# Book of Abstracts XXV Brazilian Algebra Meeting 

State University Campinas, December 3-7, 2018

## Session: Algebraic Geometry and Commutative Algebra

Room L004, Anexo 2 - IMECC

|  | December 3rd | December 4th | December 5th | December 7th |
| :---: | :---: | :---: | :---: | :---: |
| 14h00-14h50: | Polini | Vallès | Chardin | Gondim |
| 15h00-15h50: | Ramos | Malaspina | Ulrich | Naeliton |
| 16h00-16h30: | Coffee Break | Coffee Break | Coffee Break | Coffee Break |
| 16h30-16h55: | Baltazar | Marques da Costa | Novacoski | Merighe |
| 17h00-17h50: | Hassanzadeh | Boggi | Henni | Hugo |
| 18h00-18h50: |  |  | Carocca |  |

## Is the isotropy group of a derivation always an algebraic group? Rene Baltazar (Universidade Federal do Rio Grande - Brazil)


#### Abstract

: We will present the recent results on the isotropy group of a simple derivation (Ivan Pan and Luis Gustavo) and, more recently, the idea of a new demonstration given by Dan Yan. In the end, we will comment on a possible answer to Daniel Levcovitz's question about necessary and sufficient conditions for this group to be an algebraic group.


Endomorphisms of Jacobians of algebraic curves with automorphisms<br>Marcos Boggi (Universidade Federal de Minas Gerais - Brazil)


#### Abstract

: This is a joint work with Eduard Looijenga. Let $C$ be a complex smooth projective algebraic curve endowed with an action of a finite group $G$ such that the quotient curve has genus at least 3 . We prove that if the $G$-curve $C$ is very general for these properties, then the algebra of $Q$-endomorphisms of its Jacobian equals the image of the group algebra $Q_{G}$. We use this to obtain (topological) properties regarding certain virtual linear representations of a mapping class group.


## On Group representations and their actions on Surfaces and Varieties <br> Angel Carocca (Universidad de La Frontera - Chile)


#### Abstract

: In this communication we present some properties of the different linear representations induced by the action of a finite group on geometric objects associated to Riemann Surfaces and Abelian Varieties.


Multigraded Castelnuovo-Mumford regularity<br>Marc Chardin (CNRS - Sorbonne Université - France)


#### Abstract

: An extension of the notion of Castelnuovo-Mumford regularity to toric rings was given by Maclagan and Smith. A series of work are giving a more and more precise picture of the properties of this notion. Linearity of the resolutions of truncations of the given ideal or module and multigraded Betti numbers are well understood in terms of regularity. Also a generalization of this notion, beyond the one that is computing sheaf cohomology on toric varieties, have been given and studied.

In this talk, I will concentrate on the special case of standard bigraded rings and modules. In this case, some general results become more precise and easier to describe and to prove. However, the situation is pretty different than in the standard graded situation. In order to shade some light on the differences, I will present some explicit examples.


# Linearly presented perfect ideals of codimension 2 in three variables <br> Zaqueu Alves Ramos (Universidade Federal de Sergipe - Brazil) 


#### Abstract

: The subject this talk is a recent joint work with A. Dória and A. Simis about the Rees algebra and special fiber of linearly presented perfect ideals of codimension 2 in three variables. This work is inspired in a recent paper of N. P. H. Lan. Here we recover and extend his work. We apply our results to three important models: linearly presented ideals of plane fat points, reciprocal ideals of hyperplane arrangements and linearly presented monomial ideals.


The local uniformization problem<br>Josnei Antonio Novacoski (Universidade Federal de São Carlos - Brazil)


#### Abstract

: In this talk we discuss the local uniformization problem. This problem, which can be seeing as a local version of resolution of singularities for algebraic varieties, remains open in positive characteristic. We will discuss the recent developments in this problem. For instance, we will present a result by the speaker and Spivakovsky, which shows that in order to prove local uniformization in the general case, it is enough to prove it for a subclass of valuations (those of "rank one").


