Interpretação geomêtrica da derivada

consideramos a função $f(x) = x^2$

e graficamos as retas que passam pelos pontos

$(1, f(1))$ $(5, f(5))$ gráfico A

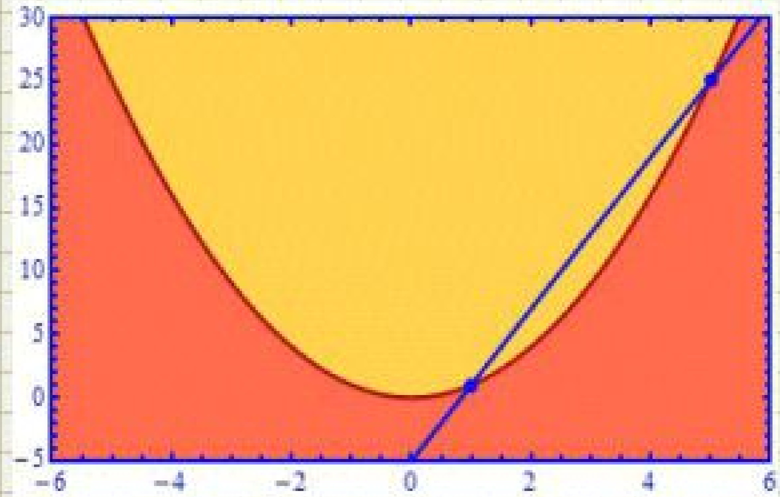
$(1, f(1))$ $(4, f(4))$ B

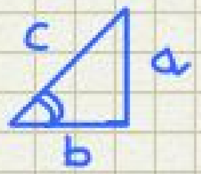
$(1, f(1))$ $(3, f(3))$ C

$(1, f(1))$ $(2, f(2))$ D

calculando o ângulo formado com o eixo x

GRÁFICO A





$c \operatorname{sen}\alpha = a$

$c \cos\alpha = b$

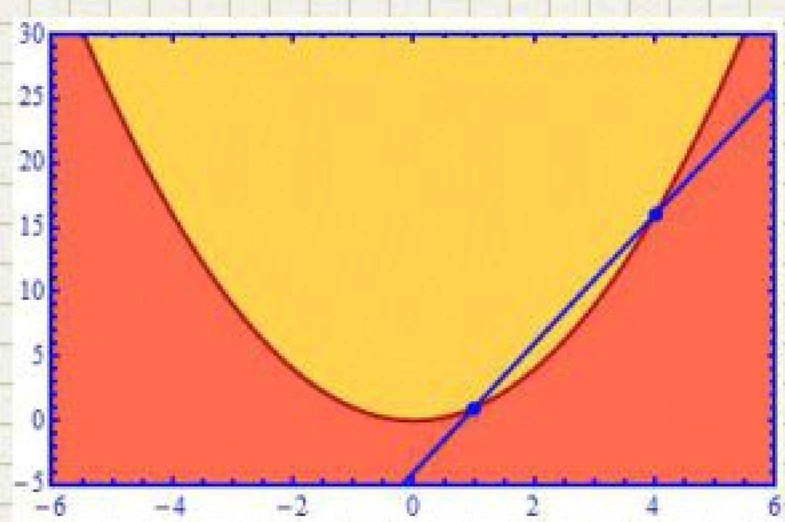
$a/b = \tan\alpha$

$a = f(5) - f(1) = 25 - 1 = 24$

$b = 5 - 1 = 4$

$\boxed{\tan\alpha = 6}$

$\alpha = 80.54°$

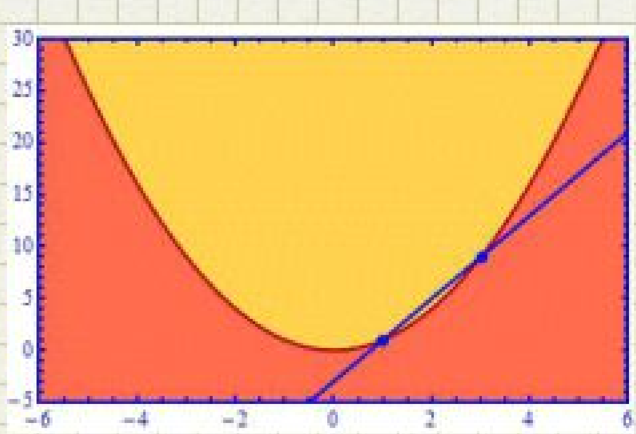


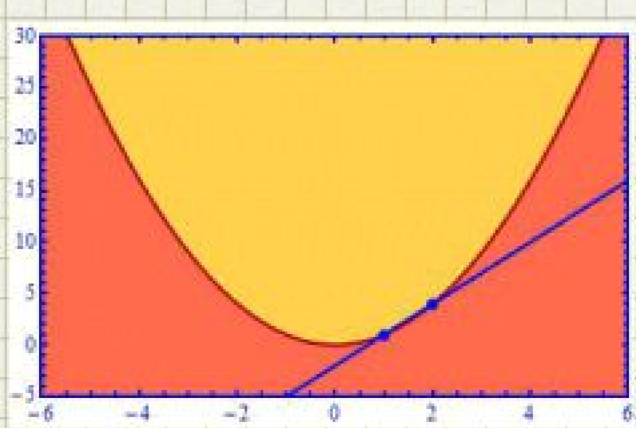
