Fermion-fermion bound state condition for scalar exchanges

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The condition for the existence of a bound state between two fermions exchanging massive scalars is derived. For low scalar mass, we reproduce the scalar field model result. The high scalar mass result exhibits a somewhat different inequality condition.

I. INTRODUCTION

Since the experimental discovery of the mass of the neutrinos [1] a legitimate question has been posed. Is there a possibility of a bound state between weakly interacting particles such as an electron and a neutrino? If the particles involved where nonrelativistic the answer would be immediate and negative. From numerical studies of the Schrödinger equation [2] the existence of a bound state produced by a Yukawa (attractive) coupling

$$V_Y(r) = -\frac{g_{eff}^2}{4\pi} \frac{\exp[-\mu r]}{r},$$
(1)

has been found to be

$$\frac{g_{eff}^2}{4\pi} \geq 0.84 \frac{\mu}{m},$$
(2)

where μ is the exchanged particle mass and m is the reduced mass. A related derivation is the use of a surrogate to the Yukawa potential, the Hulthen potential, V_H, which approximates the Yukawa potential for small r,

$$V_H(r) = -\frac{g_{eff}^2}{4\pi} \frac{2\mu}{\exp[2\mu r] - 1}.$$
(3)

Our choice of V_H is made so that the terms r^{-1} and r^0 in a series development about $r = 0$ are identical to the Yukawa potential. The Schrödinger equation with the Hulthen potential can be solved *analytically* [3] and the existence of a bound state yields a similar result to that above, i.e.

$$\frac{g_{eff}^2}{4\pi} \geq \frac{\mu}{m}.$$
(4)

High mass exchanges would necessitate extremely strong couplings, obviously unphysical for weak interactions where $\mu/m > 10^{11}$ [4]. However, if one considers the relativistic corrections to the Schrödinger equation one encounters the well-known Darwin term [5]

$$\frac{1}{8m} \nabla^2 V(\boldsymbol{r}),$$
(5)

which for a Yukawa potential $V_Y(r)$ yields

$$\frac{1}{8m^2} [\mu^2 V_Y(r) + 4\pi g_{eff}^2 \delta(\boldsymbol{r})].$$
(6)

The first term above can be summed with the potential contribution to yield an overall amplification factor

$$\left(1 + \frac{\mu^2}{8m^2}\right) V_Y(r).$$
(7)

This is what has been called Yukawa coupling amplification [6]. Here the effect must be small to comply with the very nature of correction terms. However, if one where so bold as to assume this amplification for high μ/m one would invert the resonance condition (4), i.e.

$$\frac{g_{eff}^2}{4\pi} \geq \frac{8m}{\mu},$$
(8)

which allows bound states even for the weak interactions. The problem, theoretically, now shifts to determining the resonance condition for high mass exchanges in a more rigorous manner. A method has been introduced and applied in field theory [7,8]. It consists of confronting the lowest ladder contributions (box and crossed) to the scattering amplitude at rest, with the tree diagram contribution (also in the rest frame). The requirement that the sum of the former be equal or greater than the tree contribution reproduces exactly the Hulthen condition for low μ/m within a scalar-scalar model with scalar particle exchanges. In this paper it will also be shown to be also valid in the case of fermion-fermion (f-f) interacting with scalar exchanges. More recently [9], the scalar model calculation was extended to the high μ/m limit (in either limit, approximations or numerical calculations are needed). The high μ/m result was even more restrictive than the Hulthen inequality (4), i.e. it required even larger g_{eff}^2, specifically

$$\frac{g_{eff}^2}{2\pi^2} \geq \frac{\mu^2}{m_1 m_2} \Big/ \left(\ln\frac{\mu^2}{m_1 m_2} + \frac{1 + \rho^2}{1 - \rho^2} \ln\rho\right),$$
(9)

with $\rho = m_1/m_2$. However, it was noted that since the Klein-Gordon equation lacks a Darwin term correction there is no reason to expect Yukawa amplification. In this paper, we essentially repeat our low and high μ/m limits for f-f interacting via scalar exchange. This case does

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contain a Darwin term identical to that of the well known electrostatic case although some additional corrections also exist.