# Lefschetz properties for Artinian Gorenstein algebras presented by quadrics 

Rodrigo José Gondim Neves (Universidade Federal Rural de Pernambuco - Brazil)


#### Abstract

: Cohomology rings over a field of smooth projective varieties are Artinian Gorenstein algebras . For such varieties we have Lefschetz hard Theorem, hence we spect that "good" AG algebras have the analogous Lefschetz porperty. It was conjectured by j. Migliori ans U, Nagel that Weak Lefschetz property would hold for AG algebras presented by quadrics in characteristic zero. It is NOT true. We construct a family of counterexamples using a combinatorial-algebraic-geometric construction. Joint work with Giuseppe Zappalà.


# Beyond the Koszul-Cech spectral sequence 

Hamid Hassanzadeh (Universidade Federal de Rio de Janeiro - Brazil)


#### Abstract

: In this talk, I will explain few things about the history of the Koszul and Cech complexes and the way they appear in commutative algebra. We will see how Eagon and Norcott generalized the Koszul complex and how this led to the Buchsbaum-Eisenbud family of complexes. We then turn to two new recent works; one is joint with (M. Chardin, C. Polini, A. Simis, and B. Ulrich) and the other with (Viniçius Bouça). In the first work, we will see how the infinite terms in the Koszul-Cech spectral sequence can explain the Macaulay self-duality theorem in higher dimensions (A duality theory which has some roles in the determination of the Hodge loci for families of hypersurfaces and the triviality of Abel-Jacobi map) while in the second part we will see how the Buchsbaum-Eisenbud family of complexes arisen naturally inside the Koszul-Cech spectral sequence.


On the fixed Locus of instanton sheaves on $\mathbb{P}^{3}$<br>Abdelmoubine Amar Henni (Universidade Federal de Santa Catarina - Brazil)


#### Abstract

: We give some properties of the fixed rank 2 Instanton sheaves on $\mathbb{P}^{3}$ under the natural action of the 3 -dimensional torus. This allows us to relate then to Pandharepande-Thomas stable pairs. Moreover, we classify all the supports and give a lower bound on the number of irreducible components of the fixed locus.


# Vector Bundles without intermediate cohomology, CastelnuovoMumford regularity and Beilinson type spectral sequences <br> Francesco Malaspina (Politecnico de Torino - Italy) 


#### Abstract

: A classical result by Horrocks characterizes the vector bundles without intermediate cohomology (ACM bundles) on a projective space as direct sum of line bundles. A very simple proof of this criterion use the Castelnuovo-Mumford regularity. In this talk we will discuss on ACM bundles, splitting criteria and the CM type of smooth projective varieties. In particular we provide two examples of smooth projective surfaces of tame CM type, by showing that the parameter space of isomorphism classes of indecomposable ACM bundles with fixed rank and determinant on a rational quartic scroll is either a single point or a projective line. Moreover we will schow that the Segre threefold with Picard number two is the only smooth projective varieties of wild CM-type with only


a finite number of ACM bundles which are not Ulrich. It is a joint work with G. Casnati, D. Faenzi. and G. Sanna.

# The Structure of the Disguised Residual Intersection 

Vinicius Bouça Marques da Costa (Universidade Federal de Rio de Janeiro - Brazil)


#### Abstract

: The theory of residual intersections is an important topic in Algebraic Geometry and Commutative Algebra, an can be seen as a generalization of the theory of linkage. Artin and Nagata gave the algebraic definition of aa residual intersection, and gave (wrong) conditions for a residual intersection to be Cohen-Macaulay. The correct answer was given later by Huneke and Ulrich: if $R$ is Cohen-Macaulay and $I$ is an ideal evenly linked to a Strongly Cohen-Macaulay ideal satisfying $G_{\infty}$, then $R / J$ is CohenMacaulay for every residual intersection $J=(a: I)$ of $I$. Later, Hassanzadeh, Naeliton, Chardin and Hoa tackles the same problem but dropping the $G_{\infty}$. Their methods relies on constructing a family of approximation complexes wich gives information about a "disguised residual intersection" $K \subset J$. In this talk we give an explicit description of the disguised residual intersection $K$ and show that if $I$ satisfies $S D_{1}$, then the disguised residual intersection $K$ is equal to the concrete residual intersection $J=(a: I)$, answering a conjecture from Hassanzadeh and Naeliton. This sheds some light on the structure of the colon ideals $J=(a: I)$ if $J$ is a residual intersection. This is a joint work with Hassanzadeh.