$a = f(4) - f(1) = 16 - 1 = 15$

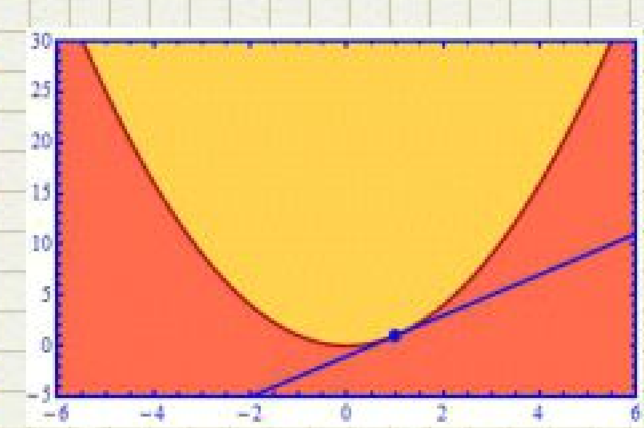
$b = 4 - 1 = 3$

$\boxed{\tan\alpha = 5}$

$\alpha = 78.69°$







$a = 9 - 1 = 8$

$b = 3 - 1 = 2$

$\boxed{\tan\alpha = 4}$

$\alpha = 75.96°$

$a = 4 - 1 = 3$

$b = 2 - 1 = 1$

$\boxed{\tan\alpha = 3}$

$71.57°$

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$f'(x)$

$\tan\alpha = f'(1)$

$$\lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{\cancel{x^2} + 2x\Delta x + \Delta x^2 - \cancel{x^2}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} (2x + \Delta x) = 2x$$

$f(x) = x^2$ $\quad$ $f'(x) = 2x$ $\quad$ $f'(1) = 2$ $\quad$ $\boxed{\tan\alpha = 2}$

