

**DERIVADA  $f(x) g(x)$**

$$\lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x)g(x+\Delta x) - f(x)g(x)}{\Delta x}$$

$$\lim_{\Delta x \rightarrow 0} \left[ \frac{f(x+\Delta x)g(x+\Delta x) - f(x+\Delta x)g(x)}{\Delta x} + \frac{f(x+\Delta x)g(x) - f(x)g(x)}{\Delta x} \right]$$

$$\lim_{\Delta x \rightarrow 0} \left[ f(x+\Delta x) \frac{g(x+\Delta x) - g(x)}{\Delta x} + \frac{f(x+\Delta x) - f(x)}{\Delta x} g(x) \right]$$

$$f(x)g'(x) + f'(x)g(x)$$

usando  $[f^n(x)]' = n f^{n-1}(x) f'(x)$

podemos inmediatamente encontrar  $f(x)/g(x)$

$$\left[ \frac{f(x)}{g(x)} \right]' = f(x) \left[ \frac{1}{g(x)} \right]' + \frac{f'(x)}{g(x)} \frac{1}{g(x)}$$

$$\hookrightarrow \left[ \frac{1}{g(x)} \right]' = -1 \frac{g''(x)}{g^2(x)} g'(x)$$

$$- \frac{f(x)g'(x)}{g^2(x)} + \frac{f'(x)}{g(x)}$$

$$\boxed{\left[ \frac{f(x)}{g(x)} \right]' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}}$$

ENCONTRAR A DERIVADA DE

$$\frac{x-1}{x-2}$$

$$\frac{(x-1)'(x-2) - (x-1)(x-2)'}{(x-2)^2} = \frac{x-2-x+1}{(x-2)^2} = \boxed{-\frac{1}{(x-2)^2}}$$

ENCONTRAR A DERIVADA DE

$$\frac{x^2-1}{x-2}$$

$$\frac{(x^2-1)'(x-2) - (x^2-1)(x-2)'}{(x-2)^2} = \frac{2x(x-2) - x^2 + 1}{(x-2)^2}$$

$$= \boxed{\frac{x^2 - 4x + 1}{(x-2)^2}}$$

GRAFIQUEMOS ESTAS FUNÇÕES

$$\frac{x-1}{x-2}$$

$$\begin{matrix} x \rightarrow -\infty \\ x \rightarrow +\infty \end{matrix}$$

$$\begin{matrix} (-\infty/-\infty = +1) \\ (+\infty/+ \infty = +1) \end{matrix}$$

$$\begin{matrix} x \rightarrow 2^- (1.9) \stackrel{+}{=} -\infty \\ x \rightarrow 2^+ (2.1) \stackrel{+}{=} +\infty \end{matrix}$$