## On Integral Closures and Multiplicities relative to an Artinian Module

Liliam Carsava Merighe (Instituto de Ciências Matemáticas e de Computação da Universidade de São Paulo - Brazil)


#### Abstract

: Let $(R, \mathfrak{m})$ be a commutative Noetherian complete local ring. Motivated by a question of Rees, in this talk we discuss the relationship between $\overline{\mathfrak{b}}$, the classical Northcott-Rees integral closure of $\mathfrak{b}$, and $\mathfrak{b}^{*(H)}$, the integral closure of $\mathfrak{b}$ relative to an Artinian $R$-module $H$, in order to study a relation between $e(\mathfrak{a} ; M)$, the multiplicity of $\mathfrak{a}$ on $M$, and $e^{\prime}(\mathfrak{a} ; H)$, multiplicity of $\mathfrak{a}$ relative to the Artinian $R$-module $H$. We conclude $\overline{\mathfrak{b}}=\mathfrak{b}^{*(H)}$ when every minimal prime ideal of $R$ belongs to $\operatorname{Att}_{R}(H)$. As an application, we show what happens when $H$ is a generalized local cohomology module.


# Cohen-Macaulayness and canonical module of residual intersections <br> José Naéliton Marques da Silva (Universidade Federal da Paraîba - Brazil) 


#### Abstract

: The concept of residual intersection was introduced by Artin and Nagata, as a generalization of linkage; it is more ubiquitous, but also harder to understand. Geometrically, let $X$ and $Y$ be two irreducible closed subschemes of a scheme $Z$ with $\operatorname{codim}_{Z}(X) \leq$ $\operatorname{codim}_{Z}(Y)=s$ and $Y \nsubseteq X$, then $Y$ is called a residual intersection of $X$ if the number of equations needed to define $X \cup Y$ as a subscheme of $Z$ is the smallest possible, i.e. s.. The theory of residual intersections is a center of interest since the 80 's, after Huneke repaired an argument of Artin and Nagata, introducing the notion of strongly Cohen-Macaulay ideal: an ideal such that all its Koszul homlogy is Cohen-Macaulay.

In this talk We show the Cohen-Macaulayness and describe the canonical module of residual intersections $J=a:_{R} I$ in a Cohen-Macaulay local ring $R$, under sliding depth type hypotheses. We also show duality results for residual intersections that are closely connected to results by Eisenbud and Ulrich, and thus establish some tight relations between the Hilbert series of some symmetric powers of $I / a$.


## On the asymptotic behavior of Hilbert-Samuel coefficients and j-multiplicity of graded local cohomology modules <br> Victor Hugo Jorge Perez (Instituto de Ciências Matemáticas e de Computação da Universidade de São Paulo - Brazil)

## Abstract:

Let $R=\oplus_{n \in \mathbb{N}_{0}} R_{n}$ be a Noetherian homogeneous ring with local base ring $\left(R_{0}, \mathfrak{m}_{0}\right)$. Let $R_{+}=\oplus_{n \in \mathbf{N}} R_{n}$ denote the irrelevant ideal of $R$ and let $M=\oplus_{n \in \mathbf{Z}} M_{n}$ be a finitely generated graded $R$-module

In this talk, some situations, we show that the Hilbert-Samuel coefficients $e_{0}\left(\mathfrak{q}_{0}, H_{R_{+}}^{i}(M)_{n}\right)$, $e_{1}\left(\mathfrak{q}_{0}, H_{R_{+}}^{i}(M)_{n}\right)$ and $e_{2}\left(\mathfrak{q}_{0}, H_{R_{+}}^{i}(M)_{n}\right)$ are polynomials for all $n \ll 0$, where $\mathfrak{q}_{0}$ is an $\mathfrak{m}_{0^{-}}$ primary ideal of $R_{0}$.

In a more general context, we also show that the $j$-multiplicity $j_{0}\left(\mathfrak{q}_{0}, H_{R_{+}}^{i}(M)_{n}\right)$ is a polynomial for all $n \ll 0$, in certain cases, where $\mathfrak{q}_{0}$ is an arbitrary ideal of $R_{0}$.

