

FLAVOR AND CHIRAL OSCILLATIONS

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We seek a quantum-theoretic expression for the probability that a “fermionic” particle which is initially in a well-defined flavor, linear combination of mass-eigenstates, will be found, at later times, in another flavor state. We approach this problem by using the Dirac equation as evolution equation for the mass-eigenstates. The Dirac formalism is useful and essential in keeping clear many of the conceptual aspects of quantum oscillation phenomena that naturally arise in a relativistic spin one-half particle theory. Our study leads to the conclusion that the fermionic nature of the particles and the interference between positive and negative frequency components of mass-eigenstate wave packets modify the standard oscillation probability, obtained by implicitly assuming a “scalar” nature of the mass-eigenstates. Nevertheless, under particular assumptions, i.e. ultra-relativistic particles, strictly peaked momentum distributions and minimal slip-page, these modifications introduce correction factors proportional to $m_{1,2}^2/p_0^2$ which are practically un-detectable by any experimental analysis.

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1. Flavor Oscillations

The purpose of this work is to analyze the additional effects in the oscillation probability formula due to the use of the Dirac equation as evolution equation for the neutrino mass-eigenstates. To simplify our discussion, we keep the assumption that the space dependence of the wave functions is one-dimensional (z -axis) and only two flavors (ν_α , ν_β) are involved in the oscillation process.

Since, in all known processes the neutrinos are produced as almost left-handed helicity eigenstates, and for ultra-relativistic states chirality and helicity coincide, we assume that the normalizable mass-eigenstate wave functions $\psi_{1,2}(z, t)$ are

created at time $t = 0$ as a -1 chiral eigenstate. In the Pauli–Dirac representation of gamma matrices,¹ the normalizable mass-eigenstate wave packets

$$\psi_r(z, t) = \int_{-\infty}^{+\infty} dp_z g(p_z) \exp(ip_z z) \times \begin{pmatrix} 0 \\ \cos[E(p_z, m_r)t] - ih_+(p_z, m_r) \sin[E(p_z, m_r)t] \\ 0 \\ -\cos[E(p_z, m_r)t] + ih_-(p_z, m_r) \sin[E(p_z, m_r)t] \end{pmatrix}, \quad r = 1, 2, \tag{1}$$

where

$$h_{\pm}(p_z, m_r) = \frac{p_z \pm m_r}{E(p_z, m_r)} \quad \text{and} \quad E(p_z, m_r) = \sqrt{p_z^2 + m_r^2},$$

satisfy the one-dimensional Dirac equation

$$i(\gamma^0 \partial_t + \gamma^3 \partial_z) \psi_r(z, t) = m_r \psi_r(z, t), \tag{2}$$

and, at time $t = 0$, guarantee the creation of -1 chiral mass-eigenstates

$$\gamma^5 \psi_r(z, 0) = -\psi_r(z, 0). \tag{3}$$

To ease calculations and derive, under certain conditions, an analytic expression for the oscillation probability, we suppose that the momentum distribution $g(p_z)$ is given by a Gaussian function peaked around the momentum p_0 , i.e.

$$g(p_z) = \left(\frac{a^2}{32\pi^3}\right)^{1/4} \exp\left[-\frac{a^2(p_z - p_0)^2}{4}\right]. \tag{4}$$

Coming back to Eq. (1), we observe that the choice of the same momentum distribution for the mass eigenstate wave packets allows us to *instantaneously* create at $t = 0$, in a localized region centered around the spatial coordinate $z = 0$, a *pure* flavor state ν_α .² In fact, recalling that the time evolution of flavor wave packets is described by

$$\begin{aligned} \Psi(z, t) &= \psi_1(z, t) \cos \theta \nu_1 + \psi_2(z, t) \sin \theta \nu_2 \\ &= [\psi_1(z, t) \cos^2 \theta + \psi_2(z, t) \sin^2 \theta] \nu_\alpha + [\psi_2(z, t) - \psi_1(z, t)] \cos \theta \sin \theta \nu_\beta \\ &= \psi_\alpha(z, t; \theta) \nu_\alpha + \psi_\beta(z, t; \theta) \nu_\beta, \end{aligned} \tag{5}$$

we immediately find

$$\psi_\alpha(z, 0; \theta) = \left(\frac{1}{2\pi a^2}\right)^{1/4} \exp\left(-\frac{z^2}{a^2}\right) \exp(ip_0 z) \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \quad \text{and} \quad \psi_\beta(z, 0; \theta) = 0.$$

