\int (1 + \sec x)^2 dx$

m) $\int \dfrac{\cos x + \sec x}{\cos x} \, dx$

11. a) Determine $\alpha$ e $\beta$ de modo que

$$\text{sen } 6x \cos x = \frac{1}{2}(\text{sen } \alpha x + \text{sen } \beta x)$$

$$\left( Sugest\tilde{a}o: \text{sen } p + \text{sen } q = 2 \text{ sen } \frac{p+q}{2} \cos \frac{p-q}{2}. \right)$$

b) Calcule $\int \text{sen } 6x \cos x \, dx$

12. Calcule

a) $\int \text{sen } 5x \cos x \, dx$

b) $\int \text{sen } 3x \cos 4x \, dx$

c) $\int \text{sen } x \cos 3x \, dx$

d) $\int \text{sen } 3x \cos 3x \, dx$

13. a) Determine $\alpha$ e $\beta$ de modo que

$$\text{sen } 3x \text{ sen } 2x = -\frac{1}{2}(\cos \alpha x - \cos \beta x)$$

$$\left( Sugest\tilde{a}o: \cos p - \cos q = -2 \text{ sen } \frac{p-q}{2} \text{ sen } \frac{p+q}{2}. \right)$$

b) Calcule $\int \text{sen } 3x \text{ sen } 2x \, dx$.

14. a) Determine $\alpha$ e $\beta$ de modo que

$$\cos 5x \cos 2x = \frac{1}{2}(\cos \alpha x + \cos \beta x)$$

b) Calcule $\int \cos 5x \cos 2x \, dx$.

15. Calcule

a) $\int \text{sen } x \text{ sen } 3x \, dx$

b) $\int \text{sen } 2x \text{ sen } 5x \, dx$

c) $\int \text{sen } 3x \cos 2x \, dx$

d) $\int \cos 5x \cos x \, dx$

e) $\int \cos 7x \cos 3x \, dx$

f) $\int \cos 2x \cos 4x \, dx$

2

2.

**4. Calcule**

a) $\int \dfrac{2}{x-3} \, dx$

b) $\int \left( \dfrac{5}{x-1} + \dfrac{2}{x} \right) dx$

c) $\int \dfrac{1}{2x+3} \, dx$

d) $\int \left( x + \dfrac{3}{x-2} \right) dx$

e) $\int \dfrac{x}{x+1} \, dx$

f) $\int \dfrac{x+2}{x-1} \, dx$

g) $\int \dfrac{2x+3}{x+1} \, dx$

h) $\int \dfrac{x^2}{x+1} \, dx$

**5. Suponha** $\alpha, \beta, m$ **e** $n$ **constantes, com** $\alpha \neq \beta$. **Mostre que existem constantes** $A$ **e** $B$ **tais que**

$$\frac{mx+n}{(x-\alpha)(x-\beta)} = \frac{A}{x-\alpha} + \frac{B}{x-\beta}$$

**6. Utilizando o Exerc. 5, calcule**

a) $\int \dfrac{1}{(x+1)(x-1)} \, dx$

b) $\int \dfrac{2x+3}{x(x-2)} \, dx$

c) $\int \dfrac{x}{x^2-4} \, dx$

d) $\int \dfrac{1}{x^2-4} \, dx$

e) $\int \dfrac{5x+3}{x^2-3x+2} \, dx$

f) $\int \dfrac{x+1}{x^2-x-2} \, dx$

g) $\int \dfrac{2}{x^2-5x+6} \, dx$

h) $\int \dfrac{x-3}{x^2+3x+2} \, dx$

**7. Seja** $a \neq 0$ **uma constante. Verifique que**

$$\int \frac{1}{a^2+x^2} \, dx = \frac{1}{a} \text{ arc tg } \frac{x}{a} + k.$$

**8. Calcule**

a) $\int \dfrac{1}{5+x^2} \, dx$

b) $\int \dfrac{2}{4+x^2} \, dx$

c) $\int \dfrac{1}{2+5x^2} \, dx$

d) $\int \dfrac{3}{5+x^2} \, dx$

e) $\int \dfrac{x}{5+x^2} \, dx$

f) $\int \dfrac{3x+2}{1+x^2} \, dx$

g) $\int \dfrac{x-1}{4+x^2} \, dx$

h) $\int \dfrac{2x-3}{1+4x^2} \, dx$

i) $\int \dfrac{1}{1+(x+1)^2} \, dx$

j) $\int \dfrac{1}{x^2+2x+2} \, dx$

l) $\int \dfrac{2}{5+(x+2)^2} \, dx$

m) $\int \dfrac{1}{x^2+4x+8} \, dx$

n) $\int \dfrac{1}{x^2+x+1} \, dx$

o) $\int \dfrac{2}{x^2+2x+2} \, dx$

3.

