Neutrino Chiral Oscillations

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A localized wave packet exhibits under certain conditions zitterbewegung. A similar phenomenon occurs for the chirality of a massive particle. In the case of massive neutrinos, since they are detected via the V-A weak charged currents, this oscillation may even explain the "missing" solar neutrino experiments. The neutrino remains a mass eigenstate but contains an almost sterile right-handed component. This qualitative discussion opens up a number of interesting physical considerations.

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I. INTRODUCTION

We are interested in chiral oscillations as a possible explanation of the missing solar neutrino problem because, *neutrinos with positive chirality are decoupled from the neutrino absorbing charged weak currents*. In recent years there has been enormous interest in the problem of the missing solar neutrinos [1,2]. With the inclusion of the latest experimental results, approximately a third of the expected solar electron neutrino flux, within the energy ranges measured, is missing in earth bound experiments [3,4]. The related search for neutrino masses was stimulated by the mass oscillations invoked as the most promising hypothesis to explain the missing neutrino data. If the neutrinos have mass then it is possible to postulate mixing between known neutrino species (electron, muon and tau) [5] or between the known neutrinos and higher mass neutrinos including sterile neutrinos [6]. Such assumptions together with the violation of individual (e, μ, τ) leptonic number permit oscillations between neutrino species. This hypothesis is extensively studied in the literature [7], and is characterised by the parameterization with the differences between the squared values of neutrino masses and, of course, by the mixing angles. As we shall see below, our chiral oscillations are quite different, and do not involve any mixing angles.

Let us forget the neutrino for a moment and recall some basic facts about zitterbewegung [8]. In first quantization, the relativistic Dirac equation predicts the existence for the spin $\frac{1}{2}$ particle, under certain conditions, of a time dependence (oscillation) in the average of the position variable (the velocity eigenvalues associated with this oscillation are $\pm c$ even for a massive particle). This phenomenon, known as zitterbewegung, is a consequence of the non commutation of $\boldsymbol{x}$ with the Hamiltonian H. Since our calculation for chirality follows an almost identical line of reasoning, we shall not give here any details of the demonstration of zitterbewegung. We recall the possibility to avoid zitterbewegung by redefining the space variable via the Foldy-Wouthuysen transformation [9]. However, this introduces a very complicated position variable and is in contrast with the justification of the Darwin term in atomic physics [10]. In fact, the existence of zitterbewegung is no stranger than the existence of negative energy solutions, on which it depends. We wish to recall that zitterbewegung is only manifest for wave functions with significant interference between positive and negative frequencies. In the following we shall talk of frequencies in plane waves and not of energies. Frequencies and energies coincide only for plane waves . For bound states and localized production the Hamiltonian is never the free particle one and frequency and energy no longer coincide. For example, the energy eigenstates of the hydrogen atom are not plane wave solutions and indeed contain both types of frequency, but, non the less, they represent definite energies.

For an electron, model calculations [11] predict an oscillating zitterbewegung frequency of around 10^{21} cycles per second, far to rapid at present for any hope of direct measurement. However, we mention as an aside that if the neutrinos are massive, one of them, probably the electron neutrino, is the least massive of all known fermions and as a consequence its zitterbewegung frequency may even be within the capacity of direct experimental time resolution. Even if we cannot envisage any practical experimental test, given the neutrino's ephemeral properties.

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The starting point for what follows is the observation that *the creation of a particle is always a localized operation*, notwithstanding the use in perturbation theory of plane waves (the asymptotic as well as unperturbed wave functions). For example a particle produced in the laboratory is obviously localized, sometimes its origin is measured to within a few microns in vertex detectors. Normally this fact is ignored or considered irrelevant. Nevertheless, the creation of a particle involves, even if for only short times, a non free Hamiltonian which fortunately we do not need to know for our present calculations. We shall simply assume the form of the wave function at creation as a gaussian, to ease calculations, hoping that this assumption at least approximates the true wave function. The average chirality of the particle then exhibits a time dependence similar to zitterbewegung, and it is this type of oscillation which we consider in this paper.