$\alpha = 63.43°$

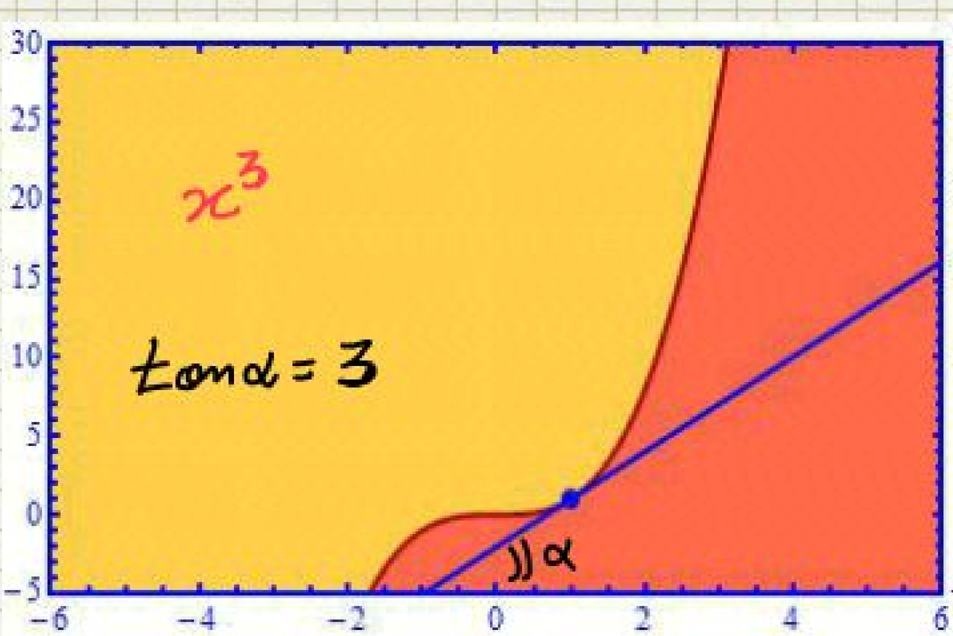
$f(x) = x^n$ $\quad$ $f'(x) = ?$

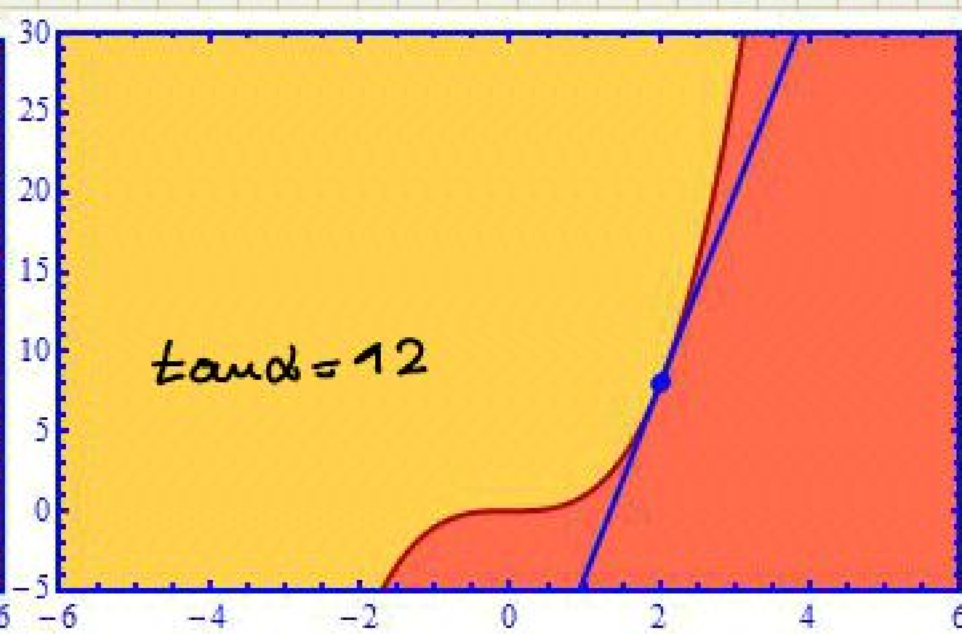
$$\lim_{\Delta x \to 0} \frac{(x+\Delta x)^n - x^n}{\Delta x} = \lim_{\Delta x \to 0} \left(n x^{n-1} + \dots \Delta x + \dots \Delta x^2 + \dots \right)$$

