Exercícios aula do dia 4 de maio

1) USANDO A EXPANSÃO DE TAYLOR ATÉ A SEGUNDA ORDEM EM $x_0 = 2$ CALCULE A INTEGRAL DE $(x^2+x^5)^{1/2}$ ENTRE 1 E 3.

$f(x) = (x^2+x^5)^{\frac{1}{2}}$ $\qquad f'(x) = \frac{1}{2}(x^2+x^5)^{-\frac{1}{2}}(2x+5x^4)$

$f''(x) = -\frac{1}{4}(x^2+x^5)^{-\frac{3}{2}}(2x+5x^4)^2 + \frac{1}{2}(x^2+x^5)^{-\frac{1}{2}}(2+20x^3)$

$f(2) = 36^{\frac{1}{2}} = 6$ $\qquad f'(2) = \frac{1}{2}36^{-\frac{1}{2}}(4+80) = \frac{84}{12} = 7$

$f''(2) = -\frac{1}{4\,36^{3/2}}(4+80)^2 + \frac{1}{2}36^{-\frac{1}{2}}(2+160) = -\frac{84\cdot 84}{4\cdot 216} + \frac{162}{12}$

$= -\frac{49}{6} + \frac{27}{2} = \frac{-49+81}{6} = \frac{32}{6} = \frac{16}{3}$

$f(x) = 6 + 7(x-2) + \frac{16}{3}\frac{(x-2)^2}{2}$

$$\boxed{f(x) = 6 + 7(x-2) + \frac{8}{3}(x-2)^2}$$

$$\boxed{g(x) = 6x + \frac{7}{2}(x-2)^2 + \frac{8}{9}(x-2)^3}$$

$g(3) = 18 + \frac{7}{2} + \frac{8}{9}$

$g(1) = 6 + \frac{7}{2} - \frac{8}{9}$

$g(3) - g(1) = 12 + \frac{16}{9} = \boxed{\frac{124}{9}}$

2) USANDO A INTEGRAÇÃO POR PARTES CALCULAR $\int dx\, x^2 \operatorname{sen} 2x$

$f = x^2 \qquad g' = \operatorname{sen} 2x$

$f' = 2x \qquad g = ?$

$\cos 2x \longrightarrow -2\operatorname{sen} 2x$

$-\frac{1}{2}\cos 2x \longrightarrow \operatorname{sen} x$

$\int fg' = fg - \int f'g$

$\int dx\, x^2 \operatorname{sen} 2x = -\frac{x^2}{2}\cos 2x - \int dx\, 2x\left(-\frac{\cos 2x}{2}\right)$

$= -\frac{x^2}{2}\cos 2x + \int dx\, x\cos 2x$

REPETIMOS A INTEGRAÇÃO POR PARTES PARA $\int dx\, x\cos 2x$

$f = x \qquad g' = \cos 2x$

$f' = 1 \qquad g = ?$

$\operatorname{sen} 2x \longrightarrow 2\cos 2x$

$\frac{1}{2}\operatorname{sen} 2x \longrightarrow \cos 2x$

$\int dx\, x\cos 2x = \frac{x}{2}\operatorname{sen} 2x - \int dx \frac{\operatorname{sen} 2x}{2}$

$= \frac{x}{2}\operatorname{sen} 2x + \frac{\cos 2x}{4}$

RESPOSTA $\boxed{-\frac{x^2}{2}\cos 2x + \frac{x}{2}\operatorname{sen} 2x + \frac{\cos 2x}{4}}$

controle

$\cancel{-\frac{2x}{2}\cos 2x} + \frac{x^2}{2}2\operatorname{sen} 2x + \cancel{\frac{1}{2}\operatorname{sen} 2x} + \cancel{\frac{x}{2}2\cos 2x} - \cancel{\frac{2\operatorname{sen} 2x}{4}}$ ✓

3) CALCULE O LIMITE $\lim_{x\to\infty} \dfrac{(e^{\frac{1}{x}}-1)\,x^2\,(\cos\frac{3}{x}-1)}{\operatorname{sen}(4/x)}$

TROCAMOS VARIÁVEL $y=\dfrac{1}{x}$

$$\lim_{y\to 0} \frac{(e^y-1)(\cos 3y-1)}{y^2 \operatorname{sen} 4y}$$

$$\lim_{y\to 0} \frac{e^y-1}{y} = \lim_{y\to 0} \frac{e^y}{1} = 1 \qquad \boxed{e^y-1 \sim y}$$

$$\lim_{y\to 0} \frac{\cos 3y-1}{y^2} = \lim_{y\to 0} -\frac{3\operatorname{sen} 3y}{2y} = \lim_{y\to 0} -\frac{9\cos 3y}{2} = -\frac{9}{2}$$

$$\boxed{\cos 3y-1 \sim -\frac{9}{2}y^2}$$

$$\lim_{y\to 0} \frac{\operatorname{sen} 4y}{y} = \lim_{y\to 0} \frac{4\cos 4y}{1} = 4 \qquad \boxed{\operatorname{sen} 4y \sim 4y}$$

$$\lim_{y\to 0} \frac{(e^y-1)(\cos 3y-1)}{y^2 \operatorname{sen} 4y} = \lim_{y\to 0} \frac{y\left(-\frac{9}{2}y^2\right)}{y^2\, 4y} = \boxed{-\frac{9}{8}}$$