II. CHIRALITY TIME DEPENDENCE

If a spin $\frac{1}{2}$ particle is massless and is produced in a chiral (γ^5) eigenstate, then its chirality is a constant of motion in addition to being Lorentz invariant. These properties no longer hold for Dirac particles with mass. The free particle Hamiltonian,

$$H = -i\boldsymbol{\alpha}\cdot\boldsymbol{\partial} + m\beta \ , \tag{2.1}$$

which we shall assume to represent the time-evolution operator for times subsequent to the creation of the spinor ($t = 0$), does not commute with the chiral operator γ^5 because of the $\beta(=\gamma^0)$ term in H. Specifically,

$$-\,i\partial_t\ \langle\gamma^5\rangle = \langle\ [H,\gamma^5]\ \rangle = 2m\ \langle\gamma^0\gamma^5\rangle \tag{2.2}$$

where m is the mass of the spinor, assumed a mass eigenstate. The mean value on the right hand side of the above equation is given explicitly by

$$\langle\gamma^0\gamma^5\rangle = \int d^3x\ \overline{\Psi}(x)\gamma^5\Psi(x)\ , \tag{2.3}$$

where Ψ may be expanded in terms of plane wave solutions in the standard way:

$$\Psi(x) \equiv \int \frac{d^3k}{(2\pi)^3}\ \frac{m}{E}\ \sum_{\alpha=1}^{2}\ [\ a_\alpha(k)u_\alpha(k)e^{-ikx} + b^*_\alpha(k)v_\alpha(k)e^{ikx}\ ]\ , \tag{2.4}$$

with E which denotes the positive quantity

$$E \equiv k^0 = \sqrt{\boldsymbol{k}^2 + m^2}\ .$$

The normalization of the spinors,

$$v^+_\alpha(k)v_\beta(k) = u^+_\alpha(k)u_\beta(k) = \frac{E}{m}\ \delta_{\alpha\beta}\ ,$$
$$-\overline{v}_\alpha(k)v_\beta(k) = \overline{u}_\alpha(k)u_\beta(k) = \delta_{\alpha\beta}\ ,$$

and the normalization condition for $\Psi(x)$,

$$\int d^3x\ \Psi^+(x)\Psi(x) = 1\ ,$$

impose the following constraint on the coefficients $a_\alpha(k)$ and $b_\alpha(k)$

$$\int \frac{d^3k}{(2\pi)^3}\ \frac{m}{E}\ \sum_\alpha\ [\ |a_\alpha(k)|^2 + |b_\alpha(k)|^2\ ]\ = 1\ . \tag{2.5}$$

Now, as for zitterbewegung, the formula (2.2) does not necessarily imply a physical effect. In it is easy to see that for wave packets made up of frequencies of only one given sign, the Dirac equation for the spinors $u(k)$ and $v(k)$

$$(\not{k} - m)\,u(k) = \overline{u}(k)\,(\not{k} - m) = 0\ ,$$
$$(\not{k} + m)\,v(k) = \overline{v}(k)\,(\not{k} + m) = 0\ ,$$

leads to a zero average in Eq. (2.2). To demonstrate this point, let us superpose only positive frequency plane waves. Computing Eq. (2.3), we find

$$\begin{aligned}\langle\gamma^0\gamma^5\rangle &= \int d^3x\ \frac{d^3q}{(2\pi)^3}\ \frac{d^3k}{(2\pi)^3}\ \frac{m^2}{EE'}\ \sum_{\alpha,\beta}\ a^*_\alpha(q)a_\beta(k)\overline{u}_\alpha(q)\gamma^5u_\beta(k)e^{i(q-k)x} \\ &= \int \frac{d^3k}{(2\pi)^3}\ \frac{m^2}{E^2}\ \sum_{\alpha,\beta}\ a^*_\alpha(k)a_\beta(k)\overline{u}_\alpha(k)\gamma^5u_\beta(k)\ . \end{aligned} \tag{2.6}$$

where $E' \equiv q^0 = \sqrt{\boldsymbol{q}^2+m^2}$. Now as a consequence of the Gordon-like *chiral* identity,

$$\overline{u}_\alpha(q)\gamma^5u_\beta(k) = \frac{1}{2m}\ \overline{u}_\alpha(q)\left(\not{q}\gamma^5+\gamma^5\not{k}\right)u_\beta(k)\ , \tag{2.7}$$

we have

$$\overline{u}_\alpha(k)\gamma^5u_\beta(k) = 0\ ,$$

and so we obtain a zero average for $\gamma^0\gamma^5$. A similar proof works out for a superposition of only negative frequency plane waves.