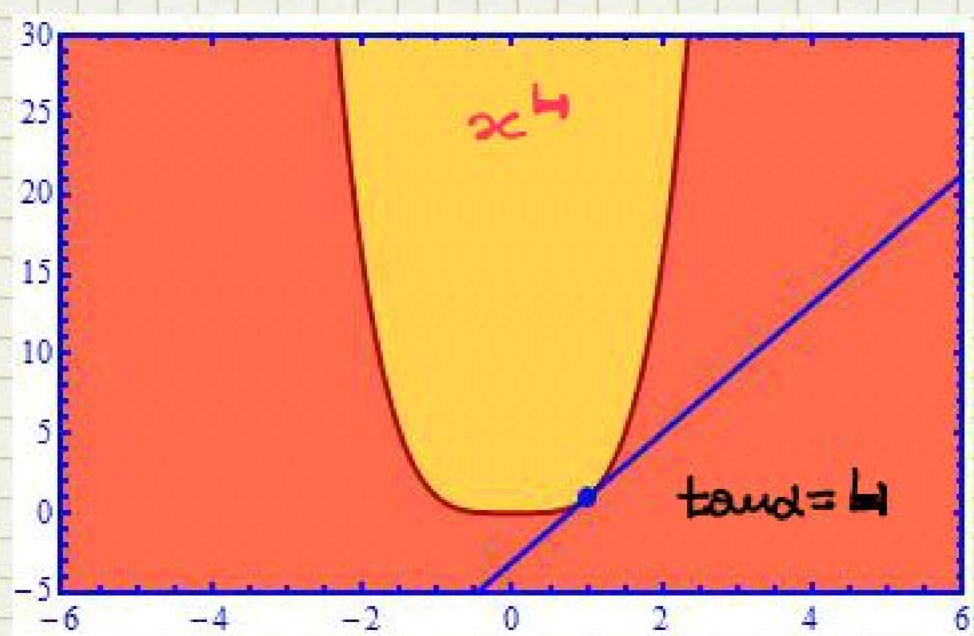
$$\boxed{n x^{n-1}}$$

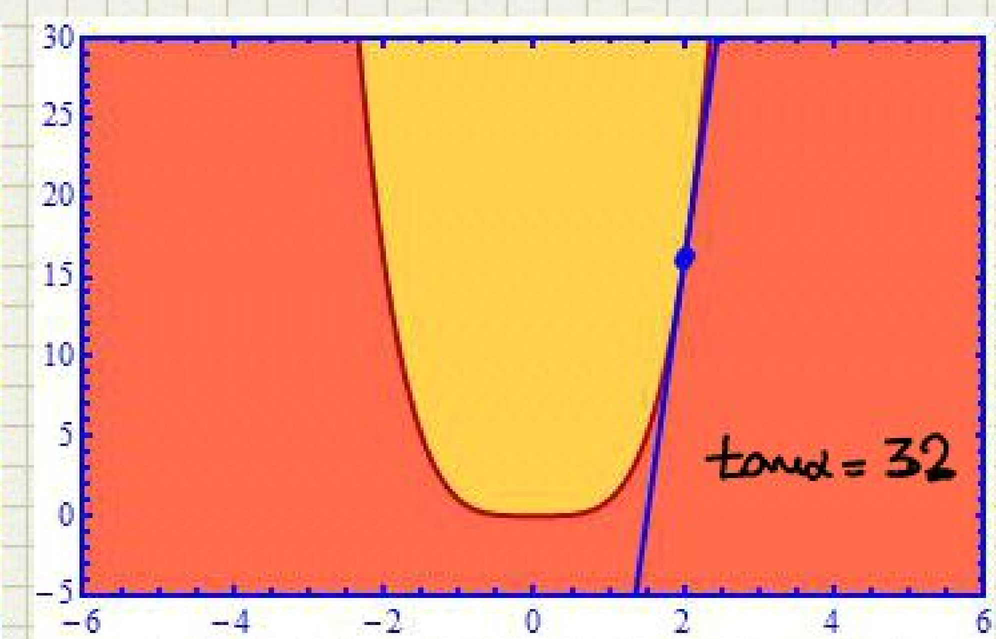
$f(x) = x^3$ $\quad$ $f'(1) = 3$ $\quad$ $f'(2) = 12$ $\quad$ $f'(x) = 3x^2$

