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# REMARKS UPON THE MASS OSCILLATION FORMULAS

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The standard formula for mass oscillations is often based upon the approximation $t \approx L$ and the hypotheses that neutrinos have been produced with a definite momentum $p$ or, alternatively, with definite energy $E$. This represents an inconsistent scenario and gives an unjustified reduction by a factor of two in the mass oscillation formulas. Such an ambiguity has been a matter of speculations and mistakes in discussing flavor oscillations. We present a series of results and show how the problem of the factor two in the oscillation length is not a consequence of Gedanken experiments, i.e. oscillations in time. The common velocity scenario yields the maximum simplicity.

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## 1. Introduction

One of the most popular fields of research in particle physics phenomenology of the past few decades has been, and still is, that of neutrino oscillations.[1−11] Publications in this field have accompanied an ever increasing and stimulating series of experiments involving either solar, atmospheric or laboratory neutrinos.[12−23] The vast majority of the theoretical studies consider the possibility of massive neutrinos distinct from the flavor eigenstates created in the various production processes. Neutrinos are not the only example of such a phenomenon. The first examples of flavor oscillations observed were in the the kaon system[24,25] where the strong interaction is involved in the particle creation. We shall continue to refer in this work to neutrinos, but the considerations are quite general. What we shall call the *factor two problem* in the neutrino oscillation formulas has already been observed and

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discussed in previous papers,[26–29] but the situation is still surprisingly confused and probably still subject to argument. We would like to close the question with this letter, but more realistically, we shall simply contribute to the general debate.

In this work we neglect the Heisenberg uncertainty principle.[30] Physically this means that the wave packet of the created particle is large enough for its mass eigenstates to be assigned (as a good approximation) definite four-momentum. Indeed, the wave packet must be so large as to allow us to legitimately assume a plane wave phase factor for each mass eigenstate.

Definite four-momentum implies definite velocities $v_n$, which are in general not the same. Now in the Lorentz phases we must eventually insert values for $x$ and $t$. Different velocities automatically imply different values of $x_n/t_n$.

At creation the mass-eigenstate wave packets must coincide if a definite flavor is created *instantaneously*. However, this is a frame-dependent condition and in any serious analysis, one must allow for a "formation time" during creation. This fact alone allows the introduction of different $t_n$ since at the laboratory the trigger will correspond (except for the equal velocity case) to nonequivalent points of the individual wave packets. This also means different $x_n$, whichever is the triggering point of the wave functions.

Our approach treats on an equal footing space–time and energy–momentum. Derivations of the oscillation formula which do *not* do this abound in the literature. The most common are of two classes, one which imposes in some way (see the next section) a common value for $x/t$ notwithstanding different velocities. The other class employs, a non-Lorentz invariant phase such that only time appears (for which a unique value is assumed) and then selects the Lorentz frame which yields the "desired" result.

We begin the next section with one of the most authorative demonstrations of the standard oscillation formula which is based upon the approximation $x \approx t = L$. We *correct* this derivation and show that it doubles the oscillation phase. In Sec. 3 a number of short calculations are presented, each valid for different assumptions for the space–time intervals and the energy–momentum eigenvalues.

We draw our conclusions in Sec. 4, where we emphasize that only situations of exactly equal ultra-relativistic velocities and for $\Delta M \ll M_n$ will the standard formula be obtained. The important historical role and as a source of both high and low energy experiments played by the neutral kaon sector is briefly discussed.

## 2. Neutrino Mixing

To focus the contents of this letter we begin by claiming that standard oscillation formula, for mixing between mass eigenstates, is often based upon the approximation $t \approx L$ and the assumptions of definite momentum or definite energy for the neutrinos created.