## On a problem of Poincaré: bounds on degrees of vector fields

Claudia Polini (University of Notre Dame - USA)

[^0]that is left invariant by a vector field in terms of the degree of the vector field. In joint work with Chardin, Hassenzadeh, Simis, and Ulrich we address this question. The question can be restated as a problem about the initial degree of the module of derivations of the coordinate ring R of the curve modulo the Euler derivation in terms of invariants of R. We exhibit lower and upper bounds for this initial degree and in several instances we are able to determine the initial degree. This talk is a more technical version of Bernd Ulrich's plenary talk. More details will be given.

Residual intersections and linear powers<br>Bernd Ulrich (Purdue University - USA)


#### Abstract

: There are numerous results that prove the Cohen-Macaulayness of the residual intersections of an ideal $I$, assuming rather strong depth conditions on the Koszul homology or on finitely many powers of $I$. We will talk about a different type of result that does not require such depth conditions: If $I$ is an ideal of analytic spread $s$, generated by forms of a single degree in a polynomial ring $S$, and if all high powers of $I$ have linear presentation, then a general ( $s-1$ )-residual intersection of $I$, though not Cohen-Macaulay in general, admits a maximal Cohen-Macaulay module that is self-dual and has a linear resolution. We apply this to residual intersections of ideals of maximal minors of certain linear matrices. Our result follows from a more general fact about homogeneous ideals of analytic spread one in a homogeneous factor ring $R$ of $S$, saying essentially that if all high powers of such an ideal are linearly presented as $S$-modules, then they are self-dual maximal Cohen-Macaulay $R$-modules with linear $S$-resolution.


This is a report on joint work with David Eisenbud and Craig Huneke.

## Jumping lines of logarithmic bundles

Jean Vallès (Université de Pau et des Pays de l'Adour - France)


#### Abstract

: On the projective plane, one can associate a vector bundle to a family of lines or to a plane curve that is called "loagrithmic bundle". Its global sections come from the vector fields tangent to the divisor defined by the lines or by more generally by the curve. Some particular lines in the plane forming a closed subset, determined by the logarithmic bundle, appear : these are the so-called jumping lines. I will study their behaviour and in the case of line arrangement the geometric relations between both set of lines.


# Posters Algebraic Geometry and Commutative Algebra 

Caracterizações Cohomologicas de fibrados vetoriais sobre Quádricas<br>Aydee López Santana

Transformações de Cremona de $\mathbb{P}^{4}$ que se fatoram através de projeções de uma interseção completa de três quádricas de $\mathbb{P}^{7}$<br>Divane Aparecida de Moraes Dantas (Pontificia Universidad Católica de Minas Gerais Brazil)


#### Abstract

: Seja $X=Q_{1} \cap Q_{2} \cap Q_{3}$ uma interseção completa de três hipersuperfícies quádricas suaves de $\mathbb{P}^{7}$ contendo dois 2 -planos $\alpha_{1}$ e $\alpha_{2}$, de tal maneira que $X$ seja suave. Fixemos dois 4-planos $\mathbb{P}_{i}^{4}$, tais que $\mathbb{P}_{i}^{4} \cap \alpha_{i}=\emptyset$, para $\mathrm{i}=1,2$. A aplicação birracional $T: \mathbb{P}_{1}^{4} \longrightarrow \mathbb{P}_{2}^{4}$, a qual se fatora por projeções de $X$ com centro $\alpha_{1}$ e $\alpha_{2}$, é uma transformação de Cremona de grau 4 cuja inversa também tem grau 4. Classificamos as transformações de Cremona considerando a posição relativa dos planos da seguinte maneira: se $\alpha_{1} \cap \alpha_{2}=\emptyset$, então T é uma transformação de Cremona determinantal, se $\alpha_{1} \cap \alpha_{2}=L$ (uma reta), então T é uma transformação de Cremona de De Jonquières e se $\alpha_{1} \cap \alpha_{2}=p$ (um ponto) T não é nem determinantal e nem de De Jonquières. Além disso, para fixar um desses três casos, daremos uma caracterização geométrica das transformações de Cremona de $\mathbb{P}^{n}$ que agem birracionalmente no conjunto dos hiperplanos que passam por um ponto.