In the next section, we illustrate the model and reduce the first order ladder contributions to a single integral in $d|\mathbf{k}| = dk$. In Sec. III, we perform the small μ/m limit and reproduce the Hulthen inequality (4). In Sec. IV, we perform the high μ/m limit. We propose a phenomenological

expression for the k integral based upon numerical simulations. In Sec. V, we draw our conclusions.
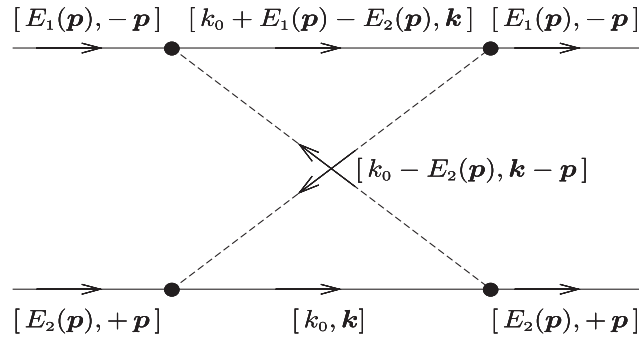
II. THE FERMIONIC MODEL

In the center of mass system and for forward scattering (see Fig. 1), the Feynman rules [4] for the amplitudes of the box ($\square$) and crossed box ($\times$) diagram yield

$$\mathcal{M}^{\square}(\mathbf{p}) = ig_1^2 g_2^2 \int \frac{d^4k}{(2\pi)^4} \frac{\bar{u}_1^{(r)}(-\mathbf{p})\{[E_1(\mathbf{p}) + E_2(\mathbf{p})]\gamma_0 - \slashed{k} + m_1\}u_1^{(r')}(-\mathbf{p})\bar{u}_2^{(s)}(\mathbf{p})(\slashed{k} + m_2)u_2^{(s')}(\mathbf{p})}{D_1^{\square}(\mathbf{p})D_2(\mathbf{p})D_0^2(\mathbf{p})} \tag{10}$$

$$\mathcal{M}^{\times}(\mathbf{p}) = ig_1^2 g_2^2 \int \frac{d^4k}{(2\pi)^4} \frac{\bar{u}_1^{(r)}(-\mathbf{p})\{\slashed{k} + m_1 + [E_1(\mathbf{p}) - E_2(\mathbf{p})]\gamma_0\}u_1^{(r')}(-\mathbf{p})\bar{u}_2^{(s)}(\mathbf{p})(\slashed{k} + m_2)u_2^{(s')}(\mathbf{p})}{D_1^{\times}(\mathbf{p})D_2(\mathbf{p})D_0^2(\mathbf{p})}$$
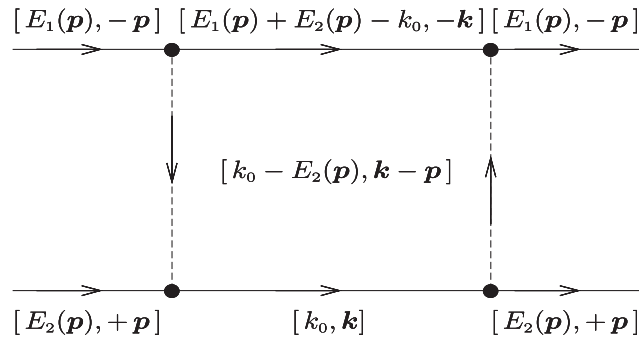
box diagram

crossed box diagram

FIG. 1. The fourth order box and crossed box diagrams in a ferminonic field model evaluated in the center of mass frame for scattering in the forward direction.

