

Timelapse[¶]

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Received 22 April, 2002

Abstract—We discuss the existence, in an arbitrary frame, of a finite time for the transformation of an initial quantum state into another, e.g., in a decay. This leads to the introduction of a timelapse $\tilde{\tau}$, by analogy with the lifetime of a particle. An argument based upon the Heisenberg uncertainty principle suggests the value of $\tilde{\tau} = 1/M_0$. Consequences for the exponential decay formula and the modifications which $\tilde{\tau}$ introduces into the Breit–Wigner mass formula are described. © 2002 MAIK “Nauka/Interperiodica”.

PACS numbers: 03.65.Ta; 14.70.Hp; 13.90.+i

The subject of this paper concerns the exponential decay law and the Breit–Wigner (BW) mass-distribution formula. These formulas are a standard part of particle physics and can indeed be connected by a simple transform, as is taught in many undergraduate physics courses [1]. In recent years, the validity of the BW has been demonstrated to an unprecedented degree by the LEP data and analysis of the Z gauge particle [2]. After substantial theoretical corrections for radiative effects, the predicted theoretical width (assuming three light neutrinos) and the experimental value, based upon a BW fit, agree to better than one percent. We will be interested later in a small discrepancy, but our first observation is that the agreement is quite impressive. These facts are somewhat surprising because neither the exponential decay law nor the BW mass curve is predicted rigorously within Quantum Mechanics (QM). On the contrary, we have precise QM objections to the former [3, 4] and only approximate derivations of the latter [5, 6]. Nor does a field theoretical treatment change the situation. To some, these QM results relegate our two formulas to little more than phenomenological games. We, on the other hand, start with these two formulas and argue for a modification which will in part reconcile the decay law with QM and provide an explanation for the discrepancy in the Z width described above.

One of the implicit assumptions in particle physics is that decays occur instantaneously. However, as Einstein has taught us, instantaneity, for anything other than a point, can at best be valid in a single Lorentz frame. Thus, even if, say, in its rest frame, a particle decayed instantaneously, a general observer would find different times for decays at different points within the wave

packet. Of course, this could only be determined in a statistical sense since the wave function is not an observable. Thus, in general, there will exist times during which the quantum state is neither the initial nor the final state but a linear combination of both that tends towards the latter with increasing time. We shall call a measure of this time interval the “timelapse.”

There is only one exception to the above observations; a measurement process may involve (ideally) the localization in space and time of a particle. This wave function collapse is instantaneous for all observers (it defines the corresponding event in each frame) and is fundamentally irreversible, since the creation of a particle at a given space-time point cannot instantaneously inflate to finite space regions without violating the limiting velocity of light. The wave function collapse is a subject of great interest in itself but will not concern us further in this paper.

Why cannot we avoid the discussion of timelapse by considering particle or state creation to be a delta function in space and time? First, because in many practical problems we know that this is not the case, e.g., as for a particle trapped in a potential well, e.g., a muon within a muonic atomic state. Second, because we often know or desire to study particles which approximate energy–momentum eigenstates, and this implies large spatial dimensions.

An earlier introduction of a type of timelapse is contained in the book by Jackson [7]. In one of the classical derivations of essentially quantum effects, Jackson introduces the “formation time” of an electron in the nuclear beta decay. Arguing that the outgoing energetic electron could be considered as accelerated from rest to its final velocity over a finite formation time, he calculates the induced spectrum of radiation by the electron

[¶]This article was submitted by the authors in English.

(inner bremsstrahlung). Jackson also notes that the same effect would occur if a charge was created during the same time interval. Invoking the uncertainty principle, he evaluates this time interval Δt as

$$\Delta t \sim 1/E, \quad (1)$$

where E is the electron energy. Now, while acknowledging the precedence of Jackson's idea of a formation time, his approach is significantly different from ours. The use of the particle energy in Eq. (1) implies that different particles take different times for acceleration. Of course, the antineutrino does not contribute to the inner bremsstrahlung and the heavy nucleon contributions are negligible, so this may appear to be an academic question. However, to us, it is obvious that the same timelapse must occur for each of the particles in the final state, independent of their final energy or momentum. It is not conceivable that the outgoing electron is created with a certainty close to unity, while the antineutrino is, perhaps, to all extents still to be created. We will take care to define for each decay a common timelapse that depends, at most, upon the kinematics of the initial system in a preferential Lorentz frame. Of course, one must also guarantee that the timelapse of the outgoing state coincides with that of the vanishing incoming state.

