REPLY TO hep-ph/0211241: "On the extra factor of two in the phase of neutrino oscillations".

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Abstract. Arguments continue to appear in the literature concerning the validity of the standard oscillation formula. We point out some misunderstandings and try to explain in simple terms our viewpoint.

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Two of the present authors (S.D.L and P.R.) together with G. C. Ducati criticized a few years ago [1] the derivations of the so-called standard oscillation formula (SOF) which was then, and remains to this date, the basis for most of the phenomenology of neutrino masses [2]. The objections then were that the derivations of the SOF in the literature were based upon invalid approximations [3]. To be more specific the plane-wave derivations (which are certainly the simplest) in general ignored the different velocities of the neutrino mass eigenstates. It is exactly these different velocities that produce *slippage* amongst the mass-eigenstate wave-packets and eventually lead to decoherence (when oscillation ceases). An example of such a calculation of the phase difference which makes the assumptions $t \approx L$ and $\bar{p} \approx \bar{E}$ is

$$\Delta\,(\,E\,t - p\,L\,) \,=\, t\,\Delta E - L\,\Delta p \,\approx\, t\,(\,\Delta E - \Delta p\,) \approx\, t\,\frac{\Delta m^2}{2\,\bar{E}}\;. \tag{1}$$

However, if one allows for the different velocities in the space interval (t constant here and $t\,\Delta v = \Delta L \neq 0$) an extra factor of two appears in the neutrino oscillation phase [1, 3]

$$\Delta\,(E\,t - p\,L) \,=\, t\,\Delta E - \bar{L}\,\Delta p - \bar{p}\,\Delta L \,=\, t\,(\,\Delta E - \bar{v}\,\Delta p - \bar{p}\,\Delta v\,) \,\approx\, t\,\frac{\Delta m^2}{\bar{E}}\;. \tag{2}$$

In ref. [1], we showed that to obtain rigorously the SOF one needed the assumption of equal velocities. At this point we committed perhaps an ingenuity by praising the *aesthetic* value of equal velocities and this has labelled us in the eyes of some as the proponents of this hypothesis. Some have even claimed that we believe both in an extra factor of two and in equal velocities, notwithstanding the fact that they are in clear contradiction.

> Independently and later the equal velocity scenario was suggested in Ref.[2](ref. [1] in this paper). The authors of Ref.[2] consider this scenario as "aesthetically the most pleasing". They proclaimed it as their "preferred choice" in particular because it leads to the frequency of neutrino oscillations twice as large as the standard one - ref. [4],

see also

> ... the scenario of equal velocities of two mass eigenstates is preferred in ref.[1] (ref. [5] in this paper) to that of equal energies ... - ref. [6].

In a recent work on wave packets [5], we identified the source of the extra factor of two in the plane-wave formalism. It is a consequence of the implicit assumption that the flavour eigenstate $\boldsymbol{\nu_\alpha}$ is identical, including hence its phase, at all points in the creation process,

$$\boldsymbol{\nu_\alpha} = \cos\theta\,\boldsymbol{\nu_1} + \sin\theta\,\boldsymbol{\nu_2}\ .$$

This may seem very reasonable, but it is not natural in the wave-packet formalism. In fact, within the wave-packet formalism, the flavor eigenstate is *not* unique at all points of creation. Each point is associated with an appropriate x-dependent phase. For example, for gaussian wave packets with spread a_n, in the $\Delta a = \Delta p = 0$ scenario (with instantaneous creation)

$$\psi(x,0)\,\boldsymbol{\nu_\alpha} = \left(\frac{2}{\pi\,a^2}\right)^{\frac{1}{4}} \exp\left[-\frac{x^2}{a^2}\right]\ \exp[\,i\,p_0\,x\,]\ \left[\,\cos\theta\,\boldsymbol{\nu_1} + \sin\theta\,\boldsymbol{\nu_2}\,\right]\ .$$

Thus, the flavour state at different points are characterized by an x-dependent phase, specifically by the plane-wave factor:

$$\exp\left[\,i\,p_0\,x\,\right]\ .$$

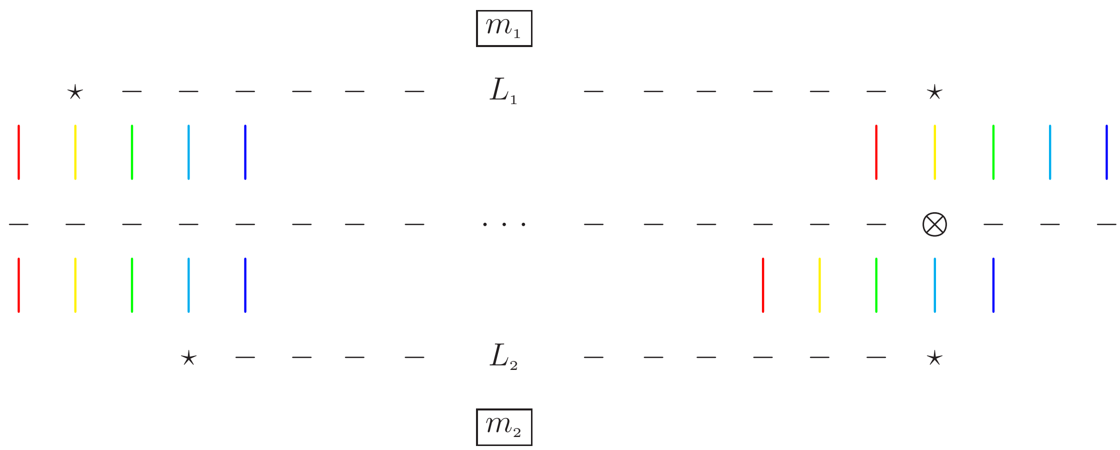
In the case of different velocities of the mass-eigenstates, interference occurs between wave packet components corresponding to different initial wave packet points. Thus, the final overlapping interference points carry with them what we call an initial phase difference. This initial phase difference compensates for the term $\bar{p}\,\Delta L$ in Eq.(2), and hence eliminates the extra factor of two, giving the standard oscillation phase (see Section III of ref. [5] for a detailed discussion).