To explain and understand the common mistakes in oscillation calculations, let us briefly recall the standard approach. The most important aspect of neutrino oscillations can be understood by studying the explicit solution for a system with only two types of neutrinos. For this two-flavor problem, the flavor eigenstates $|\nu_\alpha\rangle$ and $|\nu_\beta\rangle$ are represented by a coherent linear superposition of the mass eigenstates $|\nu_1\rangle$ and $|\nu_2\rangle$.

$$\begin{pmatrix} |\nu_\alpha\rangle \\ |\nu_\beta\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix} . \tag{1}$$

The time evolution of $|\nu_\alpha\rangle$ is determined by solving the Schrödinger equation for the $|\nu_{1,2}\rangle$ component of $|\nu_\alpha\rangle$ in the rest frame of that component

$$|\nu_n(\tau_n)\rangle = \exp[-iM_n\tau_n]\,|\nu_n\rangle, \qquad n = 1,2 \tag{2}$$

where $M_n$ is the mass of $|\nu_n\rangle$ and $\tau_n$ is the time in the mass eigenstate frame. In terms of the time $t$ and the position $L$, the Lorentz-invariant phase factor in Eq. (2) must be rewritten as

$$\exp[-iM_n\tau_n] = \exp[-i(E_n t - p_n L)]\,, \tag{3}$$

where $E_n$ and $p_n$ are the energy and the momentum of the mass eigenstates in the laboratory, or any other, frame. In the standard approach the above equation is followed by this statement: In practice, the neutrino is extremely relativistic, so the evaluation of the phase factor of Eq. (3) is calculated by making the approximation $t \approx L$. Consequently, Eq. (3) becomes in this approximation

$$\exp[-i(E_n - p_n)L]\,. \tag{4}$$

For example, in the latest presentation contained in the Review of Particle Physics,[31] the flavor oscillation probability reads

$$P(\nu_\alpha, \nu_\beta) \approx \sin^2 2\theta \sin^2\left(\frac{L}{4E}\,\Delta M^2\right) \approx \sin^2 2\theta \sin^2\left(1.27\frac{L(\mathrm{km})}{E(\mathrm{GeV})}\Delta M^2\,(\mathrm{eV}^2)\right)\,, \tag{5}$$

where $\Delta M^2 \equiv M_1^2 - M_2^2$. Equation (5) is obtained by calculating the phase factor for each mass eigenstate traveling in the $x$ direction, $\exp[-i(E_n t - p_n L)]$, with the approximation $t \approx L$ and the assumptions that $|\nu_\alpha\rangle$ has been produced with a definite momentum $p$,

$$E_n = \sqrt{p^2 + M_n^2} \approx p + \frac{M_n^2}{2p}\,,$$

or, alternatively, with a definite energy $E$,

$$p_n = \sqrt{E^2 - M_n^2} \approx E - \frac{M_n^2}{2E}\,.$$

The phase factor of Eq. (3) then reads

$$\exp\left[-i\frac{M_n^2}{2p}L\right] \quad \text{or} \quad \exp\left[-i\frac{M_n^2}{2E}L\right]\,. \tag{6}$$

"*Since highly relativistic neutrinos have $E \approx p$, the phase factors in Eq.* (6) *are approximately equal. Thus, it doesn't matter whether $|\nu_\alpha\rangle$ is created with definite* *momentum or definite energy.*" – Kayser.[31] Now this result is incorret as we shall show below. First note that $t = L$ implies for consistency

$$\frac{p_1}{E_1} = \frac{p_2}{E_2} = 1\,,$$

which, if simultaneously applied, eliminates the phase-factor completely. Null phase factors, as in all cases of equal phase factors for each mass eigenstate, preclude any oscillation phenomena. *Nor is such an approximation justified within a more realistic wave-packet presentation.*[4] Returning to the simplified plane wave discussion, one should simply write

$$\exp[-i(Et - pL)] = \exp\left[-i\left(\frac{E}{v} - p\right)L\right] = \exp\left[-i\,\frac{E^2 - p^2}{p}\,L\right] = \exp\left[-i\frac{M^2}{p}L\right]\,,$$

which differs from Eq. (6) by a *factor of two* in the argument. This simply doubles the coefficient of $\Delta M^2$ in the standard oscillation formulas.