On the other hand if both frequencies are present the interference terms yield an oscillation in chirality,

$$\begin{aligned}\langle\gamma^0\gamma^5\rangle &= \int d^3x\ \frac{d^3q}{(2\pi)^3}\ \frac{d^3k}{(2\pi)^3}\ \frac{m^2}{EE'}\ \sum_{\alpha,\beta}\ [\ b_\alpha(q)a_\beta(k)\overline{v}_\alpha(q)\gamma^5u_\beta(k)e^{-i(q+k)x} -\ \text{h.c.}\ ] \\ &= \int \frac{d^3k}{(2\pi)^3}\ \frac{m^2}{E^2}\ \sum_{\alpha,\beta}\ [\ b_\alpha(\tilde{k})a_\beta(k)\overline{v}_\alpha(\tilde{k})\gamma^5u_\beta(k)e^{-2iEt} -\ \text{h.c.}\ ]\ , \end{aligned} \tag{2.8}$$

where $\tilde{k} = (E, -\boldsymbol{k})$. The Gordon-like *chiral* identity for cross terms,

$$\overline{v}_\alpha(q)\gamma^5u_\beta(k) = -\frac{1}{2m}\ \overline{v}_\alpha(q)\left(\not{q}\gamma^5-\gamma^5\not{k}\right)u_\beta(k)\ , \tag{2.9}$$

gives us, the non null equivalence

$$\overline{v}_\alpha(\tilde{k})\gamma^5u_\beta(k) = -\frac{E}{m}\ \overline{v}_\alpha(\tilde{k})\gamma^0\gamma^5u_\beta(k)\ . \tag{2.10}$$

From now on we will work with the following (Pauli-Dirac) representation of gamma matrices

$$\begin{aligned}\gamma^0 &= \begin{pmatrix} \mathbb{1}_2 & 0 \\ 0 & \text{-}\mathbb{1}_2 \end{pmatrix}\ , \\ \gamma^i &= \begin{pmatrix} 0 & \sigma^i \\ \text{-}\sigma^i & 0 \end{pmatrix}\ , \\ \gamma^5 &= \begin{pmatrix} 0 & \mathbb{1}_2 \\ \mathbb{1}_2 & 0 \end{pmatrix}\ . \end{aligned} \tag{2.11}$$

In such a representation the spinor solutions are given by

$$\begin{aligned} u_\alpha(k) &= \left(\frac{E+m}{2m}\right)^{1/2} \begin{pmatrix} \chi_\alpha \\ \\ \frac{\sigma^ik^i}{E+m}\ \chi_\alpha \end{pmatrix}\ , \\ v_\alpha(k) &= \left(\frac{E+m}{2m}\right)^{1/2} \begin{pmatrix} \frac{\sigma^ik^i}{E+m}\ \chi_\alpha \\ \\ \chi_\alpha \end{pmatrix}\ , \end{aligned} \tag{2.12}$$

where conventionally

$$\chi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad , \quad \chi_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \ .$$

Using Eqs. (2.11) and (2.12), we easily check that

$$-\gamma^0\gamma^5 u_\beta(k) = v_\beta(\tilde{k}) \ ,$$

and so, combining this last result with Eq. (2.10), we obtain

$$\overline{v}_\alpha(\tilde{k})\gamma^5 u_\beta(k) = -\frac{E}{m}\ \overline{v}_\alpha(\tilde{k})\gamma^0\gamma^5 u_\beta(k) = \frac{E}{m}\ \overline{v}_\alpha(\tilde{k})v_\beta(\tilde{k}) = -\frac{E}{m}\ \delta_{\alpha\beta} \ . \tag{2.13}$$