Is the timelapse a function of the spatial localization of the quantum state? We believe not. There is a Δt directly connected to Δx , but this is what we might call "passage time." Consider a single particle with a sufficiently large Δx in the approximate $(E, p, 0, 0)$ eigenstate (for simplicity we take its momentum along the x axis). Using the uncertainty principle $\Delta x \Delta p \sim 1$ and the Einstein relation $E^2 = p^2 + M^2$, we find

$$\Delta x \longrightarrow \Delta p \longrightarrow \Delta E = p \Delta p / E \sim v / \Delta x;$$

hence,

$$\Delta t \sim \Delta x / v, \quad (2)$$

where v is the velocity of the particle. Thus, Δt is for our wave packet a measure of the time it takes to pass a given y - z plane, hence, the name passage time. This Δt obviously has nothing to do with a timelapse, and indeed becomes infinite in the particle rest frame. On the other hand, we expect from the approximate validity of the exponential decay law that a timelapse must be small compared to the lifetime of a particle in any frame (see below).

We believe that timelapse must be a close relative of lifetime. As for lifetime, we will define it in the rest frame of an initial single-particle state, or, more precisely, the frame in which the average velocity is null. By analogy with the lifetime τ , we will denote the timelapse for a process by $\tilde{\tau}$. What does the Heisenberg uncertainty principle tell us? We have excluded a connection to ΔE and can also exclude the halfwidth ΔM for a decay particle, since this is reserved for τ ,

$$\tau = 1/\Delta M. \quad (3)$$

This leaves us with essentially only one choice,

$$\tilde{\tau} = 1/M_0, \quad (4)$$

for a decaying particle with central mass M_0 .

Now, a decay of a composite particle such as a J/ψ may be considered an annihilation and/or interaction at the quark level. Thus, a timelapse for interactions should also be defined. The natural choice for $\tilde{\tau}$ in these cases is

$$\tilde{\tau} = 1/E_{CM}, \quad (5)$$

where E_{CM} is the center-of-mass energy. Equation (5) would automatically include Eq. (4) if it was not for the fact that a given decay may occur at a mass diverse from M_0 due to the existence of mass curves. To reconcile the two, we should modify Eq. (4) as

$$\tilde{\tau} = 1/M, \quad (6)$$

but in the subsequent applications in this paper we will employ Eq. (4) for simplicity.

How does the existence of a $\tilde{\tau}$ modify the exponential decay law? This can easily be derived after assuming a given analytic form for a state during timelapse. For simplicity and by analogy with the original decay law, we will assume this to be an exponential form. Exponential decrease $\exp[-t/\tilde{\tau}]$ is assumed for the incoming state and its complement, $1 - \exp[-t/\tilde{\tau}]$, for the corresponding outgoing state. Thus, when we consider an ensemble of $N(t)$ particles with a given lifetime τ and timelapse $\tilde{\tau}$, we must divide them into two classes: N_u , the number of undecayed particles, and N_d , those that have begun the decay process but have still a residual probability of being found in a measurement of $N(t)$. N_d would be zero if the instantaneous decay ($\tilde{\tau} = 0$) was valid.

The differential equation that governs $N_u(t)$ is the standard one

$$dN_u(t) = -\frac{1}{\tau} N_u(t) dt, \quad (7)$$

while that for $N_d(t)$ must allow for a source term proportional to $dN_u(t)$,

$$dN_d(t) = -\frac{1}{\tilde{\tau}} N_d(t) dt - dN_u(t); \quad (8)$$

the negative sign of the last term is correct since $dN_u(t) < 0$ for $dt > 0$.

