Comparando aproximações na resolucão de integrais

CONSIDERAMOS A FUNÇÃO $\sqrt{1+x}$ E CALCULAMOS A INTEGRAL DELA ENTRE 0 E 3

INTEGRAÇÃO ANALÍTICA

$$\int_0^3 dx\,(1+x)^{1/2} = \int_0^3 dx\left[\frac{2}{3}(1+x)^{3/2}\right]' = \frac{2}{3}\left[\sqrt{(1+x)^3}\right]_0^3 = \frac{2}{3}\left(\sqrt{4^3}-\sqrt{1^3}\right) = \boxed{\frac{14}{3}}$$

SÉRIE DE TAYLOR $f(x) = f(x_0) + f'(x_0)(x-x_0) + f''(x_0)(x-x_0)^2/2$

$f(x) = (1+x)^{1/2}$	1	$\sqrt{2}$	$\sqrt{3}$	2
$f'(x) = \frac{1}{2}(1+x)^{-1/2}$	$1/2$	$1/2\sqrt{2}$	$1/2\sqrt{3}$	$1/4$
$f''(x) = -\frac{1}{4}(1+x)^{-3/2}$	$-1/4$	$-1/8\sqrt{2}$	$-1/12\sqrt{3}$	$-1/32$
	$x_0=0$	1	2	3

$f_0(x) = 1 + \frac{x}{2} - \frac{x^2}{8}$

$f_1(x) = \frac{23}{16\sqrt{2}} + \frac{5x}{8\sqrt{2}} - \frac{x^2}{16\sqrt{2}}$

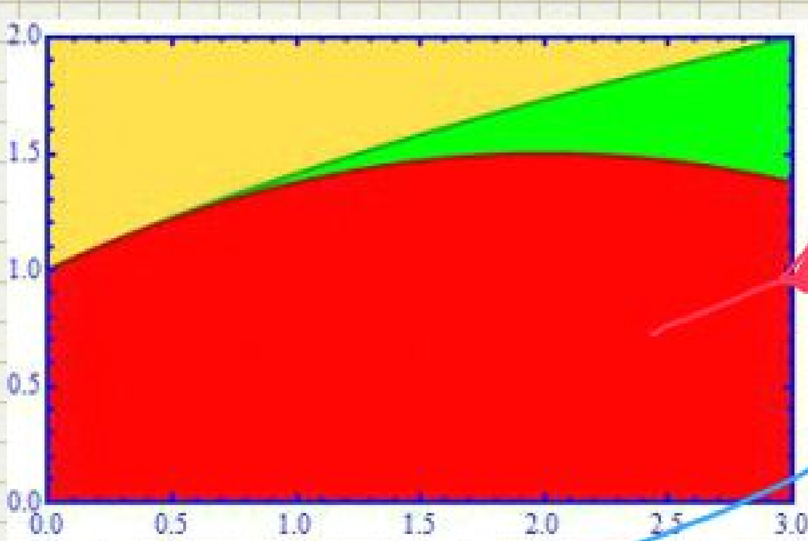
$f_2(x) = \frac{11}{6\sqrt{3}} + \frac{2x}{3\sqrt{3}} - \frac{x^2}{24\sqrt{3}}$

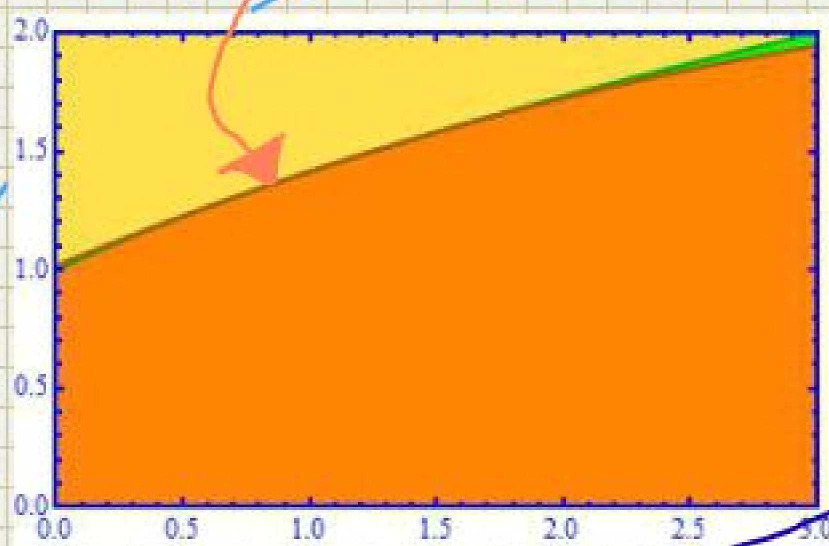
$f_3(x) = \frac{71}{64} + \frac{11}{32}x - \frac{x^2}{64}$