We emphasize that different momentum distributions imply that the mass-eigenstate wave functions are no longer equal at time $t = 0$ and consequently, in this case, it is not possible to conciliate a unique time of creation with the requirement to have a pure flavor state at time $t = 0$. Non-instantaneous creation,³ eliminating the initial difference of phase between the mass-eigenstate wave packets, avoids the unwanted flavor contamination. We do not intend to discuss here the main conceptual subtleties of this subject.⁴⁻⁶ Note that, in Gaussian models, the flavor contamination is proportional to $\exp[-(a\Delta p)^2/8]$ and $a\Delta p \gg a\delta p = 1$, we simply ignore this contamination and consider Gaussian function peaked around the same momentum p_0 . It is also important to note that, when only two flavors are involved in the oscillation process, the mixing matrix takes the simple form

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}.$$

In the case of three neutrinos, the mixing matrix may depend on more parameters, some of which may be complex, thus introducing CP violations.¹ In this paper, we shall study the two-flavor problem in the Dirac formalism and do *not* consider CP violating effects.

The probability $P_{\text{Dirac}}(\nu_\alpha \rightarrow \nu_\beta; t)$ of observing a flavor state ν_β at the instant t is equal to the integrated squared modulus of the ν_β coefficient in Eq. (5),

$$\begin{aligned} P_{\text{Dirac}}(\nu_\alpha \rightarrow \nu_\beta; t) &= \int_{-\infty}^{+\infty} dz |\psi_\beta(z, t; \theta)|^2 \\ &= \sin^2 2\theta \left[1 - \text{Re} \int_{-\infty}^{+\infty} dz \psi_1^\dagger(z, t) \psi_2(z, t) \right] / 2 \\ &= \sin^2 2\theta [1 - \text{DFO}(t)] / 2. \end{aligned} \tag{6}$$

By simple mathematical manipulations, the term $\text{DFO}(t)$ can be rewritten as

$$\begin{aligned} \text{DFO}(t) &= 4\pi \int_{-\infty}^{+\infty} dp_z g^2(p_z) \{ [1 - f(p_z, m_1, m_2)] \cos[\varepsilon_-(p_z, m_1, m_2)t] \\ &\quad + f(p_z, m_1, m_2) \cos[\varepsilon_+(p_z, m_1, m_2)t] \}, \end{aligned} \tag{7}$$

where

$$\begin{aligned} f(p_z, m_1, m_2) &= \frac{E(p_z, m_1)E(p_z, m_2) - p_z^2 - m_1 m_2}{2E(p_z, m_1)E(p_z, m_2)} \quad \text{and} \\ \varepsilon_\pm(p_z, m_1, m_2) &= E(p_z, m_1) \pm E(p_z, m_2). \end{aligned}$$

With respect to the standard treatment of neutrino oscillations done by using “scalar” mass-eigenstate wave packets and leading to a flavor conversion probability in which the only oscillating term is given by $\cos[\varepsilon_-(p_z, m_1, m_2)t]$,

$$P_{\text{Scalar}}(\nu_\alpha \rightarrow \nu_\beta; t) = \sin^2 2\theta \left[1 - 4\pi \int_{-\infty}^{+\infty} dp_z g^2(p_z) \cos[\varepsilon_-(p_z, m_1, m_2)t] \right] / 2, \tag{8}$$

we note in DFO(t) two additional terms. In the first one, the time-independent factor $f(p_z, m_1, m_2)$, coming from the spinorial form of Dirac wave packets, multiplies the *standard* oscillating term due to the interference between mass eigenstate components of positive *or* negative frequencies. In the second one, it multiplies the oscillating term $\cos[\varepsilon_+(p_z, m_1, m_2)t]$. This *new* peculiar oscillatory behavior, due to the interference between mass eigenstate components of positive *and* negative frequencies, is similar to the phenomenon referred to as *Zitterbewegung* (jittering motion) which predicts for spin one-half particles a superposition of rapid oscillations as an additional effect to the rectilinear motion of the wave packet.⁷ In atomic physics, this violent quantum fluctuation in the position of the electron makes the electron sensitive to an effective potential which explains the Darwin term in the hydrogen atom.⁸