with

$$u_{1,2}^{(s)}(\boldsymbol{q}) = \sqrt{E_{1,2}(\boldsymbol{q}) + m_{1,2}} \begin{pmatrix} \chi_s \\ \frac{\sigma \cdot \boldsymbol{q}}{E_{1,2}(\boldsymbol{q}) + m_{1,2}} \chi_s \end{pmatrix} \quad (s = 1, 2),$$

$$\chi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad \chi_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

The denominators factors are,

$$D_1^{\square}(\boldsymbol{p}) = E_1^2(\boldsymbol{k}) - [k_0 - E_1(\boldsymbol{p}) - E_2(\boldsymbol{p})]^2 - i\epsilon,$$

$$D_1^{\times}(\boldsymbol{p}) = E_1^2(\boldsymbol{k}) - [k_0 + E_1(\boldsymbol{p}) - E_2(\boldsymbol{p})]^2 - i\epsilon,$$

$$D_2(\boldsymbol{p}) = E_2^2(\boldsymbol{k}) - k_0^2 - i\epsilon, \qquad (11)$$

$$D_0(\boldsymbol{p}) = E_0^2(\boldsymbol{k} - \boldsymbol{p}) - [k_0 - E_2(\boldsymbol{p})]^2 - i\epsilon,$$

where

$$E_{1,2}(\boldsymbol{q}) = \sqrt{q^2 + m_{1,2}^2}, \qquad E_0(\boldsymbol{q}) = \sqrt{q^2 + \mu^2}.$$

At threshold ($\boldsymbol{p} \approx \boldsymbol{0}$),

$$\mathcal{M}^{\square}(\boldsymbol{0}) = i(2g_1g_2\sqrt{m_1m_2})^2 \delta_{rr'}\delta_{ss'} \int \frac{\mathrm{d}^4k}{(2\pi)^4}$$

$$\times \frac{(k_0 + m_2)(2m_1 + m_2 - k_0)}{D_1^{\square}(\boldsymbol{0})D_2(\boldsymbol{0})D_0^2(\boldsymbol{0})},$$

$$\mathcal{M}^{\times}(\boldsymbol{0}) = i(2g_1g_2\sqrt{m_1m_2})^2 \delta_{rr'}\delta_{ss'} \int \frac{\mathrm{d}^4k}{(2\pi)^4} \qquad (12)$$

$$\times \frac{(k_0 + 2m_1 - m_2)(k_0 + m_2)}{D_1^{\times}(\boldsymbol{0})D_2(\boldsymbol{0})D_0^2(\boldsymbol{0})}.$$

The poles in the lower half complex k_0 plane are at

$$k_{0,1}^{\square} = E_1(\boldsymbol{k}) + m_1 + m_2, \qquad k_{0,1}^{\times} = E_1(\boldsymbol{k}) - m_1 + m_2,$$

$$k_{0,2} = E_2(\boldsymbol{k}), \qquad k_{0,0} = E_0(\boldsymbol{k}) + m_2.$$

The box and crossed box diagrams give the following fourth-order contribution to the invariant scattering amplitude

$$\mathcal{M}^{\square}(\boldsymbol{0}) + \mathcal{M}^{\times}(\boldsymbol{0}) = \frac{(2g_1g_2\sqrt{m_1m_2})^2}{(2\pi)^3} \delta_{rr'}\delta_{ss'}$$

$$\times \int d^3k \sum_{s=0}^{2} [R_s^{\square}(\boldsymbol{k}) + R_s^{\times}(\boldsymbol{k})]$$

$$= \left(\frac{g_1g_2\sqrt{2m_1m_2}}{\pi}\right)^2 \delta_{rr'}\delta_{ss'}$$

$$\times \int_0^\infty dk k^2 [R^{\square}(k) + R^{\times}(k)]. \quad (13)$$

Below by E_s we intend $E_s(k)$ and by W and Δ we intend $m_1 + m_2$ and $m_2 - m_1$ respectively. A simple calculation shows that the explicit formulas for the residues in the k_0-plane for the box and the crossed box diagram are respectively