Then Eq. (2.2) becomes

$$\partial_t\langle\gamma^5\rangle = \int \frac{d^3k}{(2\pi)^3}\ \frac{m^2}{E^2}\ \sum_\alpha\ [\ -2iEb_\alpha(k)a_\alpha(\tilde{k})e^{-2iEt}\ +\ \text{h.c.}\ ] \ . \tag{2.14}$$

After performing the time integration, we find

$$\langle\gamma^5\rangle_{(t)} = \langle\gamma^5\rangle_{(0)} + \int \frac{d^3k}{(2\pi)^3}\ \frac{m^2}{E^2}\ \sum_\alpha\ [\ b_\alpha(k)a_\alpha(\tilde{k})\left(e^{-2iEt}-1\right)\ +\ \text{h.c.}\ ] \ . \tag{2.15}$$

Even this formula does not necessarily imply a significant physical effect. For example, we shall demonstrate below that for a gaussian initial wave function and if the initial state has average chirality zero the oscillations of the $\pm$ chiralities cancel and there is again no overall oscillation.

At this point we wish to discuss an apparent paradox. Now, for any plane wave solution of the Dirac equation with mass, the rest-frame wave function is always an equal mixture of both chiralities. This is easily seen by,

$$\begin{aligned} \psi &= \frac{1+\gamma^5}{2}\ \psi + \frac{1-\gamma^5}{2}\ \psi \\ &= \psi_+ + \psi_- \ , \end{aligned} \tag{2.16}$$

where $\psi_{+,-}$ correspond to chirality ± 1 respectively. Furthermore, we have that

$$\begin{aligned} \not{k}\psi_\pm &= +m\psi_\mp \ , \\ \not{k}\psi_\pm &= -m\psi_\mp \ , \end{aligned}$$

where $+m$ appears for positive energy plane waves and $-m$ for negative ones, and where k is the 4-momentum of the particle. Thus, in the rest-frame ($\boldsymbol{k} = 0$) we find, since $(\gamma^0)^2 = 1$, that

$$|\psi_+|^2 = |\psi_-|^2 \ . \tag{2.17}$$

Note that this result is <u>not</u> Lorentz invariant since a Lorentz boost is not a unitary transformation. Thus, while the cross section is Lorentz invariant the chiral probabilities are not. This seems to suggest that cross-section measurements are chiral independent. We seem to have an argument against the physical significance of chiral oscillations.

The reply to this objection based upon the Lorentz invariance of the cross-section is simply that in any *given* Lorentz frame chiral oscillations are manifestly important because of the chiral projection form (V-A) of the charged weak currents. The chiral probability variations produced by Lorentz transformations (even if γ^5 commutes with the Lorentz generators) are automatically compensated by the wave function normalization conditions and the Lorentz transformations of the intermediate vector bosons and other participating particles.

This situation is similar to the apparent contradiction between the following facts. *In all known processes the neutrino is produced as an almost left-handed helicity eigenstate* because created dominantly in an ultra-relativistic state (where chirality $\sim$ helicity) by the V-A interaction and hence with average chirality close to -1. But, in its rest frame the (plane wave) neutrino has average chirality zero. Viewed from the neutrino's rest frame there is no preference between $+$ and $-$ chiralities. Is this not in contradiction with the dominance of left-handed neutrinos in nature? No, because while there exists frames common to almost all neutrinos in which the chirality is almost pure -1, there exist many, but non coincident frames for which $+1$ chirality dominates for each given neutrino.

We conclude this Section with a discussion of another important question: in which frame shall we assume a given form of localization? A spherical gaussian assumption, suggested primarily by its simple integration properties, is certainly not frame independent. Thus if the sun is the relevant frame then we can assume, as we shall do below, that

the neutrinos are created as almost pure negative chiral eigenstates. Chiral oscillations then occur and this in turn implies that a measurement of the neutrino solar flux on earth will detect an "apparent" loss of neutrinos.