DERIVADA  $\neq 0$

os termos max ou min



$$\frac{x^2 - 1}{x - 2}$$

ZEROS

$$x^2 = 1$$

$$x = \pm 1$$

$\infty$

$$x = 2^\pm$$

$$\begin{array}{l} 2^-, -\infty \\ 2^+, +\infty \end{array}$$

$$\text{DERIVADA} = 0$$

$$x = 2 \pm \sqrt{3}$$

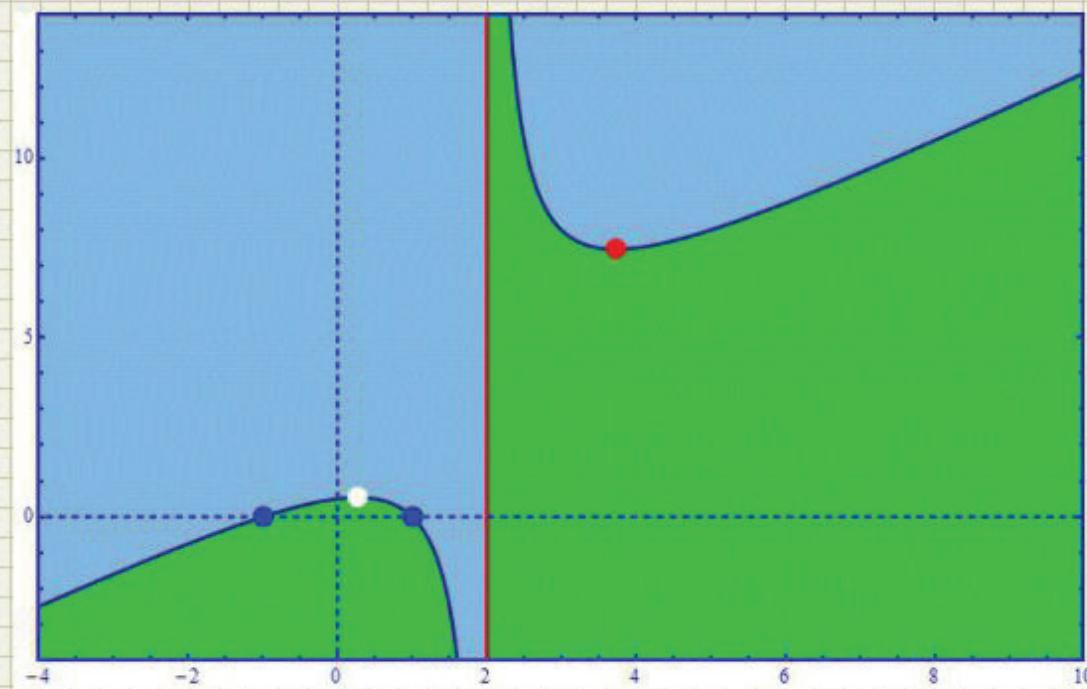
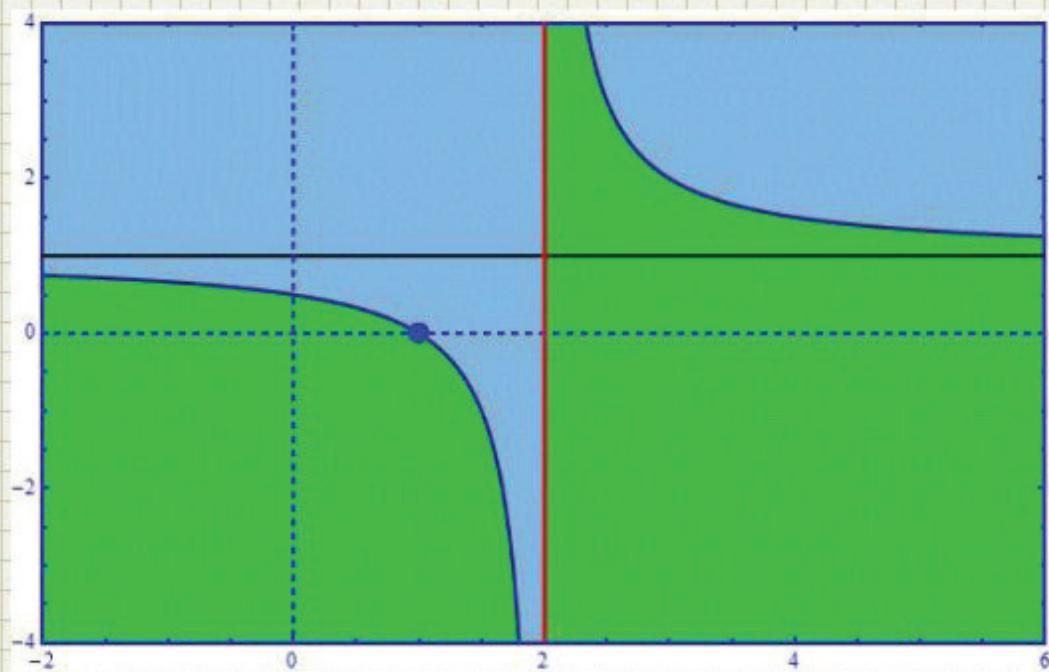
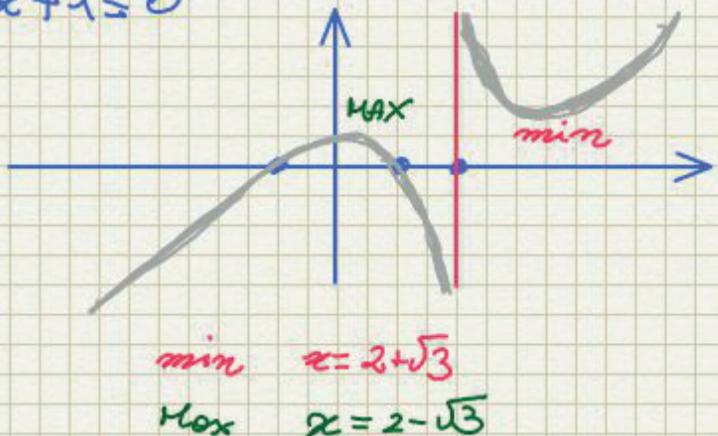
$$x^2 - 4x + 1 = 0$$

