Gabarito segunda prova

1) USANDO TAYLOR CALCULAR $\int_0^4 dx \sqrt{1+\frac{x^3}{4}}$

$$f(x) = (1+x^3/4)^{\frac12}$$
$$f'(x) = \frac12 (1+x^3/4)^{-\frac12}\frac{3x^2}{4} = \frac38 x^2\left(1+\frac{x^3}{4}\right)^{-\frac12}$$

$$f''(x) = \frac34 x\left(1+\frac{x^3}{4}\right)^{-1/2} - \frac{3}{16}x^2\left(1+\frac{x^3}{4}\right)^{-3/2}\frac{3x^2}{4}$$
$$= \frac34 x\left(1+\frac{x^3}{4}\right)^{-1/2} - \frac{9}{64}x^4\left(1+\frac{x^3}{4}\right)^{-3/2}$$

$f(2)=\sqrt3$ $\quad f'(2) = \frac{3}{2\sqrt3} = \frac{\sqrt3}{2}$ $\quad f''(2) = \frac{3}{2\sqrt3} - \frac{9}{4(\sqrt3)^3} = \frac{3}{2\sqrt3} - \frac{9}{12\sqrt3}$
$= \frac{9}{12\sqrt3} = \frac{3}{4\sqrt3} = \frac{\sqrt3}{4}$

$$f(x) = \sqrt3 + \frac{\sqrt3}{2}(x-2) + \frac{\sqrt3}{8}(x-2)^2$$
$$g(x) = \sqrt3 x + \frac{\sqrt3}{4}(x-2)^2 + \frac{\sqrt3}{24}(x-2)^3$$
$$g(4)-g(0) = 4\sqrt3 + \frac{2\sqrt3}{3} = \frac{14\sqrt3}{3} = \boxed{\frac{14}{\sqrt3}}$$

$$g(4) = 4\sqrt3 + \frac{\sqrt3}{\cancel{4}}\cancel{4} + \frac{\sqrt3}{\cancel{24}\,3}\,8$$
$$g(0) = 0 + \frac{\sqrt3}{\cancel{4}}\cdot\cancel{4} - \frac{\sqrt3}{\cancel{24}\,3}\,8$$

2) CALCULAR

$$\lim_{x\to0} \frac{\overset{x^2}{(e^{x^2}-1)}\ \overset{-\frac92 x^2}{(\cos 3x -1)}}{\underset{9x^2}{\mathrm{sen}^2\, 3x}\ \ \underset{x^2}{\mathrm{sen}\, x^2}}$$

RESPOSTA $\boxed{-\frac12}$

$\lim_{z\to0}\frac{e^z-1}{z} = \lim_{z\to0}\frac{e^z}{1} = 1$ $\quad e^z-1\sim z$ $\quad e^{x^2}-1 \sim x^2$

$\lim_{z\to0}\frac{\mathrm{sen}\, z}{z} = \lim_{z\to0}\frac{\cos z}{1}=1$ $\quad \sin z\sim z$ $\quad \mathrm{sen}\,3x\sim 3x$
$\mathrm{sen}\,x^2\sim x^2$

$\lim_{z\to0}\frac{\cos z -1}{z^2} = \lim_{z\to0}\frac{-\sin z}{2z} = -\frac12$ $\quad \cos z -1 \sim -\frac{z^2}{2}$ $\quad \cos3x-1\sim -\frac{9x^2}{2}$

3) USANDO INTEGRAÇÃO POR PARTES CALCULAR A INTEGRAL DE $4x^2e^{2x}$

$$\int fg' = fg - \int f'g$$

$f=4x^2$ $\quad f'=8x$
$g'=e^{2x}$ $\quad g=?$
$e^{2x}\to 2e^{2x}$
$\frac{e^{2x}}{2}\to e^{2x}$

$$\int 4x^2e^{2x} = 4x^2\frac{e^{2x}}{2} - \int 8x\frac{e^{2x}}{2} = 2x^2e^{2x} - \int 4xe^{2x}$$

$f=4x$ $\quad f'=4$
$g'=e^{2x}$ $\quad g=e^{2x}/2$

$$\int 4xe^{2x} = 4x\frac{e^{2x}}{2} - \int 4\frac{e^{2x}}{2} = 2xe^{2x} - 2\int e^{2x}$$
$$\downarrow$$
$$\frac{e^{2x}}{2}$$