The above result has already been noted by Lipkin,[26] who however observes this ambiguity for the case of equal three-momentum of the neutrino mass eigenstates, the "non-experiments" as he calls them, but not for his chosen equal energy scenario. *We believe that only experiment can determine if in a given situation the neutrinos are produced with the same momentum or energy or neither.* For this reason we wish to present below the differences in the various assumptions which are particularly significant for nonrelativistic velocities, admittedly not very practical for the neutrino but very important for the kaon system. In any case we emphasize that the aforementioned factor two appears not only in the scenario of common momentum but also for the equal energy assumption or "real experiments" as Lipkin[29] calls them.

In realistic situations the flavor neutrino is created in a wave packet at time $t = 0$ and thus over an extended region. We simplify our discussion by ignoring, where possible, this localization but we must note that it is essential to give an approximate significance to $L$, the distance from source to measuring apparatus, or $t$, the time of travel.

## 3. Time or Space Oscillations?

Assume that the state $|\nu_\alpha\rangle$ is created at $t = 0$ in $x \approx 0$. Introduce the Lorentz invariant plane wave factor and apply them to the mass eigenstates at a later time $t$ and for position $x$. Since the neutrino is created over an extended volume and the apparatus cannot be considered without dimension, the interference effects will involve in general amplitudes of states with different time and distance intervals. Different time intervals $t_1 \neq t_2$ may seem an unnecessary and unphysical abstraction. However, it is needed for self-consistency. Even if in a given frame the creation is considered instantaneous it will not generally appear so for another observer, given the extended dimension of the wave function. For this latter observer there will exist times when the probability of measuring the created particle is between 0 and

1. This implies the introduction, in general, of a time dependence for the growth of a wave function at each $x$ in all frames. Furthermore, if we fix $L_1 = L_2 \equiv L$, and have different velocities, $v_1 \neq v_2$, we must necessarily allow for $t_1 \neq t_2$. All this does not mean that the cases listed below are equally realistic.

Within the approximation of an effective one-dimensional treatment, we consider three broad classes:

- Common momentum, $p_1 = p_2 = p$, $E_1 \neq E_2$, $v_1 \neq v_2$;
- Common energy, $E_1 = E_2 = E$, $p_1 \neq p_2$, $v_1 \neq v_2$;
- Common velocity, $E_1 \neq E_2$, $p_1 \neq p_2$, $v_1 = v_2$.

The above cases by no means exhaust all possibilities but they are sufficient to cover almost all the assumptions made in the literature and lead to the subtle differences of the resulting formulas for $P(\nu_\alpha, \nu_\beta)$ which we are interested in.

The space–time evolution of $|\nu_\alpha\rangle$ and $|\nu_\beta\rangle$ is determined by the space–time development of the mass eigenstates $|\nu_1\rangle$ and $|\nu_2\rangle$. In the laboratory frame, we have

$$|\nu_n(L_n, t_n)\rangle = \exp[-i(E_n t_n - p_n L_n)]|\nu_n\rangle, \qquad n = 1, 2\,.$$

Consequently, the probability of observing a neutrino of a different flavor is

$$P(\nu_\alpha, \nu_\beta) = \sin^2 2\theta \sin^2\left[\frac{1}{2}\left(E_2 t_2 - E_1 t_1 - p_2 L_2 + p_1 L_1\right)\right]\,. \tag{7}$$

We can eliminate the space or time dependence in the previous formula by using the relations

$$L_1 = \frac{p_1}{E_1} t_1 \qquad \text{and} \qquad L_2 = \frac{p_2}{E_2} t_2\,.$$