At this point, it could be reasonable to postulate a wave packet made up exclusively of positive frequency plane-wave solutions, in this case the oscillation term $\cos[\varepsilon_+(p_z, m_1, m_2)t]$ vanishes. Nevertheless, it can be easily shown that initial localized states of the form

$$\psi_\alpha(z, 0) = \varphi(x)w,$$

where w is a constant spinor, *necessarily* require wave packets which contain plane wave components of both positive and negative frequencies. Consequently, in constructing such a wave packet, we cannot simply forget the contributions of negative frequency components.

We conclude this section with a brief discussion on the time-independent term $f(p_z, m_1, m_2)$. A simple study of this p_z -symmetric function shows that it goes rapidly to zero for $p_z \gg m_{1,2}$, has a minimum at $p_z = 0$ and two maxima at $p_z = \pm\sqrt{m_1 m_2}$. The maximum value of $f(p_z, m_1, m_2)$ is

$$f_{\text{max}}(\pm\sqrt{m_1 m_2}, m_1, m_2) = \frac{1}{2} \left[1 - \sqrt{1 - \left(\frac{m_1 - m_2}{m_1 + m_2} \right)^2} \right].$$

For $\Delta m \ll m_{1,2}$, it seems to have a strong evidence against any modification in the standard oscillation formula. There is, however, an important feature which we have overlooked in this section. In treating the time evolution of the spinorial mass-eigenstate wave packets, we have *completely* disregarded the chiral nature of charged weak currents and the time evolution of the chiral operator. In the next section, we aim to investigate if (and eventually how) the flavor oscillation formula could be modified by this additional effect.

2. Chiral Oscillations

It is well known that from the Heisenberg equation, we can immediately determine whether or not a given observable is a constant of motion. If neutrinos have mass, γ^5 does not commute with the mass-eigenstate Hamiltonians. This means that for massive neutrinos chirality is *not* a constant of the motion. Observing that neutrinos with positive chirality are decoupled from charged weak currents, this additional effect cannot be ignored.⁹

In the previous section, we have seen that localized states contain, in general, plane-wave components of negative and positive frequencies. As an immediate consequence of this, the interference between positive and negative frequencies, responsible for the additional oscillatory term in $DFO(t)$, will also imply an oscillation in the average of chirality. Thus, the use of Dirac equation as evolution equation for neutrino mass-eigenstate wave packets leads to an oscillation formula containing both “flavor-appearance” (neutrinos of a flavor not present in the original source) and “chiral-disappearance” (neutrinos of wrong chirality) probabilities.

In order to obtain the flavor/chiral oscillation probability formula, we observe that

$$\begin{aligned}
 & \text{Re} \int_{-\infty}^{+\infty} dz \psi_r^+(z, t) \gamma^5 \psi_s(z, t) \\
 &= -4\pi \int_{-\infty}^{+\infty} dp_z g^2(p_z) \left\{ \left[1 - f(p_z, m_r, m_s) - \frac{m_r m_s}{E(p_z, m_r) E(p_z, m_s)} \right] \right. \\
 & \quad \times \cos[\varepsilon_-(p_z, m_r, m_s)t] + \left. \left[f(p_z, m_r, m_s) + \frac{m_r m_s}{E(p_z, m_r) E(p_z, m_s)} \right] \right. \\
 & \quad \left. \times \cos[\varepsilon_+(p_z, m_r, m_s)t] \right\}, \quad r, s = 1, 2. \tag{9}
 \end{aligned}$$

