## SEQUÊNCIA DE FIBONACCI

Na matemática, os números de Fibonacci são uma sequência ou sucessão definida como recursiva pela fórmula:

$$F(n+2) = F(n+1) + F(n) \quad , \quad \text{com } n \geq 1 \text{ e } F(1) = F(2) = 1 \ .$$

Os primeiros números de Fibonacci são:

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, ...$$

Esta sequência foi descrita primeiramente por Leonardo de Pisa, também conhecido como Fibonacci, para descrever o crescimento de uma população de coelhos.

| $\odot \longrightarrow \otimes \longrightarrow \otimes\odot$ | $\star$ | $0 \longrightarrow 1 \longrightarrow 10$ |
|---|---|---|
| $\odot$ | 1 | 0 |
| $\otimes$ | 1 | 1 |
| $\otimes\odot$ | 2 | 10 |
| $\otimes\odot\otimes$ | 3 | 101 |
| $\otimes\odot\otimes\otimes\odot$ | 5 | 10110 |
| $\otimes\odot\otimes\otimes\odot\otimes\odot\otimes$ | 8 | 10110101 |
| $\otimes\odot\otimes\otimes\odot\otimes\odot\otimes\otimes\odot\otimes\otimes\odot$ | 13 | 1011010110110 |
| $\otimes\odot\otimes\otimes\odot\otimes\odot\otimes\otimes\odot\otimes\otimes\odot\otimes\odot\otimes\otimes\odot\otimes\odot\otimes$ | 21 | 101101011011010110101 |
| | $\vdots$ | |

### • 1) $F(1) + F(2) + F(3) + ... + F(n) = F(n+2) - 1$ para $n \geq 1$

**1.1)**

$n = 1$: $\quad F(1) = F(3) - 1 \ \surd$

**1.2)**

$F(1) + F(2) + F(3) + ... + F(k) = F(k+2) - 1 \ \surd$

$$\underbrace{F(1) + F(2) + F(3) + ... + F(k)}_{F(k+2)-1} + F(k+1) = F(k+3) - 1 \ ?$$

usando $F(k+1) + F(k+2) = F(k+3) \ \surd$

$$\boxed{\bullet\ \mathbf{2)}\quad F(n)=\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n-\frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n \text{ para } n\geq 1}$$

**2.1a)**

$$n=1:\quad 1=\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)-\frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)\ \surd$$

**2.1b)**

$$n=2:\quad 1=\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^2-\frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^2\ \surd$$

**2.2a)**

$$F(k)=\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^k-\frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^k\ \surd$$

**2.2b)**

$$F(k+1)=\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{k+1}-\frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^{k+1}\ \surd$$

$$F(k+2)=\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{k+2}-\frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^{k+2}\ ?$$

$$\begin{aligned}
F(k+2)&=F(k+1)+F(k)\\
&=\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{k+1}-\frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^{k+1}+\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{k}-\frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^{k}\\
&=\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{k}\left[\frac{1+\sqrt{5}}{2}+1\right]-\frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^{k}\left[\frac{1-\sqrt{5}}{2}+1\right]\\
&=\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{k}\frac{3+\sqrt{5}}{2}-\frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^{k}\frac{3-\sqrt{5}}{2}\\
&=\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{k}\left(\frac{1+\sqrt{5}}{2}\right)^{2}-\frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^{k}\left(\frac{1-\sqrt{5}}{2}\right)^{2}\ \surd
\end{aligned}$$