Now, solving these coupled equations and using initial condition $N_d(0) = 0$, we find

$$N(t) = \frac{N(0)}{\tau - \tilde{\tau}} \left[\tau \exp\left(-\frac{t}{\tau}\right) - \tilde{\tau} \exp\left(-\frac{t}{\tilde{\tau}}\right) \right]. \quad (9)$$

This is the modification of the standard exponential decay law that our timelapse $\tilde{\tau}$ introduces. Note that for

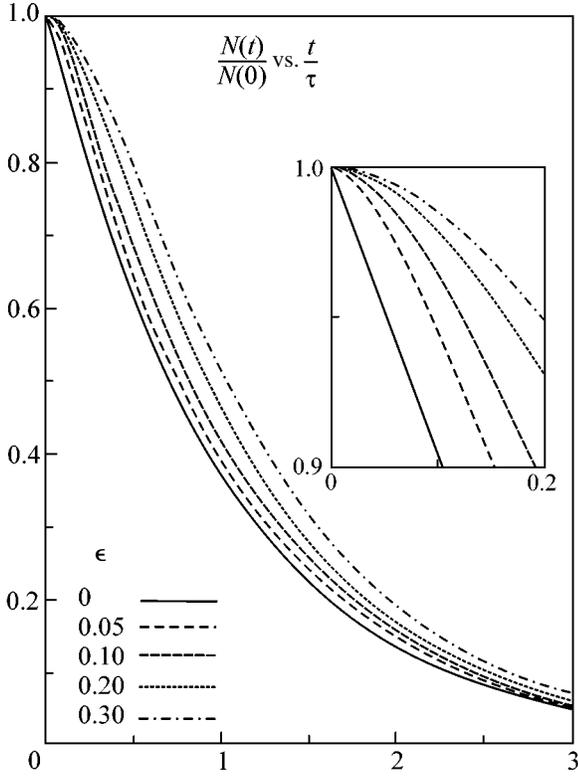


Fig. 1. The decay law $N(t)/N(0)$ versus t/τ for various values of ϵ .

$\tilde{\tau} = 0$ we obtain the single exponential form, and the limit $\tau = 0$ yields an exponential decay with lifetime $\tilde{\tau}$. Whence $\tilde{\tau} = 1/M_0$ sets a lower limit to the effective lifetime and, hence, an upper limit to the halfwidth.

A simple calculation yields, for the effective lifetime τ_{eff} ,

$$\tau_{\text{eff}} = (\tau^3 - \tilde{\tau}^3)/(\tau^2 - \tilde{\tau}^2). \quad (10)$$

For $\tilde{\tau} \ll \tau$, we can write

$$\tau_{\text{eff}} = \tau[1 + \epsilon^2 + O(\epsilon^3)], \quad (11)$$

where $\epsilon = \tilde{\tau}/\tau$. However, we note that the decay rate Γ remains connected to τ , $\Gamma = 1/\tau$. This follows from our assumption that $\tilde{\tau} \sim 1/M_0$, and, hence, it is independent of any interaction coupling constants, in contrast to Γ and τ , which obviously are directly dependent.

Another very interesting observation is that, for very small t , Eq. (9) has no linear term in t . Indeed for $t \ll \tilde{\tau}$, τ

$$N(t) = N(0) \left[1 - \frac{t^2}{2\tau\tilde{\tau}} + O(t^3) \right]. \quad (12)$$

This reconciles, for short times, our modified decay law (no longer a single exponential) with basic quantum-mechanical arguments [8–10], which have led, amongst other things, to the so-called quantum Zeno effect [11,

12]. This, at least, in principle allows $\tilde{\tau}$ to be calculated from Eq. (12) and the quantum-mechanical result $P(t) = 1 - t^2(\Delta H)^2 + \dots$ [13]; specifically $\tilde{\tau} = 1/[2\tau(\Delta H)^2] = \Gamma/[2(\Delta H)^2]$.

In Fig. 1, we show the modification of $|\psi(t)|^2 \equiv P(t)$ for various ϵ values. In this plot, the annulment of the effective lifetime is evident, as is the annulment of $dP(t)/dt$ (insert) for $t \rightarrow 0$ (source of the quantum Zeno effect). The direct measurement of $|\psi(t)|^2$ is often possible (e.g., in muon decay). However, for muons $\epsilon \sim 3 \times 10^{-16}$, so that no effect due to $\tilde{\tau}$ could ever be detected.