Obviously, any neutrino created in the sun can reasonably be considered localized within the solar core at time zero. It is not even excluded that the relevant distance scale for localization may be much smaller such as the dimensions of the nucleons within which the decaying quarks current gives rise to the neutrinos. In this case however we would have to take into account the wave function of the parent hadrons. In the extreme hypothesis, that the distance of localization (at creation) of the neutrinos is the Compton wavelength of the intermediate vector boson W, we will see that the chiral oscillation is very small.

III. A GAUSSIAN MODEL

Let us assume that at creation the neutrino wave function is given (or approximated) by a gaussian function in $\boldsymbol{x}$ with range d for the corresponding probability distribution,

$$\Psi(0,\boldsymbol{x}) = (\pi d^2)^{-3/4} \exp(-\boldsymbol{x}^2/2d^2)\ w\ , \tag{3.1}$$

with the spinor w normalized by $w^+ w = 1$. Since we have already noted that both theoretically and experimentally the ultra-relativistic neutrinos are created in almost pure -1 chiral eigenstates (massive neutrinos cannot be *rigorously* in a chiral eigenstate), we assume for w the *approximate* form

$$w \equiv \begin{pmatrix} \varphi \\ \text{-}\varphi \end{pmatrix}\ , \tag{3.2}$$

with φ a two component spinor, such that $\varphi^+\varphi = 1/2$.

Eq. (3.1) may be used to obtain the value of the coefficients $a_\alpha(k)$ and $b_\alpha(k)$,

$$a_\alpha(k) = (4\pi d^2)^{3/4} \exp(-\boldsymbol{k}^2 d^2/2)\ u^+_\alpha(k) w\ ,$$
$$b^*_\alpha(k) = (4\pi d^2)^{3/4} \exp(-\boldsymbol{k}^2 d^2/2)\ v^+_\alpha(k) w\ .$$

Note that starting with a negative chiral eigenstate we find $b^*_\alpha(k) = -a_\alpha(k)$. Eq. (2.15) then yields the time dependent value of the average of γ^5,

$$\langle\gamma^5\rangle_{(t)} = -1 + \int \frac{d^3k}{(2\pi)^3}\ \frac{m^2}{E^2}\ \sum_\alpha\ [\ a^*_\alpha(k) a_\alpha(\tilde{k}) \left(1 - e^{-2iEt}\right)\ +\ \text{h.c.}\ ]\ , \tag{3.3}$$

which after algebraic manipulations becomes

$$\langle\gamma^5\rangle_{(t)} = -1 + \text{NCO}\ , \tag{3.4}$$

where NCO (N*eutrino* C*hiral* O*scillation*) indicates the following integral

$$\text{NCO} = \int \frac{d^3k}{(2\pi)^3}\ \frac{m^2}{E^2}\ (4\pi d^2)^{3/2} \exp(-\boldsymbol{k}^2 d^2)\ [1 - \cos(2Et)]\ . \tag{3.5}$$

Alternatively we may quote the probability of finding the $(1-\gamma^5)/2$ component relevant for weak charged interactions, which coincides with its mean value:

$$\langle - \rangle \equiv \int d^3x\ \Psi^+(x)\ \frac{1-\gamma^5}{2}\ \Psi(x)\ .$$

Explicitly, we have

$$\langle - \rangle = 1 - \frac{1}{2}\ \text{NCO}\ . \tag{3.6}$$

By changing the initial chirality to $+1$,

$$w\ \rightarrow\ \tilde{w} \equiv \begin{pmatrix} \varphi \\ \varphi \end{pmatrix}\ , \tag{3.7}$$

we obtain $b^*_\alpha(k) = a_\alpha(k)$, and

$$\langle + \rangle = 1 + \frac{1}{2}\ \mathrm{NCO}\ . \tag{3.8}$$

Consequently, we find the anticipated result that for average intial chirality zero, there is no oscillation.