$f(x) = x^4$ $\quad$ $f'(1) = 4$ $\quad$ $f'(2) = 32$ $\quad$ $f'(x) = 4x^3$



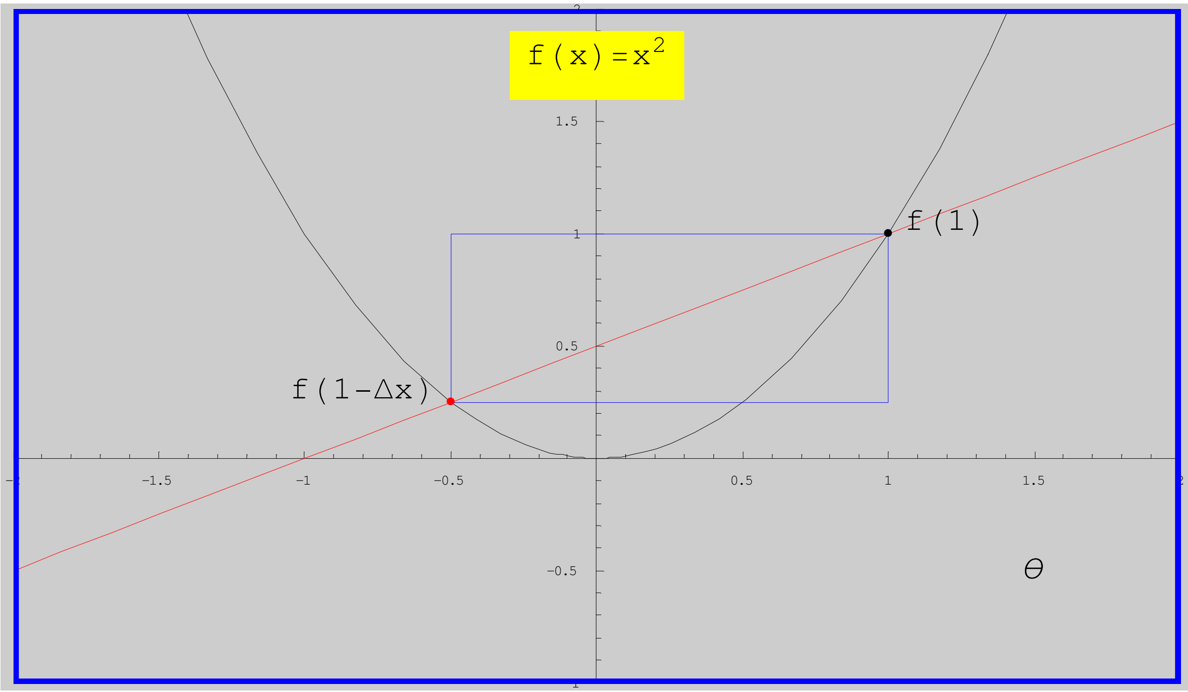


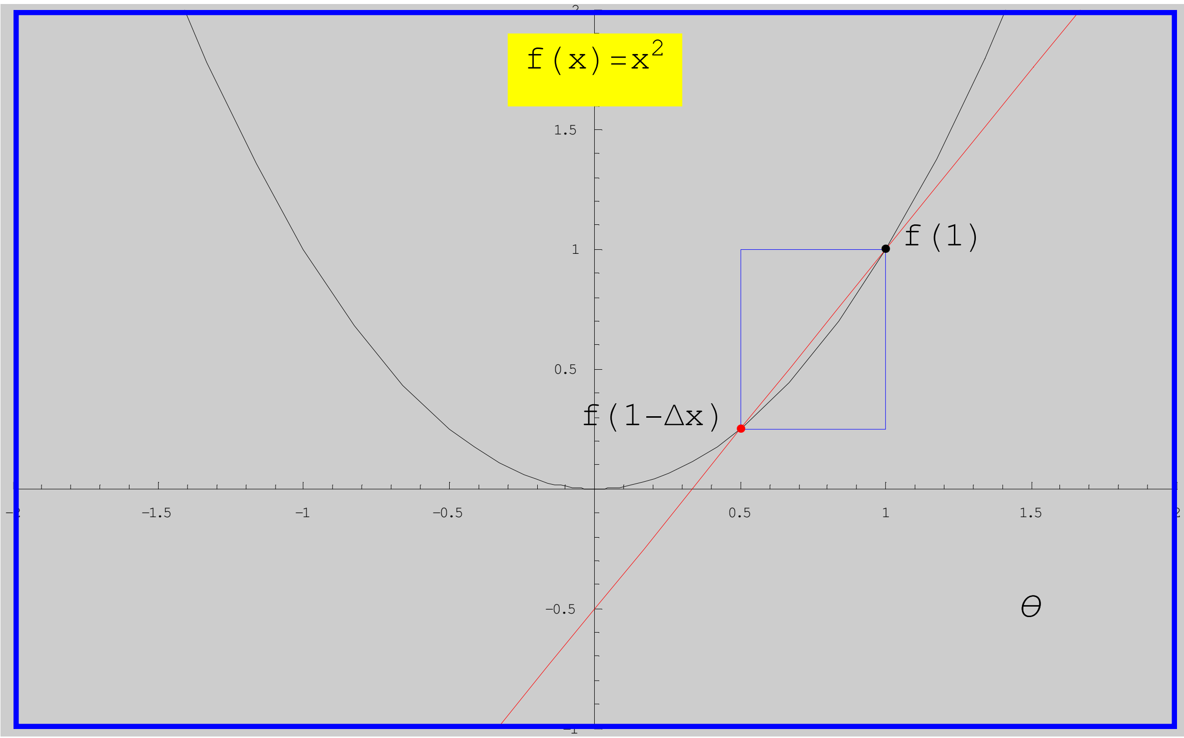


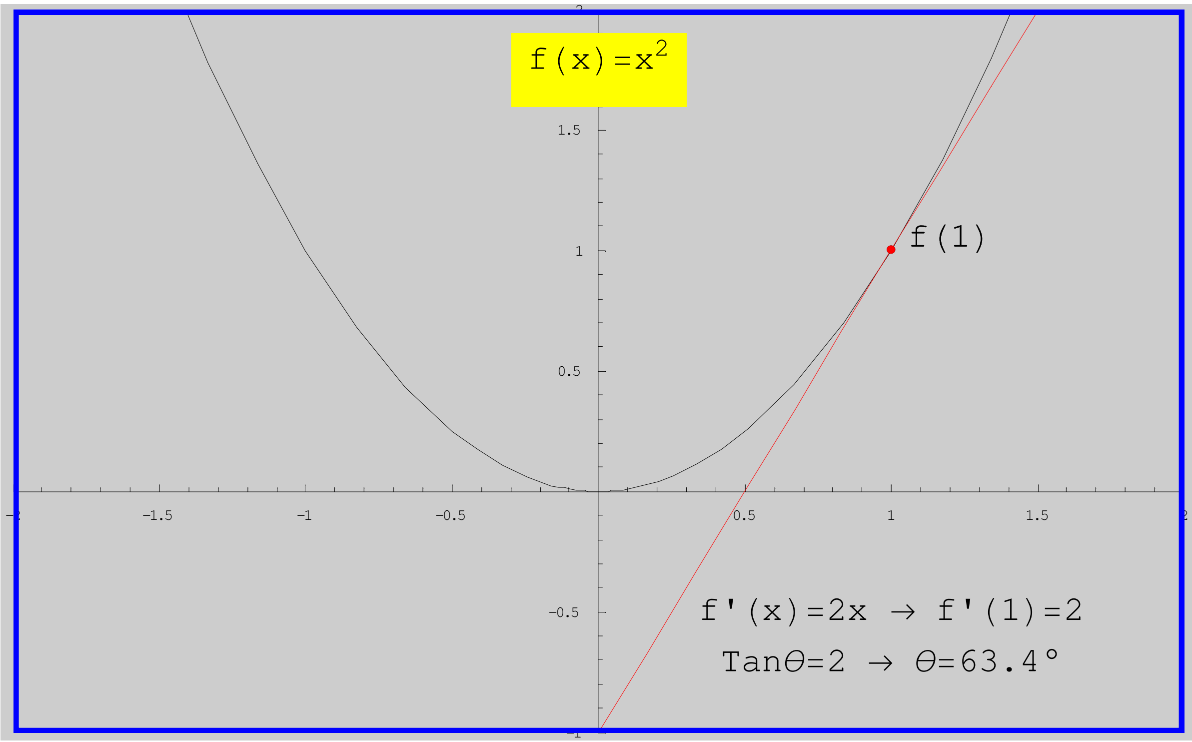


$\tan\alpha = 1$	[45°]	5	[78.690°]	16	[86.424°]
$\tan\alpha = 2$	[63.435°]	6	[80.538°]	32	[88.210°]
$\tan\alpha = 3$	[71.565°]	7	[81.870°]	64	[89.105°]
$\tan\alpha = 4$	[75.964°]	8	[82.875°]	128	[89.552°]

Derivadas e Limites







Definição de derivada (usando o conceito de limite):

$$\boxed{[\,f(x)\,]' = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}}$$

Cálculo da derivada de x^3 usando a definição anterior:

$$\begin{aligned}
(x^3)' &= \lim_{\Delta x \to 0} \frac{(x+\Delta x)^3 - x^3}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{x^3 + 3\,x^2 \Delta x + +3\,x\,(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x} \\
&= \lim_{\Delta x \to 0} [\,3\,x^2 + 3\,x\,(\Delta x)^2 + (\Delta x)^3\,] \\
&= 3\,x^2
\end{aligned}$$