Let us now calculate the modification in the standard BW mass formula produced by Eq. (9). We have

$$|\psi(t)| \propto \exp\left(-\frac{t}{2\tau}\right) \left\{ 1 - \epsilon \exp\left[\frac{t(\epsilon - 1)}{\epsilon\tau}\right] \right\}^{1/2}.$$

Hence,

$$\begin{aligned} \chi(x) &= \int_0^\infty dt \exp\left(i\frac{x}{2\tau}t\right) |\psi(t)| \\ &\propto \frac{1}{x+i} - \sum_{n=1}^\infty \frac{n!!}{2^n n!} \frac{\epsilon^n}{x+ia_n}, \end{aligned}$$

where $x = 2\tau(M - M_0)$ and $a_n = 1 + 2n(1 - \epsilon)/\epsilon$. Treating ϵ as a small quantity, we may perform an analytic calculation of $|\chi(x)|^2$ to the lowest order in ϵ . We find our modified Breit–Wigner (MBW) as follows:

$$\begin{aligned} \text{MBW} \equiv |\chi(x)|^2 &= BW \left\{ 1 - \epsilon \frac{x^2 + a_1}{x^2 + a_1^2} \right. \\ &\quad \left. + \frac{\epsilon^2}{4} \left[\frac{7}{2} + \frac{x^2 + 1}{x^2 + a_1^2} - \frac{x^2 + a_2}{x^2 + a_2^2} \right] + O(\epsilon^3) \right\}, \end{aligned} \quad (13)$$

where $BW = 2\tau M_0 / \{[\pi/2 + \arctan(2\tau M_0)](x^2 + 1)\}$ is the standard BW.

Now, ϵ is so small for almost all weak or electromagnetic decays that one might think of passing directly to the strong decays in search of the evidence of a MBW. However, of all weak processes, a special role is played by the decays of heavy intermediate vector bosons W and Z . These have widths of several GeV and, thus, correspond to ϵ of a few %. Furthermore, the data on Z are particularly precise with errors in M_z and Γ_z on the order of 10^{-5} . The LEP data yield such precise results that there are even two standard deviations from the theory in Γ_z . This is expressed by two equivalent numbers [2]

$$\Gamma_{\text{inv}} = -2.7_{-1.5}^{+1.7} \text{ MeV}$$

and/or

$$N_v = 2.9841 \pm 0.0083.$$

Now, for Z we can apply the formula given above, since $\epsilon_Z = 2.7\%$. From this formula (see also Fig. 2) we readily see that

(1) the maximum modification to the underlying BW ($\tilde{\tau} = 0$) occurs at $M = M_0$, i.e., at the peak. This effect is an increase of the peak value by $1 + 3\epsilon^2/8$. Note, however, that this underlying BW must not be confused with the best fit BW need to the data in the presence of a non-negligible $\tilde{\tau}$;

(2) the halfwidth of the underlying BW and the MBW are almost the same for small ϵ . That of the MBW is reduced by a factor of order ϵ^3 and not ϵ^2 , as might have been expected from Eq. (11) for the lifetime modification.

The smallest errors of the curve are around the peak value $M = M_0$. In a fit with a BW to data, in accord with our modified curve, one would be inclined to raise the peak value with consequently the same decrease in percentage of the halfwidth. Hence, we expect the fitted BW to yield a width lower than ours by, at most, the factor $1 - 3\epsilon^2/8$. Hence our estimate of Γ_{inv} is

$$-0.7 \text{ MeV} \leq \Gamma_{inv} < 0$$

less than 1/4 of the measured central value. It is amusing to note that, using the result $\Gamma_{eff} = (1 + \epsilon^2)\Gamma$, one might have expected an effect on Γ_{inv} in good agreement with the experimental value.

The modified BW can also be calculated numerically for any ϵ , and in Fig. 2 we show the ratio of this with the underlying BW for various ϵ values. The width is indeed reduced. From Fig. 2, we see that, for $\epsilon \leq 0.3$ and such that $1.4 > M/M_0 > 0.6$, the main modification indeed occurs at $M = M_0$ and is an increase on the order of a few % or less. This means that no significant evidence for the existence of $\tilde{\tau}$ from mass curves is possible until the individual errors of the data points are of this order or better.

Obviously, in looking for evidence for our modified BW, we are led to consider the largest ϵ values available. This means particles with strong interaction decays. For example, the $\rho(770)$, where $\epsilon \sim 20\%$. However, the best data points for ρ [14] are somewhat dated and are not yet precise enough to yield evidence for a $\tilde{\tau}$.