From this integral, it is readily seen that an initial -1 chiral mass-eigenstate will evolve with time changing its chirality. Once we know the time evolution of the chiral operator, we are able to construct an *effective* oscillation probability which takes into account both flavor and chiral conversion effects, i.e.

$$\begin{aligned}
 & P(\nu_{\alpha,L} \rightarrow \nu_{\beta,L}; t) \\
 &= \int_{-\infty}^{+\infty} dz |\psi_{\beta,L}(z, t; \theta)|^2 \left(= \int_{-\infty}^{+\infty} dz \psi_{\beta}^+(z, t; \theta) \frac{1 - \gamma^5}{2} \psi_{\beta}(z, t; \theta) \right) \\
 &= \sin^2 2\theta \left[\frac{1}{2} \sum_{r=1}^2 \int_{-\infty}^{+\infty} dz |\psi_{r,L}(z, t)|^2 - \text{Re} \int_{-\infty}^{+\infty} dz \psi_{1,L}^+(z, t) \psi_{2,L}(z, t) \right] / 2 \\
 &= \sin^2 2[\text{DCO}(t) - \text{DFCO}(t)]/2. \tag{10}
 \end{aligned}$$

As done in the previous section, the terms DCO(t) and DFCO(t) can be rewritten by using a p_z -integration,

$$\begin{aligned}
 \text{DCO}(t) &= 2\pi \sum_{r=1}^2 \int_{-\infty}^{+\infty} dp_z g^2(p_z) \{1 - c(p_z, m_r, m_r) \\
 &\quad + c(p_z, m_r, m_r) \cos[2E(p_z, m_r)t]\} \\
 &= 1 - 2\pi \sum_{r=1}^2 \int_{-\infty}^{+\infty} dp_z g^2(p_z) \left\{ \frac{m_r^2}{2E^2(p_z, m_r)} \right. \\
 &\quad \left. - \frac{m_r^2}{2E^2(p_z, m_r)} \cos[2E(p_z, m_r)t] \right\} \tag{11}
 \end{aligned}$$

and

$$\begin{aligned}
 \text{DFCO}(t) &= 4\pi \int_{-\infty}^{+\infty} dp_z g^2(p_z) \{ [1 - c(p_z, m_1, m_2)] \cos[\varepsilon_-(p_z, m_1, m_2)t] \\
 &\quad + c(p_z, m_1, m_2) \cos[\varepsilon_+(p_z, m_1, m_2)t] \}, \tag{12}
 \end{aligned}$$

where

$$c(p_z, m_r, m_s) = f(p_z, m_r, m_s) + \frac{m_r m_s}{2E(p_z, m_r)E(p_z, m_s)}.$$

The functions $c(p_z, m_r, m_s)$ have a common maximum at $p_z = 0$ which, contrary to what happened for $f(p_z, m_1, m_2)$, does not depend on the mass values,

$$c_{\max}(0, m_r, m_s) = \frac{1}{2},$$

and, following the same asymptotic behavior of $f(p_z, m_1, m_2)$, go rapidly to zero for $p_z \gg m_{1,2}$.

3. Analytic Flavor/Chiral Oscillation Formula

Now we intend to explicitly calculate the terms DCO(t) and DFCO(t). The use of free Gaussian wave packets is justified in nonrelativistic quantum mechanics because for these particular functions, the calculations can be carried out exactly. The reason lies in the fact that the frequency components of the mass-eigenstate wave packets, $E(p_z, m_r) = p_z^2/2m_r$, modify the momentum distribution into “generalized” Gaussian, easily integrated by well-known methods of analysis. The term p_z^2 in $E(p_z, m_r)$ is then responsible for the variation in time of the width of the mass-eigenstate wave packets, the so-called *spreading* phenomenon. In relativistic quantum mechanics the frequency components of the mass-eigenstate wave packets, $E(p_z, m_r) = \sqrt{p_z^2 + m_r^2}$, do not permit an immediate analytic integration. This difficulty, however, may be remedied by assuming strictly peaked momentum distributions around p_0 , $\delta p \ll p_0$ ($\Rightarrow ap_0 \gg 1$). In this case, we can conveniently truncate the power series