For time oscillations, we have

$$P(\nu_\alpha, \nu_\beta) = \sin^2 2\theta \sin^2\left[\frac{M_2^2}{2E_2} t_2 - \frac{M_1^2}{2E_1} t_1\right]\,, \tag{8}$$

whereas, for space oscillations, we obtain

$$P(\nu_\alpha, \nu_\beta) = \sin^2 2\theta \sin^2\left[\frac{M_2^2}{2p_2} L_2 - \frac{M_1^2}{2p_1} L_1\right]\,. \tag{9}$$

### 3.1. *Common momentum scenario*

For *common momentum* neutrinos productions, we shall consider two different situations, common arrival time and fixed laboratory distance,

$*$ $t_1 = t_2 = t$, $\qquad P(\nu_\alpha, \nu_\beta) = \sin^2 2\theta \sin^2\left[\frac{t}{2}\left(\frac{M_2^2}{E_2} - \frac{M_1^2}{E_1}\right)\right];$

$*$ $L_1 = L_2 = L$, $\qquad P(\nu_\alpha, \nu_\beta) = \sin^2 2\theta \sin^2\left[\frac{L}{2p}(M_2^2 - M_1^2)\right].$

As already mentioned, Lipkin[29] time oscillations represent non-experiments or Gedanken experiments because they measure time oscillations. For "real" experiments, in the scenario of common momentum neutrino production, we should use the formula

$$P(\nu_\alpha,\nu_\beta)=\sin^2 2\theta \sin^2\left[\frac{L}{2p}\Delta M^2\right]\,. \tag{10}$$

In terms of the average neutrino energy,

$$E_{\rm av}=\frac{E_1+E_2}{2}=p\left[1+\frac{M_1^2+M_2^2}{4p^2}+\mathcal{O}\left(\frac{M^4}{p^4}\right)\right]\,,$$

we can rewrite the previous equation, in the ultra-relativistic limit as

$$P(\nu_\alpha,\nu_\beta)\approx\sin^2 2\theta \sin^2\left[\frac{L}{2E_{\rm av}}\,\Delta M^2\right]\,. \tag{11}$$

Thus, we find a factor two difference between the oscillation coefficient in this formula and the standard mass oscillation formula of Eq. (5).

### 3.2. *Common energy scenario*

Let us now consider *common energy* neutrinos productions,

$*$ $t_1=t_2=t$, $\quad P(\nu_\alpha,\nu_\beta)=\sin^2 2\theta \sin^2\left[\frac{t}{2E}(M_2^2-M_1^2)\right]$;

$*$ $L_1=L_2=L$, $\quad P(\nu_\alpha,\nu_\beta)=\sin^2 2\theta \sin^2\left[\frac{L}{2}\left(\frac{M_2^2}{p_2}-\frac{M_1^2}{p_1}\right)\right]$.

Space oscillations are described by

$$P(\nu_\alpha,\nu_\beta)=\sin^2 2\theta \sin^2\left[\frac{L}{2}\Delta\left(\frac{M^2}{p}\right)\right]\,, \tag{12}$$

where

$$\Delta\left(\frac{M^2}{p}\right)=\frac{M_2^2}{p_2}-\frac{M_1^2}{p_1}\,.$$

This can be written as

$$\Delta\left(\frac{M^2}{p}\right)=\frac{\Delta M^2}{E}\left[1+\frac{M_1^2+M_2^2}{2E^2}+\mathcal{O}\left(\frac{M^4}{E^4}\right)\right]\,.$$

Consequently, in the ultra-relativistic limit, Eq. (12) becomes

$$P(\nu_\alpha,\nu_\beta)\approx\sin^2 2\theta \sin^2\left[\frac{L}{2E}\Delta M^2\right]\,. \tag{13}$$

*The factor two difference is thus also present in common energy scenarios.* The formulas in Eqs. (10) and (12) tend to the same result in the ultra-relativistic limit, Eqs. (11) and (13), and are in disagreement with the standard formula, Eq. (5). In theory, at least the differences between them may be experimentally determined, especially for nonrelativistic processes.