$$R_1^{\square}(k) = [2m_2(m_1 - E_1) - k^2]/\{4WE_1(E_1 + m_1)[\mu^2 - 2m_1(E_1 + m_1)]^2\},$$

$$R_2^{\square}(k) = [k^2 - 2m_1(E_2 + m_2)]/\{4WE_2(E_2 - m_2)[\mu^2 + 2m_2(E_2 - m_2)]^2\},$$

$$R_0^{\square}(k) = [2(E_0 + 2m_2)(2m_1 - E_0)][(E_0 - m_1)BC + (E_0 + m_2)A_{\square}C - A_{\square}B]/[A_{\square}^2B^2C^3] + 2(m_1 - m_2 - E_0)/[A_{\square}BC^2], \quad (14)$$

with $A_{\square} = 2E_0m_1 - \mu^2$, $B = -2E_0m_2 - \mu^2$ and $C = 2E_0$, and

$$R_1^{\times} = [k^2 + 2m_2(E_1 + m_1)]/\{4\Delta E_1(E_1 - m_1)[\mu^2 + 2m_1(E_1 - m_1)]^2\},$$

$$R_2^{\times} = -[k^2 + 2m_1(E_2 + m_2)]/\{4\Delta E_2(E_2 - m_2)[\mu^2 + 2m_2(E_2 - m_2)]^2\}, \quad (15)$$

$$R_0^{\times} = 2(E_0 + 2m_1)(E_0 + 2m_2)[(E_0 + m_1)BC + (E_0 + m_2)A_{\times}C - A_{\times}B]/[A_{\times}^2B^2C^3] + 2(E_0 + W)/[A_{\times}BC^2],$$

with $A_{\times} = -(2E_0m_1 + \mu^2)$. It is to be noted, and can be used in calculation, that the residues of the box and crossed residues are related by

$$R_{1,2,0}^{\times} = -R_{1,2,0}^{\square}[m_1 \to -m_1]. \quad (16)$$

However, care must be used when applying this symmetry because for example $\sqrt{m_1^2} + m_1 = 2m_1$, while, under $m_1 \to -m_1$, $\sqrt{(-m_1)^2} - m_1 = 0 \neq -2m_1$. The rule of thumb is that square root factors should be left as such before applying such symmetries.

Before passing to the actual calculation of the small and large μ/m results, we must discuss two important technical questions. The first is the question of the convergence of the k integrals. The second is the feature of real pole contributions in some of these residue integrals.

A. Convergence

Individually, the leading residues terms yield divergent integrals, both linear and logarithmic. This was not the case for the scalar model [8]. However, when summed, the

divergences cancel, specifically in the limit $k \to \infty$,

$$16m_1^2m_2^2k^2R_1^\square = -\frac{m_2^2}{W} - \frac{(\mu^2 + 2m_1m_2 - 3m_1^2)m_2^2}{Wm_1k} + O\left(\frac{1}{k^2}\right),$$

$$16m_1^2m_2^2k^2R_2^\square = +\frac{m_1^2}{W} - \frac{(\mu^2 + 2m_1m_2 - 3m_2^2)m_1^2}{Wm_2k} + O\left(\frac{1}{k^2}\right),$$

$$16m_1^2m_2^2k^2R_0^\square = +\Delta$$
$$+ \frac{(\mu^2 + 2m_1m_2)(m_1^3 + m_2^3)/W - 3m_1^2m_2^2}{m_1m_2k}$$
$$+ O\left(\frac{1}{k^2}\right).$$

Consequently,

$$16m_1^2m_2^2k^2R^\square = -\frac{m_1^2m_2^2}{2k^3} + O\left(\frac{1}{k^5}\right) \quad \text{and}$$

$$16m_1^2m_2^2k^2R^\times = +\frac{m_1^2m_2^2}{2k^3} + O\left(\frac{1}{k^5}\right).$$