5. Calcule $\int e^{-st} \operatorname{sen} t \, dt; \; s > 0$ constante.

6. Verifique que para todo natural $n > 1$ e todo real $s > 0$

$$\int t^n e^{-st} \, dt = -\frac{1}{s} \, t^n \, e^{-st} + \frac{n}{s} \int t^{n-1} e^{-st} \, dt.$$

7. Calcule

a) $\int_0^1 x \, e^x \, dx$ 

b) $\int_1^2 \ln x \, dx$

c) $\int_0^{\frac{\pi}{2}} e^x \cos x \, dx$ 

d) $\int_0^x t^2 e^{-st} \, dt \; (s \neq 0)$

8. Sejam $m$ e $n$ dois naturais diferentes de zero. Verifique que

a) $\int_0^1 x^n (1-x)^m \, dx = \dfrac{m}{n+1} \int_0^1 x^{n+1} (1-x)^{m-1} \, dx$

b) $\int_0^1 x^n (1-x)^m \, dx = \dfrac{n! \, m!}{(m+n+1)!}$

4.

1. Calcule.

a) $\int \sqrt{1 - 4x^2} \, dx$ 

b) $\int \dfrac{1}{\sqrt{4 - x^2}} \, dx$

c) $\int \dfrac{1}{\sqrt{4 + x^2}} \, dx$ 

d) $\int \dfrac{1}{4 + x^2} \, dx$

e) $\int \dfrac{x}{\sqrt{1 - x^2}} \, dx$ 

f) $\int \sqrt{3 - 4x^2} \, dx$

g) $\int \dfrac{x^2}{\sqrt{1 - x^2}} \, dx$ 

h) $\int x^2 \sqrt{1 - x^2} \, dx$

i) $\int \dfrac{1}{x \sqrt{1 + x^2}} \, dx$ 

j) $\int \sqrt{9 - (x-1)^2} \, dx$

l) $\int \sqrt{9 - 4x^2} \, dx$ 

m) $\int \sqrt{-x^2 + 2x + 2} \, dx$

n) $\int \sqrt{-x^2 + 2x + 3} \, dx$ 

o) $\int \dfrac{1}{x^2 \sqrt{1 + x^2}} \, dx$

5.

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9. $\displaystyle\int \frac{x+3}{x^2-x}\,dx$

10. $\displaystyle\int \frac{x^2+x+1}{x^2-x}\,dx$

11. $\displaystyle\int \frac{x^3+x+1}{x^2-2x+1}\,dx$

12. $\displaystyle\int \frac{x^3+x+1}{x^2-4x+3}\,dx$

13. $\displaystyle\int \frac{1}{x^2+5}\,dx$

14. $\displaystyle\int \frac{x+1}{x^2+9}\,dx$

15. $\displaystyle\int \frac{x^2+3}{x^2-9}\,dx$

16. $\displaystyle\int \frac{1}{x^2-x-2}\,dx$

6.

cícios 12.8

Calcule.