### 3.3. *Common velocity scenario*

The scenario of different momentum and energy neutrinos productions, with *common velocity*, merits special attention, because only if $v_1 = v_2$ the probability $P(\nu_\alpha, \nu_\beta)$ is valid for all times. Otherwise, $P(\nu_\alpha, \nu_\beta)$ is valid only until the wave packet for the two mass eigenstates overlap substantially. This complication does not exist for $v_1 = v_2$. Indeed, with this condition, the wave packets travel together for all observers and we may even employ a common $L$ and common $t$. Furthermore, there exists in this case a rest frame, $v = 0$, for our flavor eigenstate common to that of the mass eigenstates. This situation is implicit in all calculations that use a common proper time $\tau$. By assuming a common velocity scenario, we must necessarily require different momentum and energies for the neutrinos produced. Due to the common velocity, the time evolution for the mass eigenstates in the common rest frame is

$$|\nu_n(\tau)\rangle = \exp[-iM_n\tau]|\nu_n\rangle\,, \tag{14}$$

and only in this case is Eq. (3) really justified with its non-indexed time and distance. In fact, for common velocities, the Lorentz-invariant phase factor can be rewritten in terms of the common time, $t$, and the common position, $L$, in the laboratory frame. We can eliminate the time dependence in the previous formula by using the relations

$$L = \frac{p_1}{E_1}t = \frac{p_2}{E_2}t\,.$$

Space oscillations are thus described by

$$P(\nu_\alpha, \nu_\beta) = \sin^2 2\theta \sin^2\left[\frac{L}{2}\left(\frac{M_2^2}{p_2} - \frac{M_1^2}{p_1}\right)\right]\,. \tag{15}$$

This equation is formally equivalent to Eq. (13) and thus, at first glance, it seems to reproduce the factor two difference. *This is a wrong conclusion!* Indeed, in the scenario of common velocity,

$$p_1 = M_1\gamma_v v \qquad \text{and} \qquad p_2 = M_2\gamma_v v$$

and consequently

$$\Delta\left(\frac{M^2}{p}\right) = \frac{M_2 - M_1}{\gamma_v v} = \frac{\Delta M^2}{2p_{\rm av}}\,.$$

Space oscillations, in the common velocity scenario, are thus described by

$$P(\nu_\alpha, \nu_\beta) = \sin^2 2\theta \sin^2\left[\frac{L}{4p_{\rm av}}\,\Delta M^2\right]\,, \tag{16}$$

and this recalls the standard result with the factor four in the denominator. However, it must be noticed that for this case

$$\frac{E_1}{E_2} = \frac{p_1}{p_2} = \frac{M_1}{M_2}$$

and this may be very far from unity. Thus, the use of $p_{\rm av}$ in Eq. (16) is not exactly the $E$ intended in the standard formula, which was identical, or almost, for both neutrinos.

We conclude our discussion by giving the mass oscillation formula in terms of the "standard" phase factor

$$\frac{L}{4E_{\rm av}}\Delta M^2\,,$$

and the parameter

$$\alpha=\left(1+\frac{\gamma_1 M_1}{\gamma_2 M_2}\right)\left(\frac{1}{v_2}-\frac{1}{v_1}\frac{\gamma_2 M_1}{\gamma_1 M_2}\right)\left(1-\frac{M_1^2}{M_2^2}\right)^{-1}\,.$$

The new formula reads

$$P(\nu_\alpha,\nu_\beta)=\sin^2 2\theta\sin^2\left[\alpha\frac{L}{4E_{\rm av}}\Delta M^2\right]\,. \tag{17}$$