Álgebras de Clifford e a Teoria da relatividade<br>Elen Michele Rodrigues (Universidade Federal de Viçosa - Brazil)


#### Abstract

: No estudo da geometria analítica clássica são tratados vetores unidimensionais. Uma deficiência desta abordagem é a restrição da definição do produto vetorial apenas para espaços tridimensionais. Uma forma de generalizar tal produto para espaços em outras dimensões, incluindo o caso bidimensional,é feita pela álgebra geométrica com a introdução de bivetores, fragmentos orientados do plano, e de forma similar fragmentos de dimensões superiores.


Uma abordagem algébrica da álgebra geométrica é feita pelas Álgebras de Clifford. A álgebra geométrica ou de Clifford pode ser vista como uma generalização da álgebra
vetorial, e consiste em um poderoso formalismo quando se trata da descrição física da natureza. Assim, propomos nesse trabalho, definir a álgebra de Cliford, estudar suas propriedades e a introduzir como ferramenta para o estudo de conceitos físicos da Teoria da Relatividade Restrita.

Geometria Enumerativa de Retas Simpléticas<br>Gabriel Araújo Guedes (Universidade Federal Rural de Pernambuco - Brazil)


#### Abstract

: Neste trabalho começamos fazendo uma revisão de geometria algébrica simplé tica, dando enfase a grassmaniana lagrangeana $\mathbb{L} \mathbb{G}(n, 2 n)$. Em seguida construímos o espaço de parâmetros das hipersuperfícies de grau $d$ que contêm um par de retas reversas,como o blow-up de um subfibrado de $\mathbb{L} \mathbb{G}(2,4) \times \mathbb{L} \mathbb{G}(2,4)$ ao longo da diagonal. Como conseguimos fazer uma boa descrição da ação do grupo simplético $S_{p}(2 n)$ sobre $\mathbb{L} \mathbb{G}(2,4)$, encontramos os pontos fixos dessa ação e assim usar a fórmula de resíduos de Bott para encontrar o grau desse espaço de parâmetros.


On the dimension of the loci of curves with subcanonical points in families of 6 -semigroups.<br>John Ever Quispe Vargas (Universidade Federal de Minas Gerais - Brazil)


#### Abstract

: For any 1-parameter family of symmetric semigroup of multiplicity 6 , the dimension of the moduli space of pointed curves whose Weierstrass semigroup belongs to the family is computed. In addition, we propose some questions on moduli spaces and Weierstrass points. This is a working in progress joint with A. Contiero (UFMG).


Rings of constants of linear derivations on Fermat rings<br>Marcelo Oliveira Veloso (niversidade Federal de São João del-Rei - Brazil)


#### Abstract

: In this work we characterize all the linear $\mathbb{C}$-derivations of the Fermat ring. We show that the Fermat ring has linear $\mathbb{C}$-derivations with trivial ring of constants and construct some examples.


# The Cohomology of the Grassmannian is a gl-module <br> Parham Salehyan (Universidade Estadual Paulista - Brazil) 


#### Abstract

: The singular cohomology ring of the Grassmannian $G(r ; n)$ is a module over the Lie algebra $g l_{n}(Q)$ of the matrices $\left(a_{i j}\right)_{0 \leq i ; j<n}$, whose entries are all zero but nitely many. Using Schubert calculus techniques we explicitly describe the action by computing the generating function of the action of elementary matrices $E_{i j}$ having all zero entries but 1 in the $(i ; j)$ position. The case $r=1$ is the well known fact that each vector space is a module over its Lie algebra of endomorphism. The case $r=n=\infty$ has been treated by the Date-Jimbo-Kashiwara-Miwa, yielding the bosonic vertex representation of $g l_{\infty} Q$ ).


On limit linear series with simple basis<br>Renan da Silva Santos (Instituto de Matemática Pura e Aplicada - Brazil)


#### Abstract

: Eisenbud and Harris have defined the notion of limit linear series as some kind of degeneration of linear series over smooth curves varying in a family whose limit is a singular compact type curve. Esteves and Osserman studied the relationship between a slightly differente notion of limit linear series and the fibers of the Abel map, for nodal curves with two smooth components. To every r-dimentional limit linear series $\mathfrak{g}$ they associated a closed subscheme $\mathbb{P}(\mathfrak{g})$ of the fiber of Abel map, proved it's Cohen-Macaulay of pure dimention $r$ and calculated its Hilbert polynomial. We can also view $\mathbb{P}(\mathfrak{g})$ in a product of projetive spaces $\mathbb{P}^{r} \times \mathbb{P}^{r}$ and they showed it is a degenaration of the diagonal, thus calculating its Chow class. Our current work focus on generalize Esteves and Osserman's results to compact type curves with $n$ components, $n \geq 3$. A key point on their proof is the existence of a simples basis for the limite linear series, which is equivalent to being exact for two component curves. For curves with more components we proved that the existence of simple basis implies the exacteness condition.