$$x \rightarrow \pm \infty$$

$$\frac{\partial f}{\partial x} = 0$$

$$(-\infty, -\infty)$$

$$(+\infty, +\infty)$$



$$\begin{array}{l} \text{min} \\ (2 + \sqrt{3}, 1 + 2\sqrt{3}) \end{array}$$

Max

$$(2 - \sqrt{3}, 1 - 2\sqrt{3})$$

# Estudo de funções tipo $f(x)/g(x)$



A)  $\frac{(x-1)(x-4)}{(x-2)(x-3)}$

ZEROS

$$x=1 \text{ e } x=4$$

$\infty$

$$2^- \begin{array}{c} + \\ - \end{array}$$

$$2^+ \begin{array}{c} + \\ - \end{array}$$

$$\begin{array}{ll} (2^-, -\infty) & (2^+, +\infty) \\ (3^-, +\infty) & (3^+, -\infty) \end{array}$$

B)  $\frac{(x-2)(x-3)}{(x-1)(x-4)}$

ZEROS

$$x=2 \text{ e } x=3$$

$\infty$

$$1^- \begin{array}{c} - \\ - \end{array}$$

$$1^+ \begin{array}{c} - \\ - \end{array}$$

$$\begin{array}{ll} (1^-, +\infty) & (1^+, -\infty) \\ (4^-, -\infty) & (4^+, +\infty) \end{array}$$

$$\left[ B = \frac{1}{A} \right]$$

MÁXIMOS / MÍNIMOS

$$\frac{g'}{g} \rightarrow g'g - gg' = 0$$

$$(2x-5)(x^2-5x+4) - (2x-5)(x^2-5x+6) = 0$$

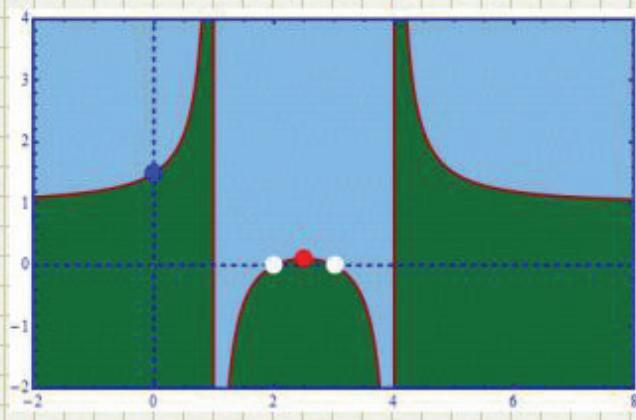
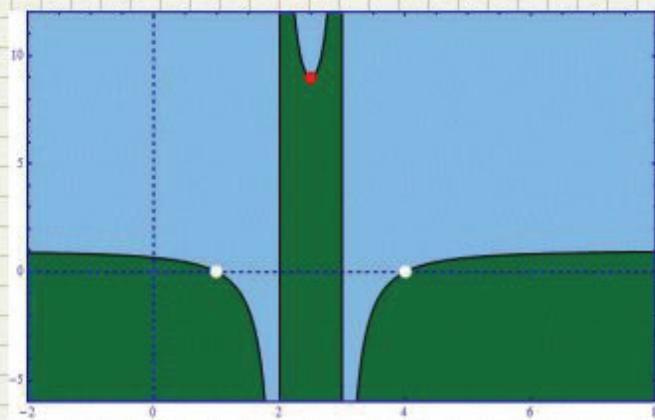
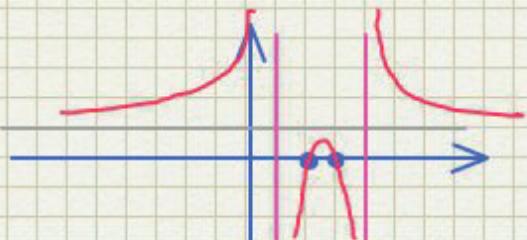
$$\left( \frac{5}{2}, \frac{9}{4} \right)$$