1. $\int \dfrac{\cos x}{4 - \text{sen}^2\, x}\, dx$

2. $\int \dfrac{1}{\text{sen}\, x + \cos x}\, dx$

3. $\int \dfrac{\text{sen}\, 2x}{1 + \cos x}\, dx$

4. $\int \dfrac{2\, \text{tg}\, x}{2 + 3\cos x}\, dx$

5. $\int \dfrac{1}{\sqrt{3}\,\cos x - \text{sen}\, x}\, dx$

6. $\int \dfrac{1}{2 + \text{sen}\, x}\, dx$

## 9 – EXERCÍCIOS DO CAPÍTULO

1. Calcule.

a) $\int \dfrac{4x^2 - 2\sqrt{x}}{x}\, dx$

b) $\int (3x - 2)\sqrt{x}\, dx$

c) $\int (x + 5)^3\, dx$

d) $\int \dfrac{2x + 3}{\sqrt{x^2 + 3x}}\, dx$

e) $\int \dfrac{(\sqrt{3} - \sqrt{x})^2}{\sqrt{x}}\, dx$

f) $\int \dfrac{1}{1 + e^{-x}}\, dx$

g) $\int \dfrac{1}{\text{sen}^2\, x}\, dx$

h) $\int \dfrac{1}{1 + \cos x}\, dx$

i) $\int \cos^3 x\, dx$

j) $\int \dfrac{x}{\sqrt{1 - x^4}}\, dx$

l) $\int \dfrac{1}{\sqrt{16 - 9x^2}}\, dx$

m) $\int \dfrac{1}{\sqrt{1 + 5x^2}}\, dx$

n) $\int \text{tg}^4\, x\, dx$

o) $\int \dfrac{x}{x^4 + 16}\, dx$

p) $\int \dfrac{3x}{\sqrt[3]{1 + 2x}}\, dx$

q) $\int \dfrac{\cos x}{4 - \text{sen}^2\, x}\, dx$

r) $\int (2x + \sqrt{1 + x^2})^2\, dx$

s) $\int \dfrac{1}{\sqrt{2 + x - x^2}}\, dx$

t) $\int \dfrac{1}{x^2 + 4x + 3}\, dx$

u) $\int \dfrac{1}{x^2 + 3x + 1}\, dx$

v) $\int \dfrac{3}{2x - x^2 - 10}\, dx$

x) $\int \text{sen}\,(\ln x^2)\, dx$

6

7.

**2.** Calcule e verifique o resultado por derivação.

a) $\int x^3 e^{-x}\, dx$

b) $\int \cos x \operatorname{sen} 2x\, dx$

c) $\displaystyle\int \frac{1}{\sqrt{1+x+x^2}}\, dx$

d) $\displaystyle\int \frac{x}{(x-3)^3}\, dx$

e) $\int (\ln x)^3\, dx$

f) $\displaystyle\int \frac{x^2+x-56}{x+1}\, dx$

g) $\int \operatorname{sen}^3 x \cos^3 x\, dx$

h) $\displaystyle\int \frac{x}{\sqrt{4-x^2}}\, dx$

i) $\displaystyle\int \frac{1}{x^3-27}\, dx$

j) $\displaystyle\int \frac{x^2-5x}{x^2+2}\, dx$

l) $\int \operatorname{sen} 3x \cos 5x\, dx$

m) $\int e^{2x} \cos 3x\, dx$

n) $\displaystyle\int \frac{1}{x^2+x-1}\, dx$

o) $\displaystyle\int \frac{x+1}{x^2+x+1}\, dx$

p) $\int \operatorname{arc\,tg} 3x\, dx$

q) $\int \operatorname{arc\,sen} 2x\, dx$

r) $\displaystyle\int \frac{1}{\cos x - \operatorname{sen} x}\, dx$

s) $\int \cos^3 x\, dx$

t) $\int x\sqrt{4+5x^2}\, dx$

u) $\int \sqrt{4+5x^2}\, dx$

v) $\displaystyle\int \frac{x^3+x}{\sqrt{4+9x^2}}\, dx$

x) $\displaystyle\int \frac{1}{1+\operatorname{sen} x}\, dx$

**3.** Seja $a$ uma constante não nula. Verifique.

a) $\displaystyle\int \frac{1}{x^2-a^2}\, dx = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + k$

b) $\displaystyle\int \frac{1}{\sqrt{x^2+a^2}}\, dx = \ln(x+\sqrt{x^2+a^2}) + k$

c) $\displaystyle\int \sqrt{x^2+a^2}\, dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2} \ln(x+\sqrt{x^2+a^2}) + k$

d) $\displaystyle\int \sqrt{a^2-x^2}\, dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2} \operatorname{arc\,sen} \frac{x}{a} + k$

e) $\displaystyle\int x^n e^{ax}\, dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax}\, dx$, com $n \in \mathbb{N}$.