$$E(p_z, m_r) = E_r + v_r(p_z - p_0) + \frac{1}{2}E_r(1 - v_r^2)(p_z - p_0)^2 + O[(p_z - p_0)^3],$$

where $E_r = \sqrt{p_0^2 + m_r^2}$ and $v_r = p_0/E_r$ and get an analytic expression for the oscillation probability. By considering terms up to $(p_z - p_0)^2$ order in $E(p_z, m_r)$, we include an analysis of spreading effects. In this preliminary study, we are, however, interested only up to first-order corrections. Thus, we approximate the frequency components by

$$E(p_z, m_r) \approx E_r + v_r(p_z - p_0). \quad (13)$$

As a consequence of this approximation, we get

$$c(p_z, m_r, m_s) \approx \frac{1}{2} \left[1 - v_r v_s + v_r v_s (v_r^2 + v_s^2 - 2) \frac{p_z - p_0}{p_0} \right],$$

$$\varepsilon_{\pm}(p_z, m_1, m_2) \approx E_1 \pm E_2 + (v_1 \pm v_2)(p_z - p_0).$$

For ultra-relativistic particle ($m_r \ll p_0$), we can also use the following expression for the central energy values (E_r) and the group velocities (v_r) of the mass-eigenstate wave packets,

$$E_r \approx p_0 + \frac{m_r^2}{2p_0} \quad \text{and} \quad v_r \approx 1 - \frac{m_r^2}{2p_0^2}.$$

This implies

$$c(p_z, m_r, m_s) \approx \frac{m_r^2 + m_s^2}{4p_0^2} \left(1 - 2 \frac{p_z - p_0}{p_0} \right),$$

$$\varepsilon_+(p_z, m_1, m_2) \approx 2p_0 \left[1 + \frac{m_1^2 + m_2^2}{4p_0^2} + \frac{p_z - p_0}{p_0} \left(1 - \frac{m_1^2 + m_2^2}{4p_0^2} \right) \right],$$

$$\varepsilon_-(p_z, m_1, m_2) \approx \frac{\Delta m^2}{2p_0} \left[1 - \frac{p_z - p_0}{p_0} \right].$$

Finally, by simple algebraic manipulations and after ‘‘Gaussian’’ integrations, we find

$$\begin{aligned} \text{DCO}(t) \approx & 1 - \frac{m_1^2}{4p_0^2} + \exp \left[- \left(\frac{2p_0^2 - m_1^2}{\sqrt{2}ap_0^2} t \right)^2 \right] \frac{m_1^2}{4p_0^2} \\ & \times \left\{ \cos \left[\frac{2p_0^2 + m_1^2}{p_0} t \right] + \frac{4p_0^2 - 2m_1^2}{a^2 p_0^3} t \sin \left[\frac{2p_0^2 + m_1^2}{p_0} t \right] \right\} \\ & - \frac{m_2^2}{4p_0^2} + \exp \left[- \left(\frac{2p_0^2 - m_2^2}{\sqrt{2}ap_0^2} t \right)^2 \right] \frac{m_2^2}{4p_0^2} \\ & \times \left\{ \cos \left[\frac{2p_0^2 + m_2^2}{p_0} t \right] + \frac{4p_0^2 - 2m_2^2}{a^2 p_0^3} t \sin \left[\frac{2p_0^2 + m_2^2}{p_0} t \right] \right\}, \end{aligned} \quad (14)$$

$$\begin{aligned}
 \text{DFCO}(t) \approx & \exp \left[- \left(\frac{\Delta m^2}{2\sqrt{2}ap_0^2} t \right)^2 \right] \left\{ \left[1 - \frac{m_1^2 + m_2^2}{4p_0^2} \right] \cos \left[\frac{\Delta m^2}{2p_0} t \right] \right. \\
 & \left. + \frac{m_1^2 + m_2^2}{4p_0^2} \frac{\Delta m^2}{a^2 p_0^3} t \sin \left[\frac{\Delta m^2}{2p_0} t \right] \right\} \\
 & + \exp \left[- \left(\frac{4p_0^2 - m_1^2 - m_2^2}{2\sqrt{2}ap_0^2} t \right)^2 \right] \frac{m_1^2 + m_2^2}{4p_0^2} \left\{ \cos \left[\frac{4p_0^2 + m_1^2 + m_2^2}{2p_0} t \right] \right. \\
 & \left. + \frac{4p_0^2 - m_1^2 - m_2^2}{a^2 p_0^3} t \sin \left[\frac{4p_0^2 + m_1^2 + m_2^2}{2p_0} t \right] \right\}. \tag{15}
 \end{aligned}$$

