Teste do dia 20 de abril

EX. 1 DESENVOLVER $\sqrt{1+x}$ ATÉ A SEGUNDA ORDEM EM SÉRIE DE TAYLOR E $x_0 = 0$ E CALCULAR A INTEGRAL ENTRE $(-1, 1)$

$f(x) = (1+x)^{\frac{1}{2}}$ $\quad f'(x) = \frac{1}{2}(1+x)^{-\frac{1}{2}}$ $\quad f''(x) = -\frac{1}{4}(1+x)^{-\frac{3}{2}}$

$f(x) = \sqrt{1} + \frac{1}{2\sqrt{1}}x - \frac{1}{4\,1^{3/2}}\frac{x^2}{2} = 1 + \frac{x}{2} - \frac{x^2}{8}$

$g(x) = x + \frac{x^2}{4} - \frac{x^3}{24} \longleftarrow$ USANDO $x^n \to x^{n+1}/n$

$g(1) - g(-1) = \left(1 + \frac{1}{4} - \frac{1}{24}\right) - \left(-1 + \frac{1}{4} + \frac{1}{24}\right) = 2 - \frac{\cancel{2}^{1}}{\cancel{24}_{12}} = \boxed{\frac{23}{12}}$

EX. 2 $\sqrt{1+x^2}$ $\quad x_0 = 0$ $\quad$ INTEGRAL ENTRE $(-1, 1)$

$f(x) = (1+x^2)^{1/2}$ $\quad f'(x) = \frac{1}{\cancel{2}}(1+x^2)^{-1/2}\cancel{2}x = x(1+x^2)^{-1/2}$

$f''(x) = (1+x^2)^{-1/2} - \frac{1}{\cancel{2}}x(1+x^2)^{-3/2}\cancel{2}x$
$= (1+x^2)^{-1/2} - x(1+x^2)^{-3/2}$

$f(x) = 1 + \frac{x^2}{2} \longrightarrow g(x) = x + \frac{x^3}{6}$

$g(1) - g(-1) = \left(1 + \frac{1}{6}\right) - \left(-1 - \frac{1}{6}\right) = \boxed{\frac{7}{3}}$

EX. 3 xe^x $\quad x_0 = 0$ $\quad$ INTEGRAL ENTRE $(-1, 1)$

$f(x) = xe^x$ $\quad f'(x) = e^x + xe^x$ $\quad f''(x) = e^x + e^x + xe^x = 2e^x + xe^x$

$f(x) = x + x^2 \longrightarrow g(x) = \frac{x^2}{2} + \frac{x^3}{3}$

$g(1) - g(-1) = \left(\frac{1}{2} + \frac{1}{3}\right) - \left(\frac{1}{2} - \frac{1}{3}\right) = \boxed{\frac{2}{3}}$

EX. 4 $x\cos x$ $\quad x_0 = \frac{\pi}{2}$ $\quad$ INTEGRAL ENTRE $\left(\frac{\pi}{4}, \frac{3}{4}\pi\right)$

$f(x) = x\cos x$ $\quad f'(x) = \cos x - x\,\mathrm{sen}\,x$ $\quad f''(x) = -2\,\mathrm{sen}\,x - x\cos x$

$f(\pi/2) = 0$ $\quad f'(\pi/2) = -\frac{\pi}{2}$ $\quad f''\left(\frac{\pi}{2}\right) = -2$

$f(x) = -\frac{\pi}{2}\left(x - \frac{\pi}{2}\right) - \left(x - \frac{\pi}{2}\right)^2 = -\frac{\pi}{2}x + \cancel{\frac{\pi^2}{4}} - x^2 - \cancel{\frac{\pi^2}{4}} + \pi x = \frac{\pi}{2}x - x^2$

$g(x) = \frac{\pi}{4}x^2 - \frac{x^3}{3} = \left(\frac{\pi}{4} - \frac{x}{3}\right)x^2$

$g\left(\frac{3}{4}\pi\right) - g\left(\frac{\pi}{4}\right) = \left(\cancel{\frac{\pi}{4} - \frac{\pi}{4}}\right)\left(\frac{3}{4}\pi\right)^2 - \left(\frac{\pi}{4} - \frac{\pi}{12}\right)\frac{\pi^2}{16} = \boxed{-\frac{\pi^3}{96}}$

EX. 5

ACHAR O RESULTADO EXATO DA INTEGRAÇÃO DO EXERCÍCIO NÚMERO 3 INTEGRANDO POR PARTES xe^x

$$\int f g' = fg - \int f' g \qquad \begin{matrix} f = x & f' = 1 \\ g' = e^x & g = e^x \end{matrix}$$

$$\int_{-1}^{1} dx\, xe^x = \left[xe^x\right]_{-1}^{1} - \int_{-1}^{1} dx\, e^x = \left[xe^x - e^x\right]_{-1}^{1} = \left[(x-1)e^x\right]_{-1}^{1} = 2e^{-1} = \boxed{\frac{2}{e}}$$

EX. 6

ACHAR O RESULTADO EXATO DA INTEGRAL DO EXERCÍCIO NÚMERO 4 INTEGRANDO POR PARTES $x\cos x$

$$\begin{matrix} f = x & f' = 1 \\ g' = \cos x & g = \operatorname{sen} x \end{matrix}$$

$$\int_{\frac{\pi}{4}}^{\frac{3}{4}\pi} dx\, x\cos x = \left[x \operatorname{sen} x\right]_{\frac{\pi}{4}}^{\frac{3}{4}\pi} - \int_{\frac{\pi}{4}}^{\frac{3}{4}\pi} dx\, \operatorname{sen} x$$

$$= \left[x \operatorname{sen} x + \cos x\right]_{\frac{\pi}{4}}^{\frac{3}{4}\pi}$$

$$= \left(\frac{3}{4}\pi \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) - \left(\frac{\pi}{4}\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$

$$= \frac{\pi}{2\sqrt{2}} - \frac{2}{\sqrt{2}} = \boxed{\frac{\pi - 4}{2\sqrt{2}}}$$