Tight Lagrangian submanifolds

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In 1991, Y.-G. Oh [3] introduced the notion of *tightness* of closed Lagrangian submanifolds in compact Hermitian symmetric spaces. Let $\left(\widetilde{M}, \omega, J\right)$ be a Hermitian symmetric space of a compact type and L be a closed embedded Lagrangian submanifold of $\widetilde{M}$. Then L is said to be *globally tight* (resp. *tight*) if it satisfies

$$\sharp\left(L \cap g \cdot L\right) = \mathrm{SB}\left(L, \mathbb{Z}_2\right)$$

for any isometry $g \in G$ (resp. close to identity) such that L transversely intersects with $g \cdot L$. Here $\mathrm{SB}\left(L, \mathbb{Z}_2\right)$ denotes the sum of $\mathbb{Z}_2$-Betti numbers of L. The concept of tightness has applications to the problem of Hamiltonian volume minimization. In particular, Oh showed that the standard $\mathbb{RP}^n$ inside $\mathbb{CP}^n$ is tight and has the least volume among its Hamiltonian deformations.

We will explore the concept of *infinitesimally tight* which we show to be equivalent to the notion of locally tight. In addition, we will prove that a Lagrangian orbit $L = \mathbb{S}^3$ of $\mathrm{U}(2)$ in the flag $\mathbb{F}(1,2)$ is infinitesimally tight.

References

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