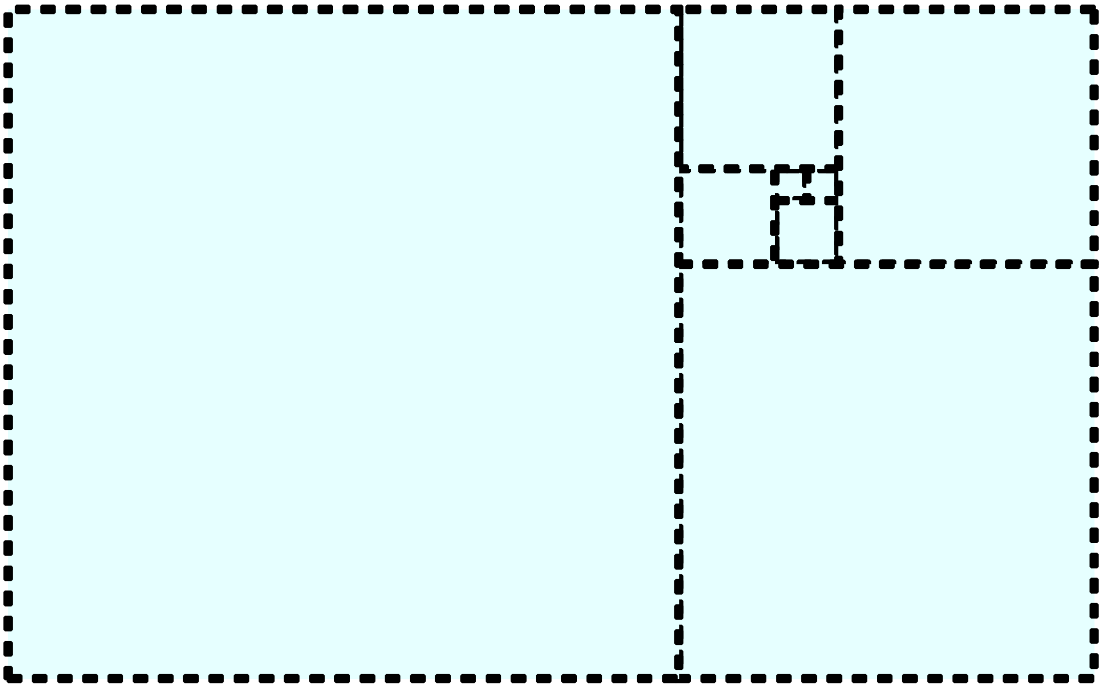
$$\boxed{\underbrace{F(n)}_{n\gg 1}\to\frac{1}{\sqrt{5}}\,\phi^n=\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n}$$

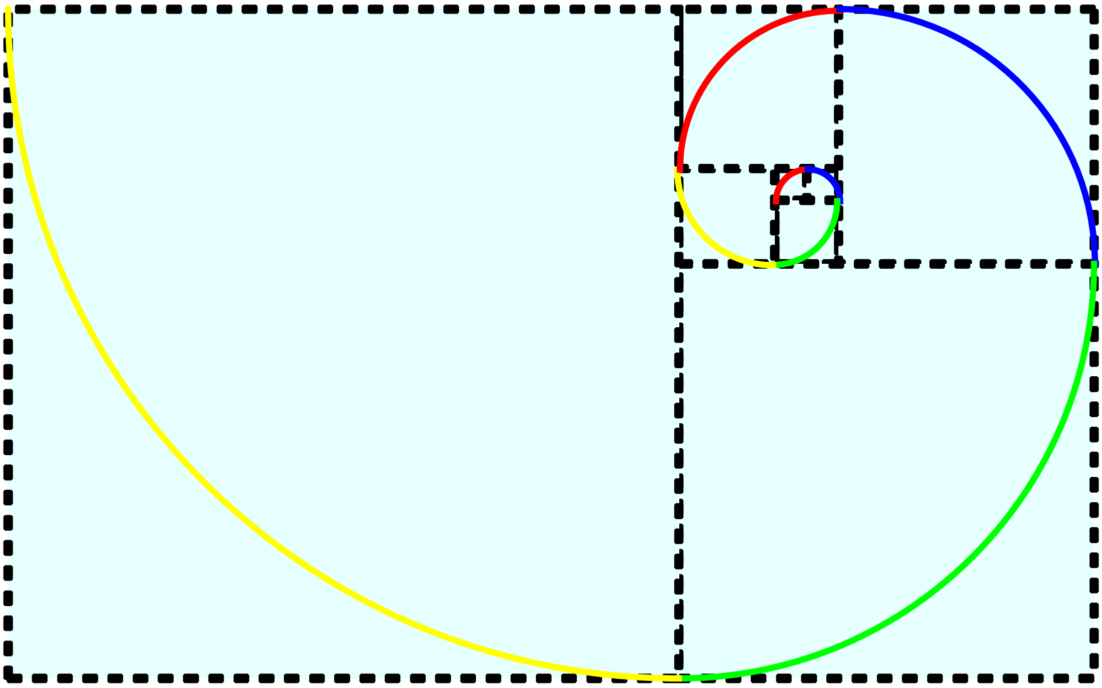
$$\underbrace{F(n+1)/F(n)}_{n\gg 1}\to\phi$$