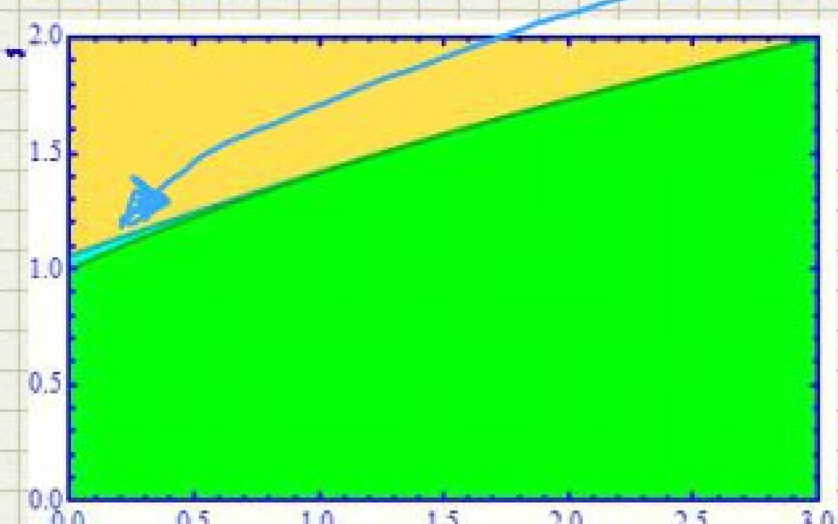
INTEGRAL ENTRE 0 E 3 DARÁ $\frac{33}{8}$, $\frac{105}{16\sqrt{2}}$, $\frac{65}{8\sqrt{3}}$, $\frac{303}{64}$

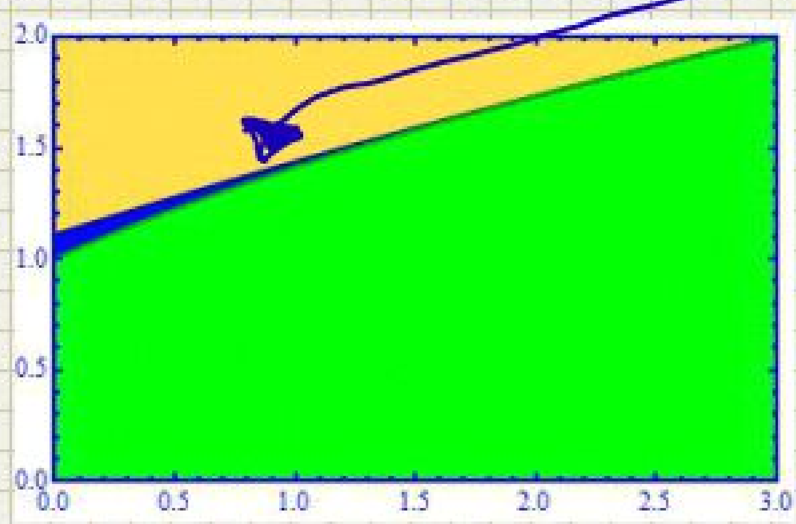
4.125 4.640 4.691 4.734

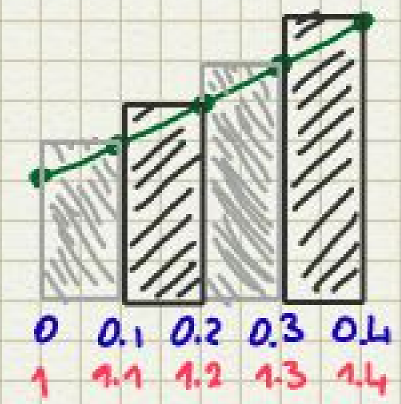
ANALITICO $\frac{14}{3} \sim 4.667$











$0.1\, f(0.1) + 0.1\, f(0.2) + 0.1\, f(0.3) + 0.1\, f(0.4)$

$0.1\, [f(0.1) + f(0.2) + f(0.3) + f(0.4)]$

$0.1\, [f(1.1) + f(1.2) + f(1.3) + f(1.4)]$

FÓRMULA $\frac{x_2 - x_1}{N} \sum_{1}^{N} i\, f\left[x_1 + \frac{i}{N}(x_2 - x_1)\right]$

CONSIDERAMOS O CASO ANTERIOR $f(x) = \sqrt{1 + x}$

$x_1 = 0 \quad x_2 = 3$

$N = 10$ $\quad \frac{3}{10} \sum_{1}^{10} i \sqrt{1 + 0 + \frac{i}{10} 3} = 0.3 \sum_{1}^{10} i \sqrt{1 + i\, 0.3}$

~ 4.815

$N = 30$ $\quad \frac{3}{30} \sum_{1}^{30} i \sqrt{1 + i \frac{3}{30}} = 0.1 \sum_{1}^{30} i \sqrt{1 + i\, 0.1}$

~ 4.716

$N = 100 \quad 4.682 \qquad N = 300 \quad 4.672$

4.667	$N=10$ 4.815 $N=100$ 4.682	$N=30$ 4.716 $N=300$ 4.672
	$f_0(x)$ 4.125 $f_2(x)$ 4.691	$f_1(x)$ 4.640 $f_3(x)$ 4.734