Both these results lead to convergent integrals, however, when summed, the leading terms again cancel and finally

$$16m_1^2m_2^2k^2(R^\square + R^\times) = -\frac{63m_1^3m_2^3}{4k^5} + O\left(\frac{1}{k^7}\right), \quad (17)$$

which is a highly convergent integrand. Notice that this leading order result is symmetric under $m_1 \leftrightarrow m_2$. We have not specified which mass, m_1 or m_2, is the lower mass and the Feynman diagrams are clearly symmetric under the interchange $m_1 \leftrightarrow m_2$. *Any final results must therefore be symmetric under this symmetry.* This feature may be used as a test of all of the following results.

B. Poles

By explicit observation the quadratic term in the denominator of $R_1^\square$ vanishes at $\mu^2 = 2m_1(E_1 + m_1)$. Poles also occur in the expression for $R_0^\square$ when $A_\square = 0$, i.e. at $\mu^2 = 2m_1E_0$. Both of these conditions correspond to the *same* value of k, which we indicate by k_p,

$$k_p^2 = \left(\frac{\mu^2}{2m_1}\right)^2 - \mu^2 \quad (18)$$

No other residues have poles. Thus, $R_1^\square$ and $R_0^\square$ exhibit double and single poles on the real axis at k_p. However, when summed *all pole contributions cancel.* This is demonstrated in some detail in the Appendix. The cancellation of the double pole is simple to show. That of the single pole which receives a contribution from $R_1^\square$ and four contributions from $R_0^\square$, one from each term in the last line of Eq. (14), is more cumbersome to see. However, it must be proved since it would otherwise dominate the large μ/m calculation, and radically change our conclusions.

III. THE EXCHANGE OF SMALL MASS SCALARS

For incoming fermions with mass m_1 and m_2 interacting by the exchange of a scalar with mass $\mu \ll m_{1,2}$, $R^\square$ and $R^\times$ contribute to the invariant scattering amplitude only for value of $k \ll m_{1,2}$ (indeed of the order of μ). In this small μ limit, we may use the approximation

$$E_{1,2} = \sqrt{k^2 + m_{1,2}^2} \approx m_{1,2} + \frac{k^2}{2m_{1,2}}.$$

We note, as an aside that for small μ ($\ll m_{1,2}$) there are no poles on the real axis. Now it is easy to show that

$$R_1^\square/R_2^\square = O[(\mu/m)^8] \ll 1.$$

Whence in the rest of this section $R_1^\square$ will be neglected. The other residue contributions yield

$$k^2R_2^\square \approx -\frac{2m_1m_2}{W}\frac{1}{E_0^4} + \frac{1}{2W}\frac{k^2}{E_0^4} - \frac{m_1}{m_2W}\frac{k^4}{E_0^6},$$

$$k^2R_2^\times \approx -\frac{2m_1m_2}{\Delta}\frac{1}{E_0^4} - \frac{1}{2\Delta}\frac{k^2}{E_0^4} - \frac{m_1}{m_2\Delta}\frac{k^4}{E_0^6},$$

$$k^2R_1^\times \approx +\frac{2m_1m_2}{\Delta}\frac{1}{E_0^4} + \frac{1}{2\Delta}\frac{k^2}{E_0^4} + \frac{m_2}{m_1\Delta}\frac{k^4}{E_0^6},$$

$$k^2R_0^\square \approx +\frac{3}{4}\frac{k^2}{E_0^5} - \frac{\Delta}{2m_1m_2}\frac{k^2}{E_0^4} + \frac{\mu^2\Delta}{2m_1m_2}\frac{k^2}{E_0^6},$$

$$k^2R_0^\times \approx -\frac{3}{4}\frac{k^2}{E_0^5} - \frac{W}{2m_1m_2}\frac{k^2}{E_0^4} + \frac{\mu^2W}{2m_1m_2}\frac{k^2}{E_0^6}.$$