In spherical coordinates, Eq. (3.5) becomes, after angular integration,

$$\mathrm{NCO} = \frac{4d^3}{\pi^{1/2}} \int_0^\infty d\mathrm{k}\ \frac{(m\mathrm{k})^2}{E^2}\ \exp(-\mathrm{k}^2 d^2)\ [1 - \cos(2Et)]\ , \tag{3.9}$$

where $\mathrm{k} \equiv |\boldsymbol{k}|$. For future considerations, we prefer to rewrite Eq. (3.9) after the following change of variable, $\rho = \mathrm{k}d$,

$$\mathrm{NCO} = \frac{4}{\pi^{1/2}} \int_0^\infty d\rho\ \rho^2 \left(\frac{\rho^2}{m^2 d^2} + 1\right)^{-1} \exp(-\rho^2) \left\{1 - \cos\left[2mt\left(\frac{\rho^2}{m^2 d^2} + 1\right)^{1/2}\right]\right\}\ .$$

This equation is particularly simple when md may be considered large,

$$\mathrm{NCO} \sim 1 - \cos(2mt)\ .$$

Then the expectation value for the negative chirality average becomes

$$\langle - \rangle \sim \frac{1 + \cos(2mt)}{2} \qquad [\ md \gg 1\ ]\ . \tag{3.10}$$

The neutrino in this limit *oscillates between negative and positive chiral eigenvalues.* When in the chiral +1 eigenstate the neutrino only participates in elastic neutral current interactions and is thus in practice "missing". To obtain a lower value for the neutrino mass we must determine the value of d. If our gaussian wave function and correspondent gaussian probability for neutrino production is identified with the luminosity [12,13] of the solar core, we find consistently a value of d of $\sim 0.1\ R_\odot$ (where $R_\odot$ is the solar radius). This yields a neutrino mass $\gg 3 \times 10^{-15}$ eV. In any model of oscillations we must allow for the fact that the arrival times of the solar neutrinos varies. In a purely particle language (very short range localization) we would say that this depended upon the exact point of the sun in which the neutrino is produced. If the oscillations are of a period much shorter than this uncertainty, then the above formula for observing the neutrino must be averaged over one or more cycles. The result would be the apparent loss of half of the neutrino flux. This is not yet incompatible with the solar neutrino data. This situation occurs for $md \gg 2\pi$ in agreement with the condition for the validity of the simplification made in Eq. (3.10).

Let us now consider two other ranges for md. For $md \ll 1$, we have

$$\mathrm{NCO} \sim \frac{4}{\pi^{1/2}}\ m^2 d^2 \int_0^\infty d\rho\ \exp(-\rho^2) \left[1 - \cos\left(2\ \frac{t}{d}\ \rho\right)\right]\ . \tag{3.11}$$

hence the conclusion that for very stringent localizations (e.g. $d \sim 1/M_W$) the oscillating term is negligible, as anticipated in the previous Section.

When $md \sim 1$ ($d \sim$ neutrino Compton wavelength), we must resort to a numerical calculation. However, in this case, we can restrict NCO and find its value for large time. To this aim, we set $md = 1$ and perform the following change of variable

$$y = 2mt\ (\rho^2 + 1)^{1/2}$$

in the time dependent part of NCO, explicitly

$$\mathrm{NCO} = \mathcal{I} + \frac{4e}{\pi^{1/2}} \int_{2mt}^\infty dy \left(\frac{1}{4m^2t^2} - \frac{1}{y^2}\right)^{1/2} \exp(-y^2/4m^2t^2)\ \cos y\ ,$$

where

$$\mathcal{I} \equiv \frac{4}{\pi^{1/2}} \int_0^\infty d\rho\ \frac{\rho^2}{\rho^2 + 1}\ \exp(-\rho^2)\ .$$

With simple algebraic operations it is easy to show that NCO is limited by

$$\mathcal{I} - \frac{1}{mt} \;<\; \text{NCO} \;<\; \mathcal{I} + \frac{1}{mt} \;.$$

Thus, in the limit of $t \to \infty$, we obtain

$$\lim_{t\to\infty} \text{NCO} \;=\; \mathcal{I} \sim 0.48 \;.$$

In this case the expectation value for the negative chirality average is

$$\langle - \rangle \stackrel{t\to\infty}{\longrightarrow} 0.76 \qquad [\; md = 1 \;] \;.$$

To the extent that this limit corresponds to the peak passage of the neutrino flux through the earth, roughly 25% of the neutrino flux would be invisible.