# Multilinear mapping versus homogeneous polynomials <br> Thiago Ginez Velanga Moreira (Universidade Estadual de Campinas - Brazil) 


#### Abstract

: Homogeneous polynomials and multilinear operators have been exhaustively investigated in the last decades under many different viewpoints. In Functional Analysis, polynomials are the pillars of the theory of holomorphic functions and in Multilinear Algebra, multilinear operators are the usual tools in different settings. In this proposal, we prove some


properties that were apparently overlooked in the literature. For instance, every multilinear mapping is a homogeneous polynomial.

# What is the local cohomology module? 

Thiago Henrique de Freitas


#### Abstract

: In this work we introduce, by a simple way, important tool in commutative algebra and algebraic geometry that is the concept of local cohomology modules. We will show some key results of this theory and recent generalizations. (Algebraic geometry and commutative algebra)


The indeterminacy locus of the Voisin map<br>Giosuè Muratore (Universidade Federal de Minas Gerais - Brazil)


#### Abstract

: Beauville and Donagi proved that the variety of lines $\mathrm{F}(\mathrm{Y})$ of a smooth cubic fourfold Y is a hyperkähler variety. Recently, C. Lehn, M.Lehn, Sorger and van Straten proved that one can naturally associate a hypekähler variety $\mathrm{Z}(\mathrm{Y})$ to the variety of twisted cubics on Y. Then, Voisin defined a degree 6 rational map: $\psi: F(Y) \times F(Y)-->Z(Y)$. We will show that the indeterminacy locus of $\psi$ is the locus of intersecting lines


Bridgeland Stability on 3-folds<br>Victor do Valle Pretti (Universidade Estadual de Campinas - Brazil)


#### Abstract

: In this work we present the concepts of Bridgeland stability, Wall-Crossing on Surfaces and 3 -folds. Let $X$ be a projective smooth surface, a Bridgeland stability condition on $X$ is a stability condition on the derived category $D^{b}(X)$ and the space of Bridgeland stability conditions $\operatorname{Stab}(X)$ defines a complex manifold of dimension equal to the rank of the numerical Grothendieck Group $K_{n u m}(X)$. The space $\operatorname{Stab}(X)$ admits a decomposition in locallyfinite chambers, where the set of stable objects is constant inside each chamber. Using a ample divisor $w \in N S(X)$ in the Neron-Severi group and a real divisor $\beta \in N S_{\mathbb{R}}(X)$ we can parametrize a half-plane of stability conditions $\left\{\left(\mathcal{A}_{s \beta, w}, Z_{s \beta, t w}\right) \mid s \in \mathbb{R}\right.$ and $\left.t>0\right\}$ in $\operatorname{Stab}(X)$, and in this half-plane the decomposition in chambers is given by Bertram's Nested Wall Theorem. It's our objective to study the structure of the Walls for $X$ being


a projective 3 -fold.

# Gonality of Rational Curves of Maximal Weight 

Vinícius Lara Lima (Universidade Federal de Minas Gerais - Brazil)


#### Abstract

: In this problem we study rational curves with a unique unibranch genus- $g$ singularity, which is of $\kappa$-hyperelliptic type in the sense of F . Torres, Weierstrass points and double coverings of curves with application: symmetric numerical semigroups which cannot be realized as Weierstrass semigroups; we focus on the cases $\kappa=0$ and $\kappa=1$, in which the semigroup associated to the singularity is of (sub)maximal weight. We obtain a partial classification of these curves according to the linear series they support, the scrolls on which they lie, and their gonality.


[^0]:    Abstract:
    In 1891, Poincaré asked if it is possible to bound the degree of a projective plane curve