$$\frac{g'}{g} \rightarrow g'g - gg' = 0$$

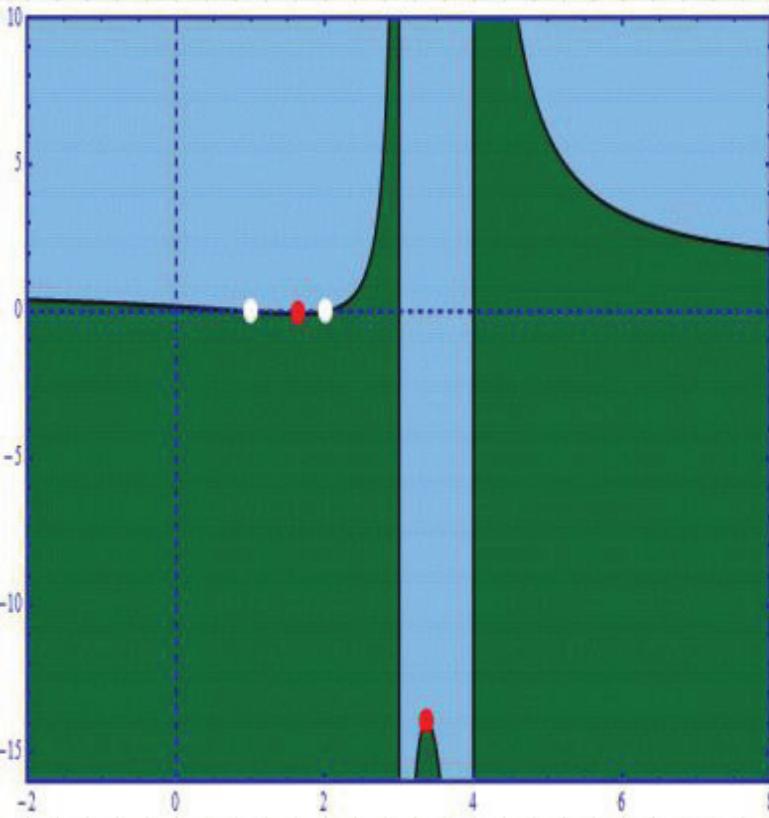
$$\left( \frac{5}{2}, \frac{1}{4} \right)$$



