

# Bifurcation and Local Rigidity of Homogeneous Solutions to the Yamabe Problem on Maximal Flag Manifolds

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In this work, we construct 1-parameter families of well known solutions to the Yamabe problem, namely, families of homogeneous metrics obtained by scaling in the direction of the fibers the original metric defined on the total space of a Riemannian submersion with totally geodesic fibers. We consider as total spaces maximal flag manifolds provided with a normal metric. Thereafter, we determine bifurcation instants for these families looking at changes of the Morse index of these metrics when the parameter varies on the interval  $[0, 1]$ . A bifurcation point for such families is an accumulation point of others solutions to the Yamabe problem conformal to homogeneous solutions. Already a local rigidity point is an isolated solution to this problem in its conformal class, i.e., is not a bifurcation instant.

## References

- [1] ALEKSEEVSKY, D. V. *Flag Manifolds*. Yugoslav Geometrical Seminar, Divcibare, 10-17 October 1996, 3-35.
- [2] ALEKSEEVSKY, D. V.; ARVANITOYEORGOS, A. *Riemannian flag manifolds with homogeneous geodesics* Trans. Am Math. Soc. 359 (8), 3769-3789, 2007.
- [3] ARVANITOYEORGOS, A. *Geometry of Flag Manifolds* International Journal Of Geometric Methods in Modern Physics, Vol. 3, (2006) pp 957-974.
- [4] ARVANITOYEORGOS, A.; CHRYSIKOS, I.; *Invariant Einstein Metrics on Flag Manifolds With Four Isotropy Summands* Ann Glob Anal Geometry,(2010) 37 pp 185-219.
- [5] ARVANITOYEORGOS, A.; CHRYSIKOS, I.; SAKANE, Y. *Homogeneous Einstein Metrics on  $G_2/T$*  Proceedings of the American Mathematical Society, 141, (2013) pp 2485-2499.
- [6] BERGER, M. *Sur Le Première Valeurs Propre des Variété Riemannienne* Lecture Notes in Math., Vol. 194, Springer (1971).

- [7] BERGERY, L. B.; BOURGUIGNON, J. P. *Laplacians and Riemannian Submersions With Totally Geodesic Fibres*. Illinois Journal of Mathematics Vol. 26, No. 2, Summer (1982).
- [8] BESSE, A. L. *Einstein Manifolds*. Classics in Mathematics, Springer-Verlag Berlin Heidelberg, 1987.
- [9] BESSON, G.; BORDONI, M. *On the Spectrum of Riemannian Submersions With Totally Geodesic Fibers*. Rendiconti Lincei Matematica e Applicazioni, Vol. 1, 1990, n. 4, p. 335-340.
- [10] BETTIOL, R. G.; PICCIONE, P. *Multiplicity of Solutions to the Yamabe Problem On Collapsing Riemannian Submersions*. Pacific Journal of Math. Vol. 266, No. 1, (2013).
- [11] BETTIOL, R. G.; PICCIONE, P. *Bifurcation and Local Rigidity of Homogeneous Solutions to the Yamabe Problem On Spheres*. Calculus of Variations Vol. 47, Springer-Verlag (2013), pag. 789-807.
- [12] BURSTALL, F. E.; RAWNSLEY, J. H. *Twistor Theory for Riemannian Symmetric Spaces*. Lectures Notes in Mathematics, Springer-Verlag, 1980.
- [13] COHEN, N.; GRAMA, L.; NEGREIROS, C. J. C. *Equigeodesics on flag manifolds*. Houston Journal of Mathematics. 37 (2011) 113-125.
- [14] DE LIMA, L.L.; PICCIONE, P.; ZEDDA, M. *On bifurcation of solutions of the Yamabe problem in product manifolds*. Annales de l'I.H.P. Analyse non linéaire, Volume 29 (2012) no. 2, pp. 261-277.
- [15] FALCITELLI, M.; IANUS, S.; PASTORE, A. M.; *Riemannian Submersions and Related Topics*. World Scientific Publishing Co. Pte. Ltd., 2004.
- [16] HELGASON, S. *Groups and Geometric Analysis*. American Mathematical Society, Mathematical Surveys and Monographs, Vol. 83, 1984.
- [17] HELGASON, S. *Differential Geometry, Lie Groups and Symmetric Spaces*. Academic Press, Inc., 1978.
- [18] HUMPHREYS, J. E. *Introduction to Lie Algebras and Representation Theory*. Springer-Verlag New York, Thirdy Edition, 1972.
- [19] IKEDA, A.; TANIGUCHI, Y., *Spectra and Eigenforms of the Laplacian on  $S^n$  and  $\mathbb{C}P^n$* . Osaka J. Math. 15 (1978), 515-546.

- [20] KIMURA, M. *Homogeneous Einstein metrics on certain Kähler C-spaces* Recent topics in differential and analytic geometry, 303-320, Adv. Stud. Pure Math., 18-I, Academic Press, Boston, MA, 1990.
- [21] KOBAYASHI, M. *Transformation Groups in Differential Geometry* Classics in Math., Band 70, Srpinger, Berlin (1995).
- [22] KRAMER, M; *Sphärische Untergruppen In Kompakten Zusammenhängenden Liegruppen.* Compositio Mathematica, tome 38, Number 2, 1979, p. 129-153.
- [23] NAGANO, T. *On the Minimum Eigenvalues of the Laplacians in Riemannian Manifolds.* University of Tokyo, 1961.
- [24] PANELLI, F.; PODESTÀ, F. *On the First Eigenvalue of Invariant Kähler Metrics.* Mathematische Zeitschrift (2014).
- [25] SAKANE, Y. *Homogeneous Einstein Metrics on Flag manifolds* Lobachevskii J. Math., Vol. 4, 1999, 71-87.
- [26] SAN MARTIN, L. A. B. *Álgebras de Lie* 2.ed. Editora Unicamp, 2010.
- [27] SAN MARTIN, L. A. B. *Grupos de Lie*, 2015.
- [28] SCHOEN, R. *Variational Theory for the Total Scalar Curvature Functional for Riemannian Manifolds and Related Topics* Topics in Calculus of Variations, Lecture Notes in Mathematics, 1365 (1989), 120-154.
- [29] SUGIURA, M. *Representation of compact groups realized by spherical functions on symmetric spaces.* Proc. Japan Acad., 38 (1962) 111-113.
- [30] URAKAWA, H. *The First Eigenvalue of the Laplacian for a Positively Curved Homogeneous Riemannian Manifold.* Compositio Mathematica, tome 59, Number 1, 1986, p. 57-71.
- [31] WANG, M.; ZILLER, W. *Existence and non-existence of homogeneous Einstein metrics,* Invent. Math., 84 (1986), p. 171-194.
- [32] WARNER, F. W. *Foundations Of Differentiable Manifolds And Lie Groups.* Scctot, Foresman and Company, Glenview, Illinois, 1971.
- [33] YAMAGUCHI, S. *Spectra of Flag Manifolds.* Memois of the Faculty of Science, Kyushu University, Ser. A, Vol. 33, Number 1, 1979.