6. Calcule.

a) $\int x \sec^2 x \, dx$

b) $\int x^2 \arcsin x \, dx$

c) $\int \dfrac{1}{1 + \cos x + \sin x} \, dx$

d) $\int x \sqrt[3]{2 + x} \, dx$

e) $\int \dfrac{x^2 - 4x - 4}{(x - 2)(x^2 + 4)} \, dx$

f) $\int \dfrac{3x^2 - 1}{(x - 1)^2(x + 2)} \, dx$

g) $\int \dfrac{x^2 - x + 1}{x^3 + 3x^2 - 4} \, dx$

h) $\int \sin^3 x \cos^2 x \, dx$

i) $\int s \sqrt{3 + 5s^2} \, ds$

j) $\int s^2 \sqrt{1 + s^2} \, ds$

l) $\int s^3 \sqrt{1 + s^2} \, ds$

m) $\int \dfrac{1}{p^4 + p^2} \, dp$

n) $\int \dfrac{\sqrt{x}}{1 + \sqrt[4]{x}} \, dx$

o) $\int \dfrac{u + 1}{1 + \sqrt{u + 1}} \, du$

p) $\int \sqrt{1 + \dfrac{1}{x}} \, dx$

q) $\int \sqrt{1 + \dfrac{1}{x^2}} \, dx \quad (x > 0)$

r) $\int \dfrac{1}{e^x + e^{-x}} \, dx$

s) $\int \dfrac{1}{x\sqrt{1 + x + x^2}} \, dx$

t) $\int \dfrac{\sqrt{4 - x^2}}{x^4} \, dx$

u) $\int \dfrac{1}{x\sqrt{x - 1}} \, dx$

7. Calcule e verifique o resultado por derivação.

a) $\int \dfrac{1}{(1 + e^x)^2} \, dx \quad$ (Faça $u = 1 + e^x$)

b) $\int \dfrac{1}{(1 + u^2)^2} \, du \quad$ (Faça $u = \operatorname{tg} \theta$)

c) $\int \sqrt{\dfrac{1 + x}{1 - x}} \, dx \quad \left(\text{Faça } \dfrac{1 + x}{1 - x} = u^2, u > 0\right)$

d) $\int \dfrac{1}{\sqrt{x + 1} + \sqrt[3]{x + 1}} \, dx \quad$ (Faça $x + 1 = u^6, u > 0$)

e) $\int (1 + \sqrt{x})^5 \, dx \quad$ (Faça $x = u^2, u > 0$)

f) $\int \dfrac{1}{\sqrt{2 + \sqrt{x}}} \, dx$

g) $\int \sqrt{1 + e^x} \, dx$

h) $\int \dfrac{dx}{\sqrt{1 + e^{2x}}} \quad$ (Faça $1 + e^{2x} = u^2, u > 1$)

i) $\int \dfrac{\sqrt{x}}{1 + \sqrt[3]{x}} \, dx \quad$ (Faça $x = u^6, u > 0$)

j) $\int \arcsin \sqrt{x} \, dx$

l) $\int \dfrac{\sin x}{1 + \sin x} \, dx$

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9.

9. Calcule.

a) $\int \dfrac{1}{x\,(x^2+1)^2}\,dx$

b) $\int \dfrac{x^3}{\sqrt{x^2-1}}\,dx$

c) $\int x^4\sqrt{1-x^2}\,dx$

d) $\int \dfrac{1}{x^2\sqrt{4x^2+1}}\,dx$

e) $\int \dfrac{x^2+1}{\sqrt{4x^2+25}}\,dx$

f) $\int x^3\sqrt{25-4x^2}\,dx$

g) $\int x^2\sqrt{4x^2-25}\,dx$

h) $\int \operatorname{sen}^3 x \cos(\cos x)\,dx$

i) $\int \operatorname{sen} x \operatorname{arc\,tg}(\cos x)\,dx$

j) $\int \dfrac{dx}{x-\sqrt{x^2-1}}$

l) $\int \sqrt{\dfrac{2+3x}{x-1}}\,dx$

m) $\int \sqrt{4x-x^2}\,dx$

n) $\int \dfrac{x \operatorname{arc\,sen} x}{\sqrt{1-x^2}}\,dx$

o) $\int \operatorname{arc\,sen}\sqrt{\dfrac{x}{x+2}}\,dx$

p) $\int \dfrac{dx}{\sqrt[3]{x}\sqrt{1+\sqrt[3]{x}}}$

q) $\int \operatorname{sen} 5x \cos 3x\,dx$

r) $\int \dfrac{\operatorname{sen}(\operatorname{tg} x)}{\cos^2 x}\,dx$

s) $\int \sqrt{(x-1)(x-2)}\,dx$

t) $\int \dfrac{x+\sqrt{x}}{\sqrt[3]{x}+3}\,dx$

u) $\int \dfrac{\cos x}{1+\cos x}\,dx$

10.