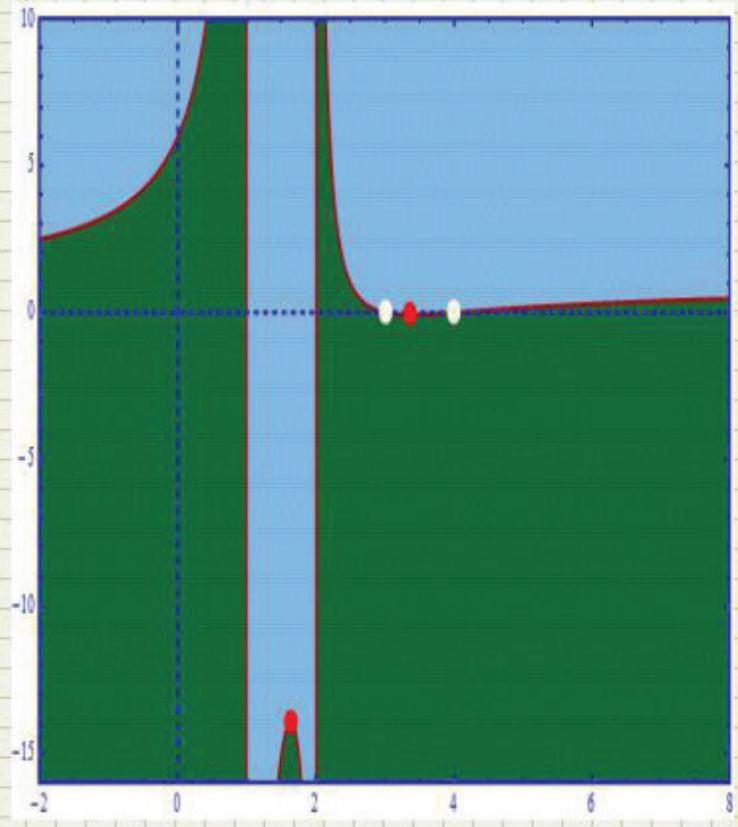
$$\lim_{x \rightarrow \pm\infty} \frac{x^2}{x^2} = 1$$



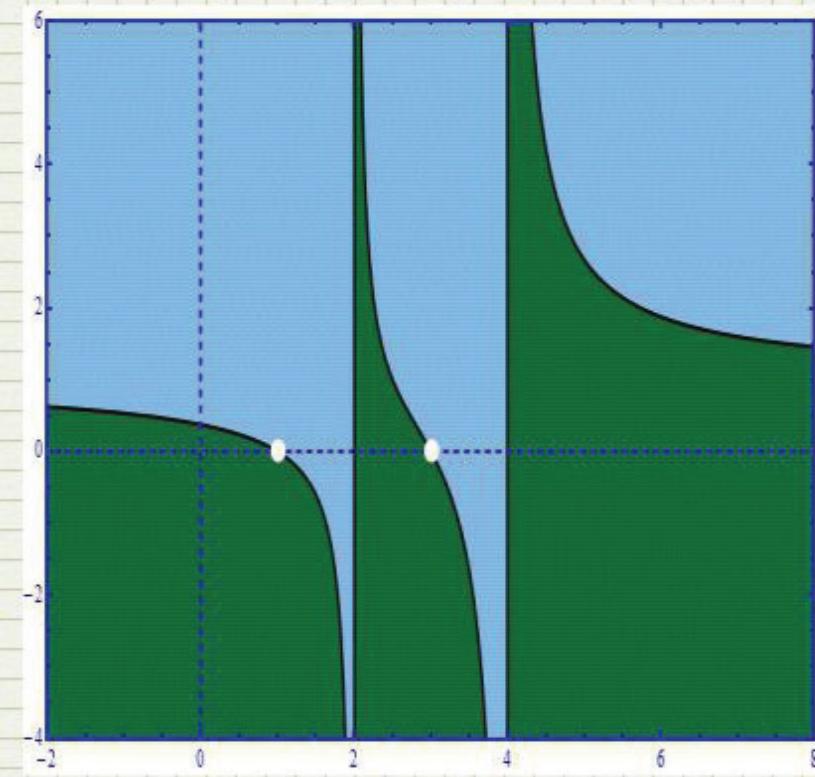
$$\frac{(x-1)(x-2)}{(x-3)(x-4)}$$



$$\frac{(x-3)(x-4)}{(x-1)(x-2)}$$



$$\frac{(x-1)(x-3)}{(x-2)(x-4)}$$



$$\frac{(x-2)(x-4)}{(x-1)(x-3)}$$