Thus,

$$k^2[R^\square(k) + R^\times(k)] \approx -2m\frac{1}{E_0^4} + \left(\frac{1}{2W} - \frac{1}{m_1}\right)\frac{k^2}{E_0^4}$$
$$+ \left(\frac{1}{W} + \frac{1}{m_1}\right)\frac{k^4}{E_0^6} + \frac{\mu^2}{m_1}\frac{k^2}{E_0^6}, \quad (19)$$

and, by making use of the elementary integrals

$$\frac{4\mu^3}{\pi}\int_0^\infty \frac{dk}{E_0^4} = \frac{4\mu}{\pi}\int_0^\infty \frac{k^2dk}{E_0^4} = \frac{16\mu}{3\pi}\int_0^\infty \frac{k^4dk}{E_0^6}$$
$$= \frac{16\mu^3}{\pi}\int_0^\infty \frac{k^2dk}{E_0^4} = 1,$$

we find that

$$\mathcal{M}^\square + \mathcal{M}^\times \approx 2m_1m_2\left(\frac{g_1g_2}{\pi}\right)^2\left(-\frac{\pi}{2}\frac{m}{\mu^3} + \frac{5\pi}{16}\frac{1}{W\mu}\right). \quad (20)$$

Comparing now this fourth-order total scattering amplitude,

$$\mathcal{M}^\square + \mathcal{M}^\times \approx -\frac{g_1^2g_2^2}{\pi}\frac{m_1^2m_2^2}{W\mu^3}\left(1 - \frac{5}{8}\frac{\mu^2}{m_1m_2}\right), \quad (21)$$

with the one boson exchange amplitude (tree diagram)

$$- 4m_1m_2 \frac{g_1 g_2}{\mu^2}, \tag{22}$$

we find that the fourth-order amplitude is greater or comparable to the second-order amplitude when

$$\frac{g_1 g_2}{4\pi} \geq \frac{\mu}{m}\left(1 + \frac{5}{8}\frac{\mu^2}{m_1 m_2}\right), \tag{23}$$

which, to the leading order, reproduces exactly the Hulthen inequality, where $g_{\text{eff}}^2 = g_1 g_2$. We have explicitly calculated and exhibited the correction term in the above inequality, and we will refer to this factor in our conclusions.

IV. THE EXCHANGE OF HIGH MASS SCALARS

The high μ/m limit is more difficult to treat and we rely upon numerical tests of the following expressions. We have three masses in our calculation of $\mathcal{M}^{\square,\times}$ so if we consider an adimensional expression, it can only be a function of μ/m_1 and μ/m_2 or alternatively of

$$\omega = \frac{\mu^2}{m_1 m_2} \quad \text{and} \quad \rho = \frac{m_1}{m_2}.$$

Indeed,

$$- \mu^2 \int_0^\infty dk k^2 [R^\square(k) + R^\times(k)] = F(\omega, \rho), \tag{24}$$

and this can be tested numerically. Now we try to parametrize $\mathcal{M}^{\square,\times}$ by a form derived in the scalar model case. We write,

$$- \mu^2 \int_0^\infty dk k^2 [R^\square(k) + R^\times(k)]$$
$$= \frac{\alpha}{\omega}\left(\ln\omega + \frac{1 + \rho^2}{1 - \rho^2}\ln\rho\right). \tag{25}$$