The results found in this Section are shown in Fig. 1, where NCO (and consequently $\langle - \rangle$) is plotted for the various cases of md as a function of time.

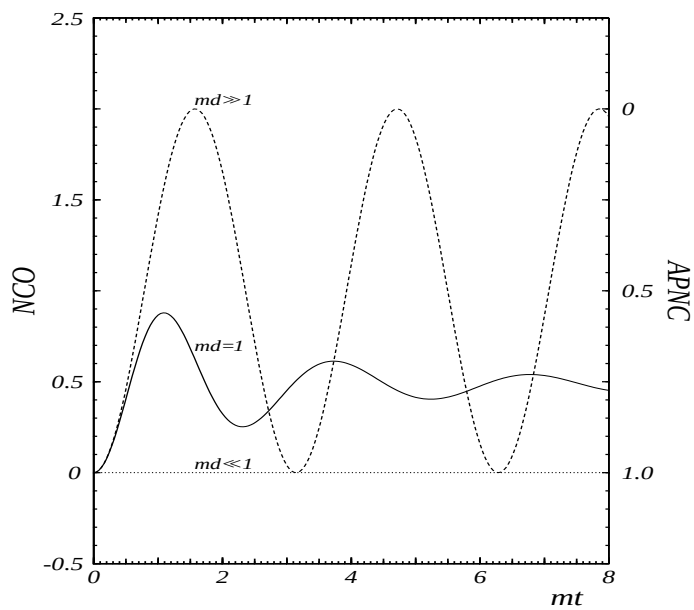

FIG. 1. The term NCO as a function of time for $md \gg 1$ (dashed), $md = 1$ (solid) and $md \ll 1$ (dotted, practically zero). The right-hand scale represents $\langle - \rangle$, the Average Probability of Negative Chirality (APNC), as a function of time.

IV. CONCLUSIONS

In this work we have suggested that a spin $\frac{1}{2}$ particle produced in a localized condition is subject to chiral oscillations reminiscent of zitterbewegung. Normally this effect is of little or no interest, but for neutrinos whose absorption involves the V-A charged current the effect may be physically significant. We have suggested that this phenomenon may well explain the missing solar neutrino problem. Specifically the effect is negligible for $md \ll 1$. It predicts $\sim 25\%$ "loss" for $md \sim 1$, and after averaging over multiple oscillations $\sim 50\%$ "loss" for $md \gg 1$. With the identification of d of the previous Section, the critical neutrino mass $m = 1/d$ becomes 3×10^{-15} eV.

Only one of our basic results coincides with more traditional oscillation models: *the neutrino must have a mass for this effect to exist*. Apart from this the differences are notable. Our oscillations have periods determined, for large md, by the particles Compton wavelength and ignores or is in addition to any neutrino mixing. There may be objections to the apparent non Lorentz invariance of our final formulas, but we have already noted that the initial localized form is clearly frame dependent. We recall that any result specified in a particular frame is by definition Lorentz invariant (and can always be expressed in a manifest invariant form). Explicitly, we have chosen an inertial Lorentz frame centered on the sun for the initial gaussian neutrino wave function.

We do not intend in this paper to confront all neutrino oscillation data. However, since the oscillation period is practically inversely proportional to the mass (see Fig. 1), we expect that the period of the muon neutrino be much smaller than that of the electron neutrino. Thus the atmospheric neutrino data is not incompatible with neutrino

masses such that the atmospheric distance corresponds to one or more oscillations for the muon neutrino while far too short for any significant electron neutrino oscillation. This argument could be used to determine an upper limit to the latter mass and the order of magnitude of the muon neutrino mass, once the localization distance d_{atm} is fixed (no longer related of course to the sun's core). Rather than speculate further on this, we wish to emphasize that the choice of an initial gaussian probability distribution was justified by its integrability, not on the basis of physical arguments and should therefore be treated with caution. On the other hand the considerations in this work invite a serious study of the appropriate creation Hamiltonian for weak interactions so as to permit the derivation of the true energy eigenvalue solutions. After which a quantitative calculation, even if only numerical, can be performed.