In ref. [5], we concluded that within the wave packet formalism the standard oscillation formula is not only exact in the case of equal velocities (no slippage) but also a good approximation in all cases in which *minimal slippage* occurs between the mass wave packets. Also in ref. [5], we confronted a long standing diatribe in the literature between equal momentum advocates and equal energy advocates of which Okun is the most fervent proponent. The $\Delta p = 0$ hypothesis has a mathematical "advantage", it allows one (in the wave packet formalism) to create at a given instant a flavour eigenstate wave packet over an extended region. Flavour eigenstate creation is the starting point in oscillation phenomena (for kaons strong hypercharge plays the role that lepton flavour plays for neutrinos). *Unfortunately there is no physical frame in which* $\Delta p = 0$, as can easily be shown [5]. In the physical cases in which $\Delta p \neq 0$ it is by no means trivial to create a pure flavour eigenstate wave function. In fact, one *must* allow for creation times which depend upon the creation point, so that at no fixed instant will we have a flavour eigenstate at all points (the "other" part of the wave packet having evolved).

The $\Delta E = 0$ frames do exist and so are a legitimate choice of frame, even if they don't happen to coincide with the laboratory frame in any of the experiments, as shown by simple kinematics. As an aside, without implying *any* preference, we note that only the equal velocities case $\Delta v = 0$ is frame independent. This is a consequence of the fact that the Lorentz transformation of velocities is mass independent. If decoherence never occurs for one observer it never occurs for any observer. The $\Delta E = 0$ case is a choice of frame, and since oscillation measurement must be frame independent, we see no reason why calculations are not made in a manifestly Lorentz invariant manner i.e. in an arbitrary frame.

Contrary to another of the criticisms of Okun et. al. [6], we have never assumed interference between wave packets at different space-time points. We have always assumed and stated that the measurement process is made at a single space-time point (an idealization). At most, the theoretician will have to average over the mass eigenstate wave-functions. However, the same cannot be said about the creation process. The wave function is extended in space. Indeed the use of a plane-wave (which is a four momentum eigenstate) implicitly assumes a sufficient spatial extension of the wave function to permit one to ignore the Heisenberg momentum uncertainties ($\delta p = 0$). As for the time needed for the creation of a wave packet, this also exists in general. In fact even if there existed a frame in which creation where instantaneous, another observer would "see" a finite time for creation. This is a direct consequence of Lorentz transformations for non-point-like entities. Hence the origin of the appearance of multiple times and distances in our papers. Obviously, with different velocities it is impossible to use a single distance-interval and time-interval. Furthermore, to create a pure flavour eigenstate we are obliged, in general, to use both multiple distances and times. There is in this no contradiction to quantum mechanics.

In the figure, we illustrate this in pictorial form. The two sets of lines represent parts of a wave function for different mass eigenstates. The vertically separation is only for design purposes. They must be imagined initially overlapping. The cross on the axis represents the measuring instrument in the laboratory. The slippage of wave packets leads, at the time of measurement, to the situation shown on the RHS where horizontal slippage has occurred. Even assuming a common time of creation $t = 0$ and of measurement $t = T$, it is obvious that there are two different spatial intervals L_1 and L_2 as displayed in the figure. There is no sense in a common "spatial velocity" v_s in contradiction with different particle velocities v_1 and v_2, as considered by Okun et al. [6] in their appendix (item 2).



We have also emphasised in our preprint [5] that the oscillation phase, and hence oscillation formula, should depend upon the details of the wave-packet shape and dimensions, things about which we have little information. Again only the equal velocity case stands out as an exception to this. This means that a single oscillation formula will not be valid for all experiments. This should be remembered if inconsistencies with the SOF are encountered before invoking more exotic solutions (such as sterile neutrinos). We believe the SOF is a good approximation in the case of minimal slippage between the wave-functions i.e. when

$$t\,\Delta v (\approx L\,\Delta v) \ll a \qquad \text{[where } a \text{ is the wave spread].}$$

Otherwise one uses the SOF only on faith.

Finally, with respect to the criticism that in a discussion about pion decay into muon and neutrino we have adopted a mixed flavour neutrino,

> Another erroneous statement of [1] (ref. [5] in this paper) is that in the decay $\pi \to \mu\nu$, the ν denotes a mixture of ν_μ and ν_e - ref. [6],

this is simply not true. It is an incredible criticism since the major part of our article [5] is *devoted* to the question of guaranteeing pure flavour creation.

References

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