The value $\alpha = 1$ reproduces the scalar model result. This phenomenological form has been tested for a wide but limited range of ω and ρ values, specifically for

$$\omega = 10^6, 10^7, 10^8 \quad \text{and} \quad \rho = 2, 10, 50.$$

In the following table

ω	ρ	Phen/Num
10^6	2	.995
10^6	10	.989
10^6	50	.978
10^7	2	1.005
10^7	10	1.000
10^7	50	.993
10^8	2	1.012
10^8	10	1.008
10^8	50	1.003

we give the comparison of phenomenological/numerical (Phen/Num) results for a best fit value of α,

$$\alpha = 0.663. \tag{26}$$

We see that to within a few percent the agreement is good. We could of course improve the comparison if we included a constant term $\ln\beta$ in the brackets which could correspond to the renormalization of the logarithmic terms. However, we consider this an excessive fitness. The important point is that the large μ/m behavior is similar to the scalar model result. The high μ resonance inequality thus reads

$$\frac{g_{\text{eff}}^2}{2\pi^2} \geq \frac{\mu^2}{\alpha m_1 m_2}\bigg/\left(\ln\frac{\mu^2}{m_1 m_2} + \frac{1 + \rho^2}{1 - \rho^2}\ln\rho\right). \tag{27}$$

V. CONCLUSIONS

We have applied in this paper a field theoretic approach to the determination of the coupling strengths needed for the existence of a fermion-fermion bound state via scalar boson exchanges. For low μ/m, we again find the Hulthen inequality [3,8] as seen in the scalar model. For high μ/m, we obtain an even more restrictive condition (27), a result again similar to the scalar field model [9]. The similarity between the scalar field model and this calculation suggests that the bound state inequality condition depends essentially upon the exchanged particles rather than the incoming ones. This was by no means obvious since the numerators of the residues are different in the two cases. Indeed at first sight the fermion-fermion model seemed to yield divergent results as a simple power count of the k-integral suggests. We have shown in this paper that the individual divergence contributions cancel. We have also shown that the real pole contributions to $\mathcal{M}^\square$ also cancel both for the double and single poles. Again it is not clear if this would happen with, say, vector particle exchanges and it must be said that a contribution from a simple pole would completely alter our high μ/m results. For the existence of a relativistic bound state such a contribution could even be desirable.

There is however a problem with our results for small μ/m and the arguments based upon the relativistic corrections to the Schrödinger equation mentioned in the introduction. The Dirac equation with a scalar potential contains a Darwin term as does the better known electrostatic case [5]. This lead us to expect, at least for small μ/m (nonrelativistic) a coupling amplification. We have purposefully kept the $O(\mu^2/m^2)$ corrections in the small μ/m case and as can be seen in the result (23) the corrections terms correspond to a coupling deamplification. The coupling constants must be somewhat *increased* to compensate the correction terms. This result is consistent with the tougher large μ/m inequality. We predict that the Hulthen inequality is a lower limit inequality for any μ/m. Is this disagreement between our field theory calculation and the nonrelativistic reduced mass equation serious? This may well be a matter of opinion but some observations are in order:

(i) The Hulthen inequality is *not* exactly in agreement with the Yukawa numeric inequality. So, we have a formal discrepancy even neglecting the relativistic correction terms;

(ii) The higher order Feynman diagrams cannot be parametrized by a simple Yukawa potential. However, the Coulomb potential works admirably well for Hydrogen like atoms except for one of the supreme successes of field theory, the Lamb shift. Unfortunately, we known of no direct way to derive the potential bound state spectrum from field theory;

(iii) It must also be remembered that not all the fourth order Feynman diagrams have been calculated.

Nevertheless, we remain troubled by this result. At the very least, we must moderate any expectations for a weak interaction calculation in which intermediate vector particles are exchanged. We expect the same low μ/m inequality (except perhaps for the correction term) but hope for a very different high μ/m result.

Our results have one physical consequence, we predict that weak interacting fermion-fermion (or scalar-scalar) particles cannot produce a bound state simply by Higgs boson exchanges [1]. It is our intention to tackle the full weak interaction case in the near future.