For common velocity, momentum and energy, the parameter $\alpha$ becomes

$$\alpha_v\equiv\alpha[v_1=v_2]=1/v\,,$$

$$\alpha_p\equiv\alpha[p_1=p_2]=(v_1+v_2)/v_1v_2\,,$$

$$\alpha_E\equiv\alpha[E_1=E_2]=2\left(1+\frac{1}{v_1v_2}\right)\Big/(v_1+v_2)\,.$$

Finally, in the ultra-relativistic limit, by killing the $\mathcal{O}(\frac{M^4}{p^4},\frac{M^4}{E^4})$ terms, we obtain

$$\alpha_v\approx 1+(M_1^2/p_1+M_2^2/p_2)/4p_{\rm av}\,,$$

$$\alpha_p\approx 2+(M_1^2+M_2^2)/2p^2\,,$$

$$\alpha_E\approx 2+(M_1^2+M_2^2)/E^2\,.$$

## 4. Conclusions

The creation of a particle may differ from process to process, therefore only experiment can decide which, if any, of the above situations are involved.[32] However, we wish to point out that *the assumptions of same momentum, p, or same energy, E, can only be valid in, at most, one reference frame.* It seems to us highly unlikely that this frame happens to coincide with our laboratory frame. This means that if the common velocity scenario, *which is frame-independent*, is not satisfied, we may legitimately doubt that any of the popular hypotheses coincides with any given experimental situation.

The plane wave treatment of mass oscillations gives the result,

$$P(\nu_\alpha,\nu_\beta)\approx\sin^2 2\theta\sin^2\left[\frac{L}{4E_{\rm av}}\,\Delta M^2\right]\,,$$

*under certain conditions*, i.e. the different mass eigenstates have a common ultra-relativistic velocity, $v_1 = v_2 = v$,

$$P(\nu_\alpha, \nu_\beta) = \sin^2 2\theta \sin^2 \left[ \frac{L}{4vE_{\rm av}}\, \Delta M^2 \right] \approx \sin^2 2\theta \sin^2 \left[ \frac{L}{4E_{\rm av}}\, \Delta M^2 \right] .$$

However, it immediately raises a number of conceptual questions. Why should the different mass eigenstates have a common velocity? We have shown in this work that, for ultra-relativistic neutrinos, the scenario of common momentum or common energy *doubles* the oscillation amplitude, yielding the standard oscillation formula

$$P(\nu_\alpha, \nu_\beta) \approx \sin^2 2\theta \sin^2 \left[ \frac{L}{2E_{\rm av}}\, \Delta M^2 \right] .$$

*The difference between the scenarios of common momentum and common energy may be experimental determined for nonrelativistic process*, *i.e. process which involves the kaon system.*

In a recent paper,[33] it was observed that the equal velocity prescription for neutrino oscillations is forbidden because for known production processes $E_1/E_2 \approx 1$ while $M_1/M_2$ may be extremely small or extremely large. Nevertheless, almost equal neutrino masses are *not* ruled out experimentally. In fact, for the solution of the solar neutrino problem through the matter-enhanced neutrino oscillations, we have $\Delta M^2_{\rm so} \approx 10^{-5}$ eV$^2$ (for the vacuum oscillations $\Delta M^2_{\rm so} \approx 10^{-10}$ eV$^2$).[34] The explanation of atmospheric neutrino experiments through the neutrino oscillations requires $\Delta M^2_{\rm at} \approx 10^{-3}$ eV$^2$.[35] The hypothesis $\Delta M \ll M_{\rm av}$, which is true for the kaon system, implies $M_{\rm av} \gg 2.2 \times 10^{-3}$ eV ($0.7 \times 10^{-5}$ eV) for solar neutrinos and $M_{\rm av} \gg 2.2 \times 10^{-2}$ eV for atmospheric neutrinos. If $\Delta M \ll M_{\rm av}$, we get $M_1/M_2 \approx 1$ which in the equal velocity prescription implies $E_1/E_2 \approx 1$. Thus, to recover the standard formula for mass oscillations, we need almost equal masses in addition to the *exact* common velocity scenario. In all the other cases, we find a doubling of the oscillation phase.