$$\boxed{\bullet\ \mathbf{3)}\quad F(n)<\left(\frac{7}{4}\right)^n \text{ para } n\geq 1}$$

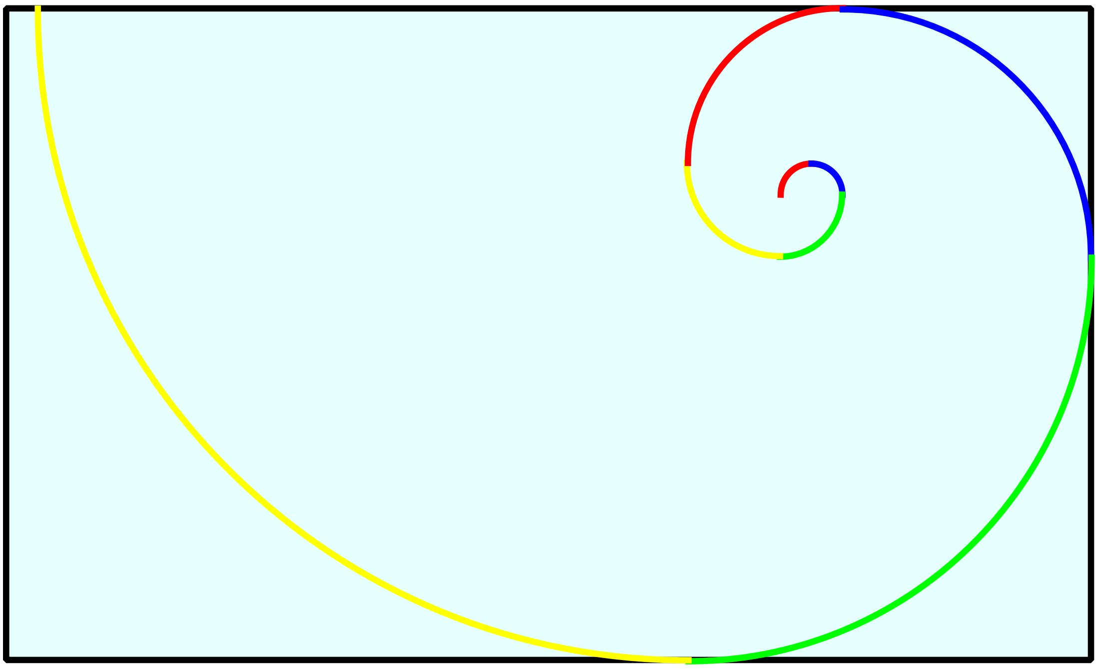
$$\boxed{\bullet\ \mathbf{4)}\quad \sum_{p=1}^{2n-1}F(p)F(p+1)=\left[F(2n)\right]^2\qquad \text{para } n\geq 1}$$

$$\boxed{\bullet\ \mathbf{5)}\quad \sum_{p=1}^{2n}F(p)F(p+1)=\left[F(2n+1)\right]^2-1\qquad \text{para } n\geq 1}$$