APPENDIX: THE POLE CONTRIBUTIONS

In this appendix, we calculate the pole contributions of $R_1^{\square}$ and $R_0^{\square}$ at

$$k_p = \sqrt{\left(\frac{\mu^2}{2m_1}\right)^2 - \mu^2}.$$

For convenience, we define the functions $F(k)$, $G(k)$ and $\boldsymbol{H}(k)$ by

$$k^2 R_1^{\square}(k) = F(k), \qquad (A1)$$

$$k^2 R_0^{\square}(k) = G(k) + \sum_{n=1}^{3} H_n(k), \qquad (A2)$$

where

$$G(k) = 2k^2(E_0 + 2m_2)(2m_1 - E_0)(E_0 - m_1)/(A_{\square}^2 BC^2),$$

$$H_1(k) = 2k^2(E_0 + 2m_2)(2m_1 - E_0)(E_0 + m_2)/(A_{\square}B^2C^2),$$

$$H_2(k) = -2k^2(E_0 + 2m_2)(2m_1 - E_0)/(A_{\square}BC^3),$$

$$H_3(k) = 2k^2(m_1 - m_2 - E_0)/(A_{\square}BC^2).$$

The first pole terms in the MacLaurin series of these functions are

$$\{F(k), G(k), \boldsymbol{H}(k)\} = \left\{\frac{F^{(-2)}(k_p)}{(k - k_p)^2} + \frac{F^{(-1)}(k_p)}{k - k_p}, \frac{G^{(-2)}(k_p)}{(k - k_p)^2}\right.$$
$$\left. + \frac{G^{(-1)}(k_p)}{k - k_p}, \frac{\boldsymbol{H}^{(-1)}(k_p)}{k - k_p}\right\} + \mathrm{O}(1),$$

where $F^{(-2)}(k_p)$ is the coefficient of $(k - k_p)^{-2}$ in $F(k)$ and so forth.

Now for the double pole, we find that the only two contributions are

$$F^{(-2)}(k_p) = -G^{(-2)}(k_p) = \frac{(2m_1^2 - \mu^2)(4m_1m_2 + \mu^2)}{16m_1^2 W \mu^4} k_p^2,$$
$$(A3)$$

whence their sum cancels.

The single pole contributions can be written as

$$\{F^{(-1)}(k_p), G^{(-1)}(k_p), \boldsymbol{H}^{(-1)}(k_p)\}$$
$$= \frac{k_p}{32m_1^2 W^2 \mu^6} \{f_p^{(-1)}, g_p^{(-1)}, \boldsymbol{h}_p^{(-1)}\}, \qquad (A4)$$

and in Table I we list the factors in graph brackets above as a series in even powers of μ. For example,

$$f_p^{(-1)} = -2W\mu^6 - (m_2 + \Delta)W\frac{\mu^4}{2m_1} - m_2 W \frac{\mu^2}{8m_1^3}.$$

The important point is contained in the last line of Table I. All single pole contributions also cancel. Thus, in conclusion, there are no real axis poles in $k^2(R^{\square} + R^{\times})$.

TABLE I. The coefficients of powers of μ^2 in the factors $f_p^{(-1)}$, $g_p^{(-1)}$, and $\boldsymbol{h}_p^{(-1)}$ for the single pole contributions. These factors are defined in Eq. (A4).

	μ^6	$\mu^4/2m_1$	$\mu^2/8m_1^3$	$\mu^0/3m_1^5 m_2$
$f_p^{(-1)}$	$-2W$	$-(m_2 + \Delta)W$	$-m_2 W$	0
$g_p^{(-1)}$	$m_1 + 2W$	$2\Delta W - 2m_1^2 + 3m_1 m_2$	$2m_2\Delta$	$-(m_1 + W)$
$h_{p,1}^{(-1)}$	$-m_1$	$2m_1^2 - 3m_1 m_2$	$m_2(2m_1 - \Delta)$	m_2
$h_{p,2}^{(-1)}$	0	$-m_1 W$	$-\Delta W$	W
$h_{p,3}^{(-1)}$	0	$2m_1 W$	ΔW	0
Sum	0	0	0	0

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