We also observe that in the scenario of different velocities, by assuming $L_1 = L_2 = L$, $t_1$ and $t_2$ could be *significantly* different, the extreme case is seen in Refs. 36 and 37, then there is no interference. The predictions of the equal velocity scenario is therefore dramatically different from the other scenarios in long baseline experiments.

In our discussion, we have used plane wave amplitudes as approximations for our calculations. A complete understanding of neutrino oscillations requires the treatment of localization of the microscopic process by which a neutrino is produced and detected. This localization, appropriately described by a wave packet treatment, is essential to give a significance to the distance from source to measuring apparatus. In such a picture, the flavor eigenstate is created not as a simple two-state system but rather as a superposition of two wave packets, one for each mass eigenstate. The coherence properties of the neutrino flux have been examined in terms of the *length of the wave packet* resulting from the electron capture process, first by Nussinov[2] and more recently re-examined by him and collaborators.[8]

In neutron physics, where the coherence properties of particle beams can be particularly well studied, there have been several discussions as to whether and how it could be possible to determine or observe the wave packet properties of a beam.[38] In recent papers[8,11] we also find an interesting discussion about the impossibility of telling the difference between beams with the same energy spectrum consisting of a mixture of long and short wave packets. Unfortunately, *it is not clear what determines the size of the wave packet at the moment of creation or even if it makes sense to talk of a precise time of creation.*[7] So, a clear and consistent discussion of neutrino oscillations by wave packets still represents an open question in this research field.[39,40]

After the completion of this work, two of the authors have reconsidered the known data upon the neutral kaon system.[41] The initial objective of this revision was to locate in the early papers[24,25] upon this subject and the origin and justification of the so-called standard oscillation formula.

As shown in Eq. (16), the standard formula is obtained if $E_1 \approx E_2 \approx E_{\rm av}$, i.e. in the case of $\Delta M \ll M$ (valid for the neutral kaons but improbable for neutrinos) and furthermore if one assumes the *equal velocities* hypothesis. This equal velocities hypothesis indeed dominates the literature including the most recent papers used by the Particle Data Group[31] for its estimate of $\Delta M$ even if these are not based upon oscillation measurements. This fact is deducible from the appearance of a unique proper time, $\tau$, multiplying the difference in mass in the appropriate formulas, such as for asymmetries in charged kaon semileptonic decays. We anticipate some of the findings of this research.

In particular a long-standing "paradox" in the literature may have as a solution the factor two discussed here. In the paper by Fujii *et al.*[42] upon neutral kaon decays, it reads "... there is a marked tendency for experiments of the first type [oscillations] to give significantly higher values [for $\Delta M$] than those of the second type [regeneration] ...".

We begin by noting that these higher values, within the experimental errors, are larger by about a factor of two. We also recall that the regeneration experiments are analyzed with the hypothesis of equal energies for the outgoing kaon mass eigenstates. Indeed, this fact can be found in the book by Okun.[43] Our observation is simply that if the original neutral kaon is not a common velocity scenario, then an overestimate of $\Delta M$ by a factor of two follows automatically from the incorrect use of the standard oscillation formula. Indeed the fitted $\Delta M$ would then have to compensate the extra factor of two in the denominator. This fact was first noted by Srivastava *et al.*[44]

This example shows that in the neutral kaon system there is already evidence for the questions posed in this letter. Neutral kaons also offer the practical possibility of performing experiments with nonrelativistic particles and hence of distinguishing between the various scenarios discussed in this work. We can suggest that an interesting experiment would be the $\Delta M$ derived with regenerated kaons in two alternative studies one with time-space oscillations of the kind considered here and the other by forward intensity measurements or interference effects due to regeneration in two or more plates. In this way one is certain that the scenario, even if in doubt, is the same for the two experiments.

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