Explicit soliton for the modified Laplacian co-flow on an almost Abelian Lie group

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The G_2-flows arise as a tool in the search of torsion-free G_2-structures, by varying a non-degenerate 3-form on an oriented and spin 7–manifold M towards some $\varphi \in \Omega^3 := \Omega^3(M)$ such that the *torsion* $\nabla^{g_\varphi}\varphi$ vanishes. In particular, the modified Laplacian co-flow was introduced by Grigorian in [2]

$$\frac{\partial}{\partial t}\psi = \Delta\psi + 2d\Big((C - \operatorname{tr} T)\varphi\Big), \tag{1}$$

this modification fixed the non-parabolicity of the Laplacian co-flow $\frac{\partial}{\partial t}\psi = \Delta\psi$ introduced by Karigiannis et al [4]. Since the flow (1) is weakly parabolic in the direction of co-closed G_2-structures, the short-time existence and uniqueness for this flow is guaranteed. This flow was studied in [3] for two explicit examples of co-closed G_2–structures with symmetry, namely for warped products of an interval, or a circle, with a compact 6-manifold N which is taken to be either a nearly Kähler or a Calabi-Yau manifold and recently, in [1] for the 7–dimensional Heisenberg group. To understand the long time existence and the singularity behaviour of the flow (1), we approach the issues formulated for a geometric flow on homogeneous spaces [5] to the flow (1), especially we are interested in the Lie bracket flow, it is a dynamical system defined on the variety of Lie algebras, corresponding to an invariant geometric flow under a natural change of variables. It was introduced in [5] as a tool for the study of regularity and long-time behaviour of solutions. As a result, we exhibit an explicit soliton for the flow (1) on an almost Abelian Lie group, solitons are G_2–structures which, under the flow, simply scale monotonically and move by diffeomorphisms. In particular, they provide potential models for singularities of the flow, as well as means for desingularising certain singular G_2–structures, both of which are key aspects of any geometric flow.

References

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