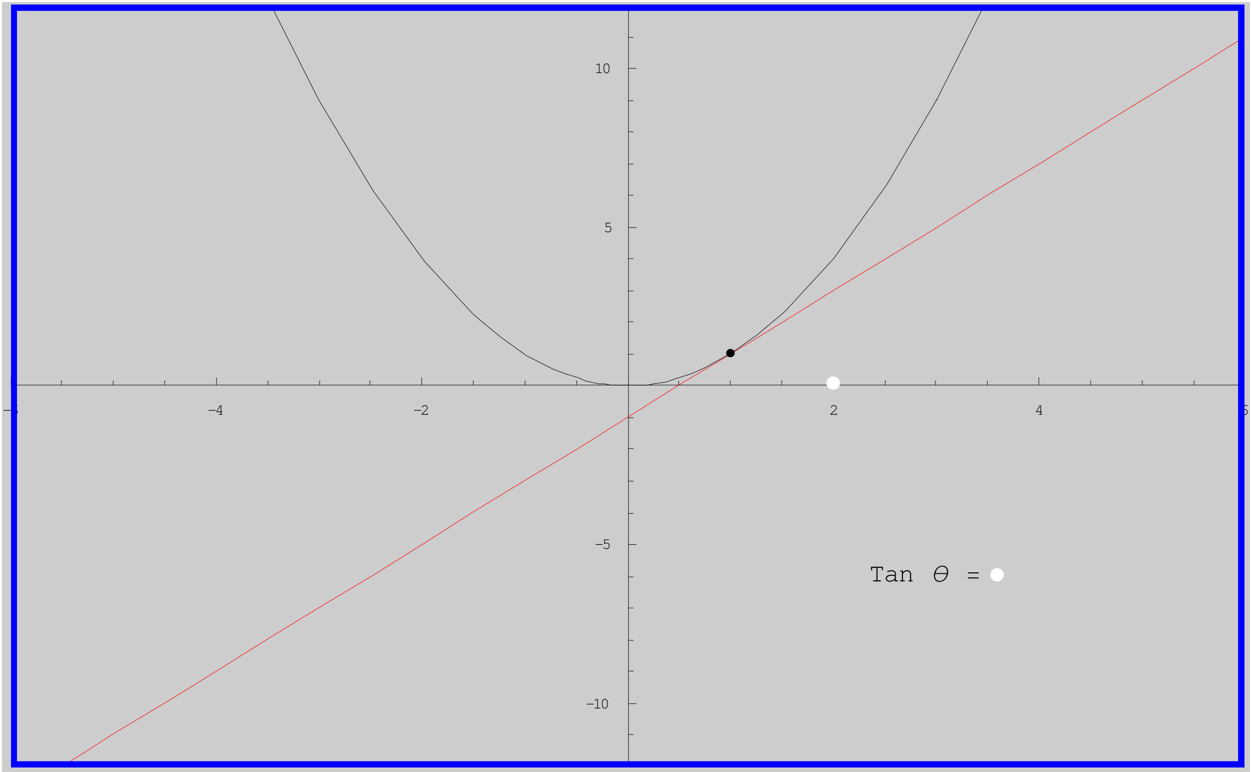
$$\boxed{(x^3)' = 3\,x^2}$$

Derivada do produto de duas funções:

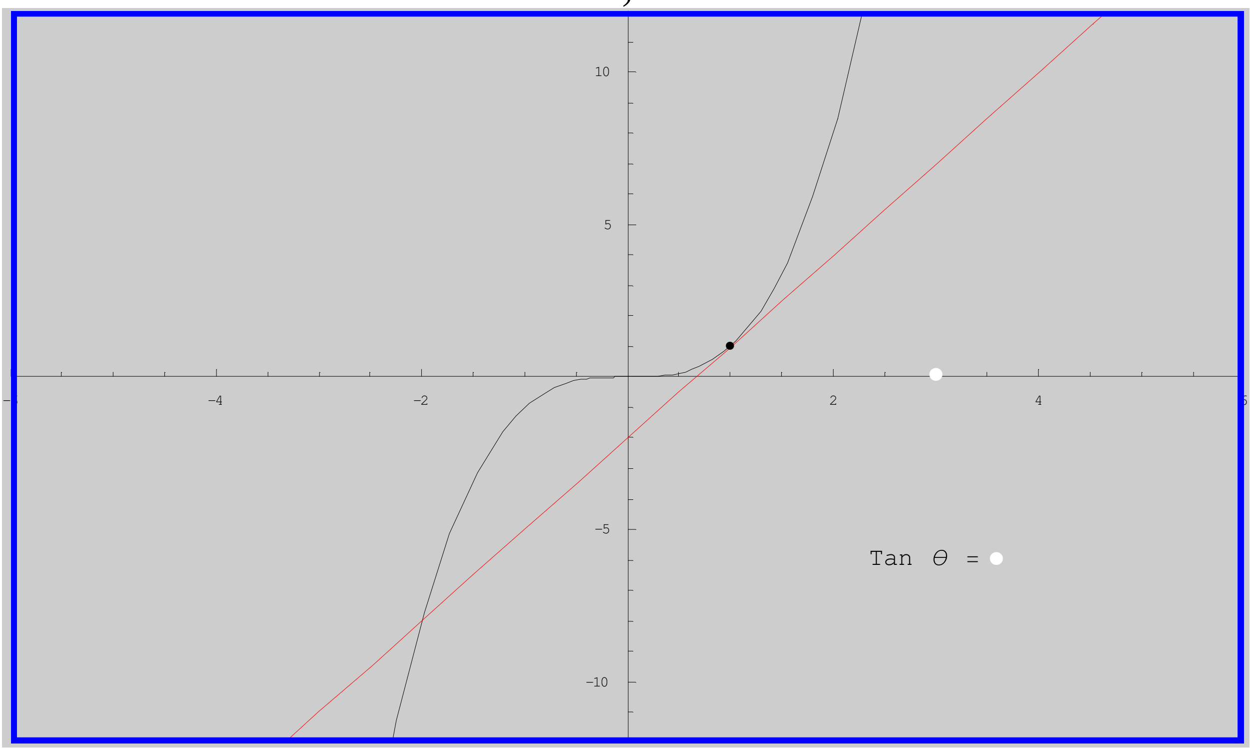
$$\begin{aligned}
[\,f(x)\,g(x)\,]' &= \lim_{\Delta x \to 0} \frac{f(x+\Delta x)\,g(x+\Delta x) - f(x)\,g(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{f(x+\Delta x)\,g(x+\Delta x) - f(x)\,g(x+\Delta x) + f(x)\,g(x+\Delta x) - f(x)\,g(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \left[\frac{f(x+\Delta x) - f(x)}{\Delta x}\; g(x+\Delta x) + f(x)\; \frac{g(x+\Delta x) - g(x)}{\Delta x} \right] \\
&= [\,f(x)\,]'\,g(x) + f(x)\,[\,g(x)\,]'
\end{aligned}$$