$$\int 4x^2e^{2x} = 2x^2e^{2x} - 2xe^{2x} + e^{2x} = (2x^2-2x+1)e^{2x}$$

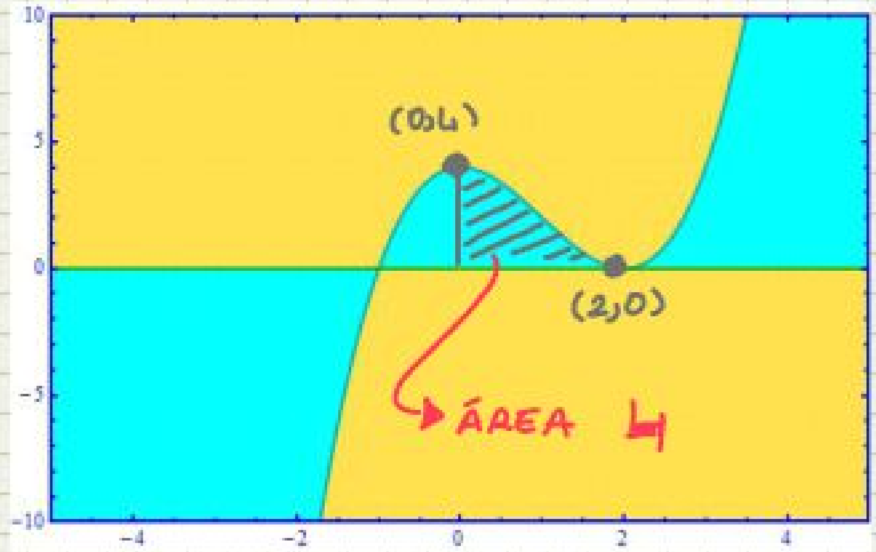
CONTROLE $\quad (2x^2-2x+1)'e^{2x} + (2x^2-2x+1)(e^{2x})'$

$$(4x-2)e^{2x} + (2x^2-2x+1)2e^{2x} = (\cancel{4x}-\cancel{2}+4x^2-\cancel{4x}+\cancel{2})e^{2x}$$

$4x^2e^{2x}$ ✓

4) ENCONTRAR A ÁREA ENTRE O MAX E O MIN DO POLINÔMIO DE 3º GRAU $\quad x^3-3x^2+4$

$3x^2-6x=0 \Rightarrow x=0 \quad x=2$ MAX = (0,4) MIN = (2, 8-12+4)
(2,0)



$$\int_0^2 dx\,(x^3-3x^2+4)$$

$$g(x) = \frac{x^4}{4} - \frac{\cancel{3}x^3}{\cancel{3}} + 4x$$

$$g(2) = 4 - \cancel{8} + \cancel{8}$$

$$g(0) = 0$$

$g(2)-g(0)$

$\boxed{4}$

5) ESTUDAR AS FUNÇÕES

$$f(x) = \frac{\sqrt{2x-3}}{x-2}\,e^x$$

$$g(x) = \frac{\sqrt{3-2x}}{x-2}\,e^x$$

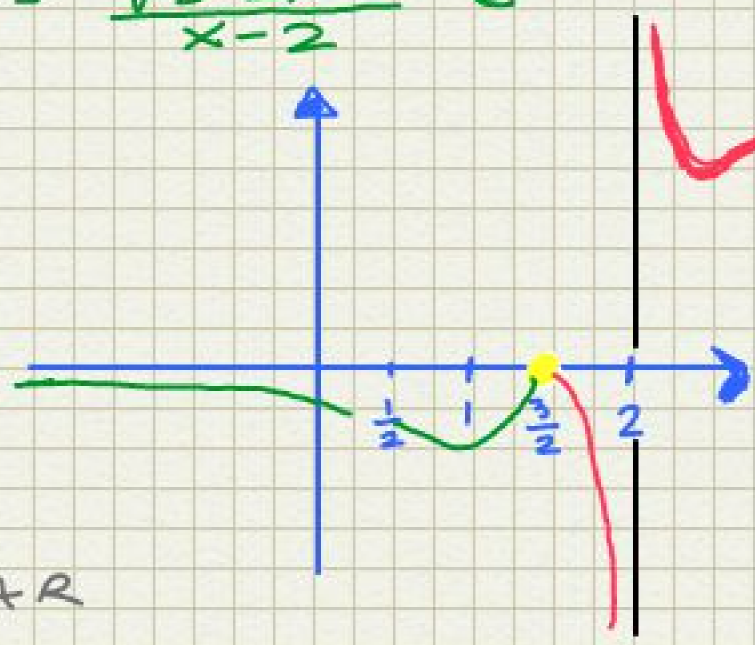
DÓMINIO ZEROS

$f(x)$ $x>3/2$ $x=3/2$
$g(x)$ $x<3/2$ $x=3/2$

ASSÍNTOTAS LIMITES $x\to\pm\infty$

$f(x)$ $x=2$ $x\to+\infty$ $f(x)\to+\infty$
$g(x)$ $\cancel{x=2}$ $x\to-\infty$ $g(x)\to 0^-$
FORA DO DOMÍNIO

$(2^+, +\infty)$ $(2^-, -\infty)$



CALCULAMOS A DERIVADA PARA ENCONTRAR MÁXIMOS E MÍNIMOS

$$\frac{\sqrt{2x-3}\;e^x}{x-2} \rightarrow \frac{\left(\frac{\cancel{2}}{\cancel{2}\sqrt{2x-3}} + \sqrt{2x-3}\right)\cancel{e^x}(x-2) - \sqrt{2x-3}\;\cancel{e^x}}{(\cancel{x-2})^2} = 0$$

NUM.: $[1 + 2x-3](x-2) - (2x-3) = 0$

$$(2x-2)(x-2) - 2x + 3 = 0$$

$$2x^2 - 6x + 4 - 2x + 3 = 2x^2 - 8x + 7 = 0$$

$$x = \frac{4 \pm \sqrt{16-14}}{2} = \frac{4\pm\sqrt{2}}{2} = 2 \pm \frac{1}{\sqrt{2}}$$

$\rightarrow$ 2.71 MIN
$\rightarrow$ 1.29 MIN

$\simeq 0.707$