4. Conclusions

In the standard treatment of neutrino oscillation, it is implicitly assumed that the use of scalar mass-eigenstate wave packets made up exclusively of positive frequency plane-wave solutions. Even though the standard oscillation formula could give the correct result when *properly* interpreted, a satisfactory description of fermionic (spin one-half) particles requires the use of the Dirac equation as evolution equation for the mass-eigenstates. In this paper, we have investigated how the the spinorial form and the interference between positive and negative frequency components of the mass-eigenstate wave packets modify the flavor oscillation probability. We have also observed that being the chiral operator not a constant of motion for massive particles, additional disappearing effects will be found in the Dirac conversion probability formula.

In this preliminary study, we have assumed an initial Gaussian localization and performed integrations by considering strictly peaked momentum distributions and ultra-relativistic mass-eigenstates. In such a case, we have been able to obtain an analytic expression for the oscillation probability formula. In the hypothesis of minimal slippage between the mass-eigenstate wave packets ($\Delta vL \ll a$), and for long distance between source and detector ($L \gg a$), i.e.

$$1 \ll \frac{L}{a} \ll \frac{p_0^2}{\Delta m^2},$$

the standard flavor oscillation probability¹⁰ is then reproduced,

$$\begin{aligned}
 & P(\nu_{\alpha,L} \rightarrow \nu_{\beta,L}; L) \\
 & \approx \sin^2 2\theta \left[1 - \frac{m_1^2 + m_2^2}{4p_0^2} \right] \left\{ 1 - \left[1 - \left(\frac{\Delta m^2}{2\sqrt{2}ap_0^2} L \right)^2 \right] \cos \left[\frac{\Delta m^2}{2p_0} L \right] \right\} / 2 \\
 & \approx \sin^2 2\theta \left\{ 1 - \cos \left[\frac{\Delta m^2}{2p_0} L \right] \right\} / 2 = \sin^2 2\theta \sin^2 \left[\frac{\Delta m^2}{4p_0} L \right]. \tag{16}
 \end{aligned}$$

The recent years have seen an increasing number of theoretical papers discussing oscillation phenomena. In particular, the standard wave plane treatment has been reviewed in view of the more appropriate *intermediate*^{11,12} and *external*^{13,14} wave packet frameworks. A common argument against the use of the intermediate wave packet formalism is that oscillating neutrinos are neither prepared or observed. Consequently, the calculation of transition probabilities between the observable particles involved in the production and detection process should better elucidate what happens in oscillation phenomena. In such a framework, the oscillating particle (described by internal line of a Feynman diagram) propagates between the source and target described by external wave packets. However, it can be shown¹⁵ that the overlap function of the incoming and outgoing wave packets in the external wave packet model is mathematically equivalent to that obtained in the intermediate wave packet formalism, and so a first analysis can be done in this more simple framework. In this paper, we have discussed only a few aspects of the theory. It would be desirable to discuss the problem within a *full* quantum field theoretic framework and from this point of view the intermediate wave packet approach used in this work only represents a preliminary step towards a real understanding of oscillation phenomena. In particular, it has been shown that the use of *fermionic* instead of *scalar* mass-eigenstates includes new physical effects (interference between positive and negative frequencies and chiral oscillation) and, consequently, some modifications in the oscillation formula. In this spirit, we propose the use of fermionic instead of scalar relativistic propagators in the external wave packet formalism. The main difficulty in carrying out a full field theoretical treatment of the oscillation phenomena is the appropriate definition of a Fock space for the weak (flavor) eigenstates.¹⁶

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