We conclude with a short list of the questions which this work has stimulated. First, what is the creation Hamiltonian of the weak interactions? How does this Hamiltonian produce a well known but often ignored fact, common to all created particles: the localization of the wave function? Furthermore, is it correct after the first instance to use the free Hamiltonian as the time evolution operator (as assumed in this work)? How are the negative frequency parts of the wave function to be interpreted if the reply to the previous question is affirmative? Finally, can all the known oscillation data be explained within the context of chiral oscillations?

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[1] J. N. Bahcall, *Neutrino Astrophysics* (Cambridge U. P., Cambridge, 1989).

[2] P. Langacker, *Solar Neutrinos*, Talk given at International School of Subnuclear Physics: *From Superstring to Present-day Physics*, Erice, Italy 1994 (UPR-0640-T, Penn U., USA 1994 - hep-ph/9411339); P. Langacker, *Neutrino Physics*, Talk given at 4th International Conference on *Physics Beyond the Standard Model*, Lake Tahoe, CA 1994 (UPR-0652-T, Penn U., USA 1994 - hep-ph/9503327).

[3] R. Davis Jr., D. S. Harner and K. C. Hoffman, Phys. Rev. Lett. **20**, 1205 (1968).

[4] A. I. Abazov *et al.*, Phys. Rev. Lett. **67**, 3332 (1991); K. S. Hirata *et al.*, Phys. Rev. D **44**, 2241 (1991); P. Anselmann *et al.*, Phys. Lett. **285B**, 376 (1992); J. N. Abdurashitov *et al.*, *ibid.* **328B**, 234 (1994); P. Anselmann *et al.*, *ibid.* **357B**, 237 (1995).

[5] B. Pontecorvo, Zh. Eksp. Teor. Fiz. **53** 1717 (1967) [Sov. Phys. JETP **26** 984 (1968)]; V. Gribov and B. Pontecorvo, Phys. Lett. **28B**, 493 (1969); L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); S. P. Mikheyev and A. Yu. Smirnov, Sov. J. Nucl. Phys. **42**, 913 (1986).

[6] M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity: Proceedings of Supergravity Workshop* Stony Brook, New York, edited by P. Van Nieuwenhuizen and D. Z. Freedman (Amsterdam: North-Holland, 1979); T. Yanagida, in *Proceedings of the Workshop on Unified Theory and the Baryon Number on the Universe*, Tsukuba, Ibaraki, Japan 1979, ed. by O. Sawada and A. Sugamoto (KEK Report 79-18, Tsukuba, Japan 1979).

[7] N. Hata and P. Langacker, Phys. Rev. D **50**, 632 (1994); *ibid.* **52**, 420 (1995); G. L. Fogli, E. Lisi and G. Scioscia, *ibid.* **52**, 5334 (1995); J. N. Bahcall, Nucl. Phys. B (Proc. Suppl.) **38**, 98 (1995); P. Langacker, *ibid.* **38**, 152 (1995); G. Gelmini and E. Roulet, Rep. Prog. Phys. **58** 1207 (1995); J. N. Bahcall and A. Ulmer, Phys. Rev. D **53**, 4202 (1996); and references therein.

[8] J. J. Sakurai, *Advanced Quantum Mechanics* (Addison-Wesley, New York, 1987).

[9] L. L. Foldy and S. A. Wouthuysen, Phys. Rev. **78**, 29 (1950).

[10] C. G. Darwin, Proc. Roy. Soc. (London) **A118**, 654 (1928).

[11] C. Itzykson and J. B. Zuber, *Quantum Field Theory* (McGraw-Hill, Singapore, 1985).

[12] K. R. Lang, *Astrophysical Data: Planets and Stars* (Springer-Verlag, New York, 1992).

[13] S. Turck-Chiéze, S. Cahen, M. Casse and C. Doom, Asthrophys. J. **335**, 415 (1988).