$$\boxed{[\,f(x)\,g(x)\,]' = [\,f(x)\,]'\,g(x) + f(x)\,[\,g(x)\,]'}$$

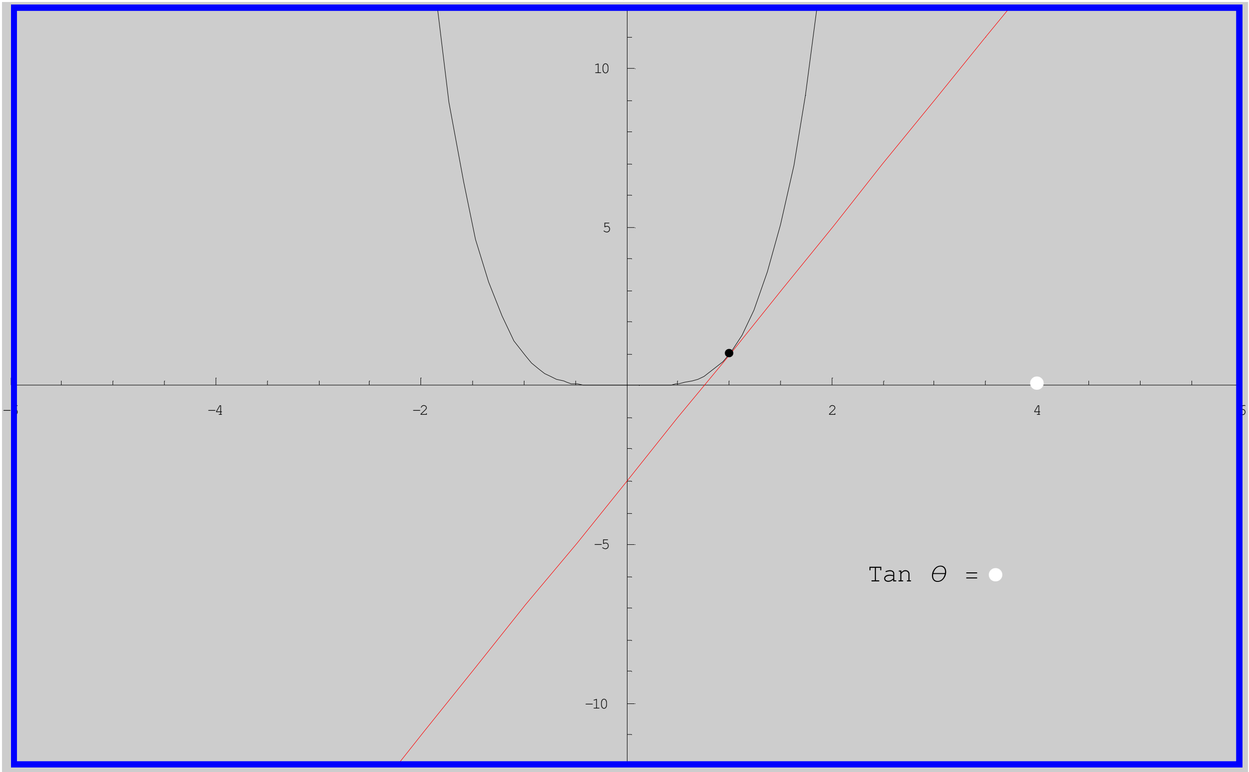
63,43



71,57



75,96



78,69