# ESPIRAL DE FIBONACCI



# NÚMERO ÁUREO

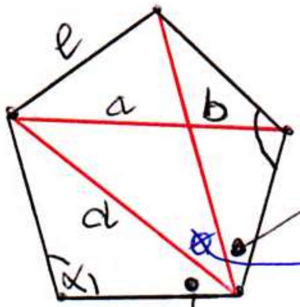
**_1,61803……_**

| | | |
|---|---|---|
| 1. | 16 | 24 |
| 2. | 23 | 19 |
| 3. | 39 | 43 |
| 4. | 62 | 62 |
| 5. | 101 | 105 |
| 6. | 163 | 167 |
| 7. | 264 | 272 |
| 8. | 427 | 439 |
| 9. | 691 | 711 |
| 10. | 1118 | 1150 |
| | $2904 = 264 \times 11$ | $2992 = 272 \times 11$ |

1. $A$
2. $B$
3. $A+B$
4. $B+(A+B) = A+2B$
5. $(A+B)+(A+2B) = 2A+3B$
6. $(A+2B)+(2A+3B) = 3A+5B$
7. $(2A+3B)+(3A+5B) = 5A+8B$
8. $(3A+5B)+(5A+8B) = 8A+13B$
9. $(5A+8B)+(8A=13B) = 13A+21B$
10. $(8A+13B)+(13A+21B) = 21A +34B$

Adding up all 10 numbers in the chain gives us a grand total of $55A+88B$ – check it yourself. But look at the seventh number in your column... this line is $5A+8B$. It is exactly the total of the chain but divided by 11!

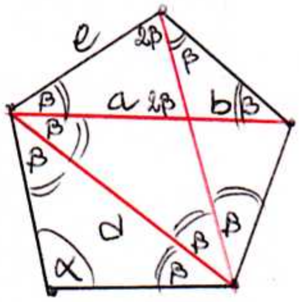
$\alpha = 3\pi/5$




$(\pi - \alpha)/2 = \pi/5$

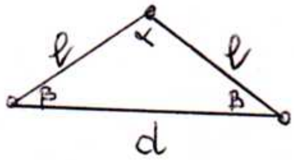
$\alpha - 2\dfrac{\pi}{5} = \dfrac{\pi}{5}$

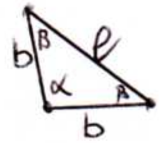
$(\pi - \alpha)/2 = \pi/5$




$\alpha = 3\pi/5 \qquad \beta = \pi/5$

$a + b = d \qquad \cancel{\boxed{a = e}}$

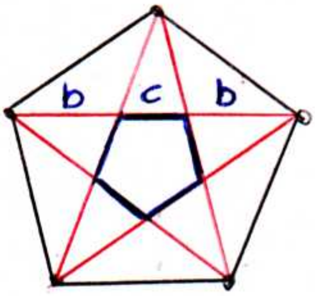



$\dfrac{d}{2} = \ell \cos\beta \qquad \dfrac{\ell}{2} = b \cos\beta$

$$2\cos\beta = 2\,\frac{1+\sqrt{5}}{4} = \phi$$

$$\boxed{\frac{d}{\ell} = \phi} \qquad \boxed{\frac{\ell}{b} = \phi}$$




$b + c = a = \ell \qquad c = \ell - b = \ell\,\dfrac{\phi - 1}{\phi}$

$$\frac{\ell}{c} = \frac{\phi}{\phi - 1}$$

Propriedade de $\phi$

$\phi^2 - \phi - 1 = 0$

$\phi^2 - 1 = \phi \qquad \phi^2 = \phi + 1$

$(\phi - 1)(\phi + 1) = \phi$

$\phi + 1 = \dfrac{\phi}{\phi - 1}$

$\phi^2 = \dfrac{\phi}{\phi - 1}$

$$\boxed{\frac{\ell}{c} = \phi^2}$$