It is natural to extend the concept of timelapse from the realm of decays to interactions in general. We have already anticipated that in these cases $\tilde{\tau} = 1/E_{CM}$. However, what is $\tilde{\tau}$ to be compared with? What plays the role of τ , here? The only thing available is $\sqrt{\sigma}$ the square root of the cross section. For numerical compar-

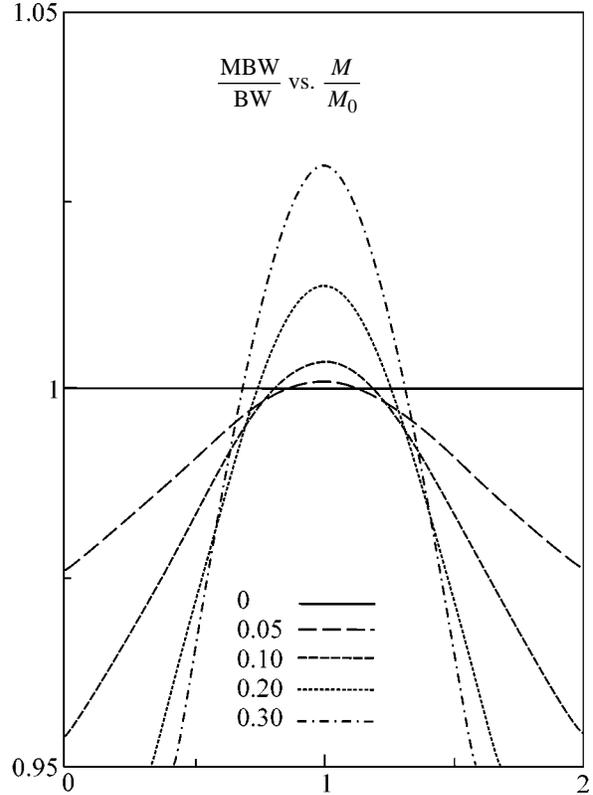


Fig. 2. Numerical calculation of the ratio of the modified and standard BW mass formulas MBW/BW versus M/M_0 for different values of ϵ .

isons, we recall that $\sqrt{60 \text{ mb}} \sim 10^{-23} \text{ s}$, while $1 \text{ GeV} \sim 6.6 \times 10^{-25} \text{ s}$.

Finally, we wish to discuss briefly our motive for this investigation, which seems at first sight far removed from the content of this paper. In oscillation studies, some authors insist that a single time interval is involved. The argument is essentially that both the creation of, say, a flavor neutrino and its detection, possibly as a different flavor, occur at fixed times. Now, as we explained in our introduction, such a situation, instantaneous creation, could at most be valid in a unique Lorentz frame, which improbably coincides with the laboratory. However, the existence of an intrinsic timelapse $\tilde{\tau}$ would imply that instantaneous creation is a myth in any frame. In practice, this means that in interference studies we must deal with multiple times in a similar way that the slippage of interfering wave packets obliges us to consider multiple distance intervals between creation and observation.

In conclusion, we have argued that, in an arbitrary frame, a wave packet will take a finite time to “grow” to or decay from its full normalized value. This encouraged us to postulate the existence, in the preferential center of a mass frame, of an intrinsic timelapse $\tilde{\tau}$ by analogy with τ . Such an assumption leads to a modifi-

cation of the decay formula and, consequently, of the BW mass formula. We have also suggested that $\tilde{\tau} = 1/E_{CM}$ on the basis of the Heisenberg uncertainty principle. In practice, the modification of the decay formula is not experimentally detectable. However, it has, as an aside, reconciled for small times the decay law with basic quantum-mechanical arguments (at least within the hypothesis of an exponential dependence upon $\tilde{\tau}$). The BW mass formula is a more practical tool for detecting a $\tilde{\tau}$. Comparing the fits to the data upon ρ decay suggest that, with improved experiments (precision on the order of 10^{-3}), we could distinguish between the standard and modified BW. Note that with $\epsilon = \Delta M/M_0$ the modified version has no extra free parameters. We may simply compare the best χ^2 fits of both to the data. At the moment, the most promising source for evidence of a $\tilde{\tau}$ appears to be in the Z decay. Timelapse provides a justification for the existence of a negative Γ_{inv} . But we must remember that this is experimentally only a two sigma effect.

We thank R. Anni and G. Co' for interesting discussions. One of the authors (SdL) is grateful for the hospitality of the Physics Department of Lecce University, where the paper was written. This work was supported in part by the FAEP (UniCAMP).

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