Taylor expansion

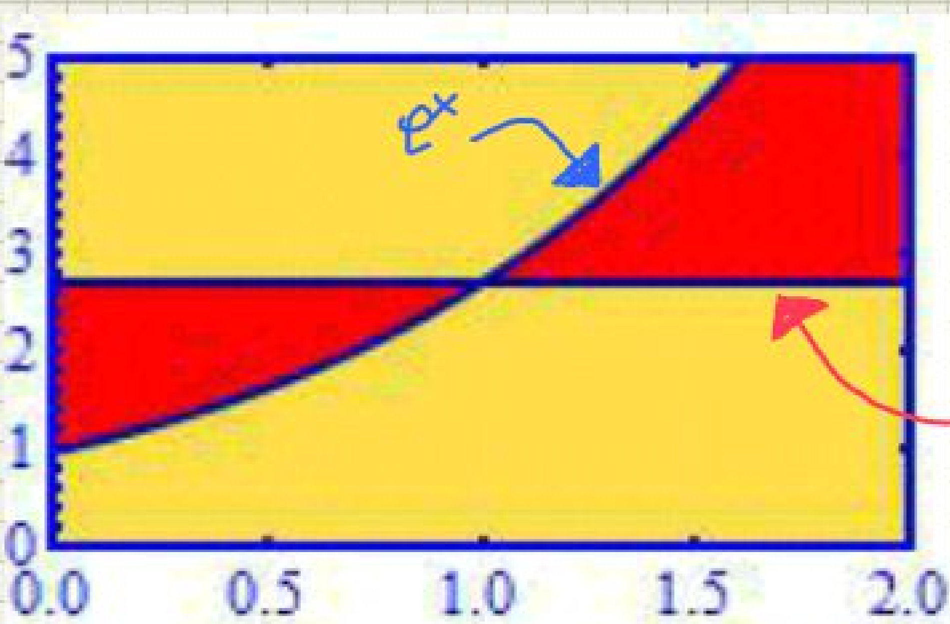
$$f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2}(x-x_0)^2 + \dots$$

EXEMPLOS : e^x E $\sqrt{1+x}$

ESCOLHEMO e^x E DESENVOLVEMOS ELA EM $x_0 = 1$
(PODE SER ESCOLHIDO QUALQUER x_0!)

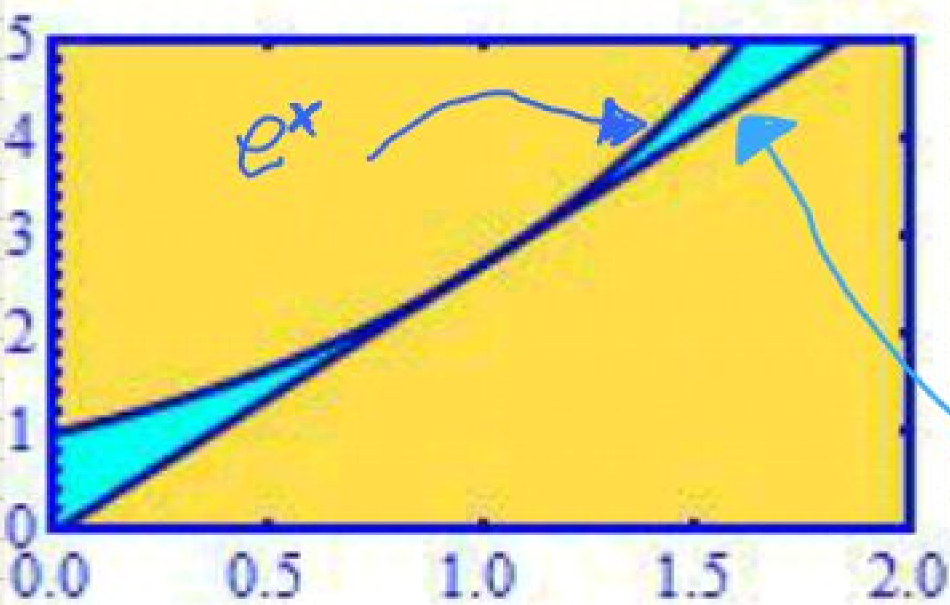
$$\boxed{e^x = e^1 + e^1(x-1) + \frac{e^1}{2}(x-1)^2 + \dots}$$

$f(x) = e^x \qquad f'(x) = e^x \qquad f''(x) = e^x$



ZERO ORDER EXPANSION $f(x_0)$

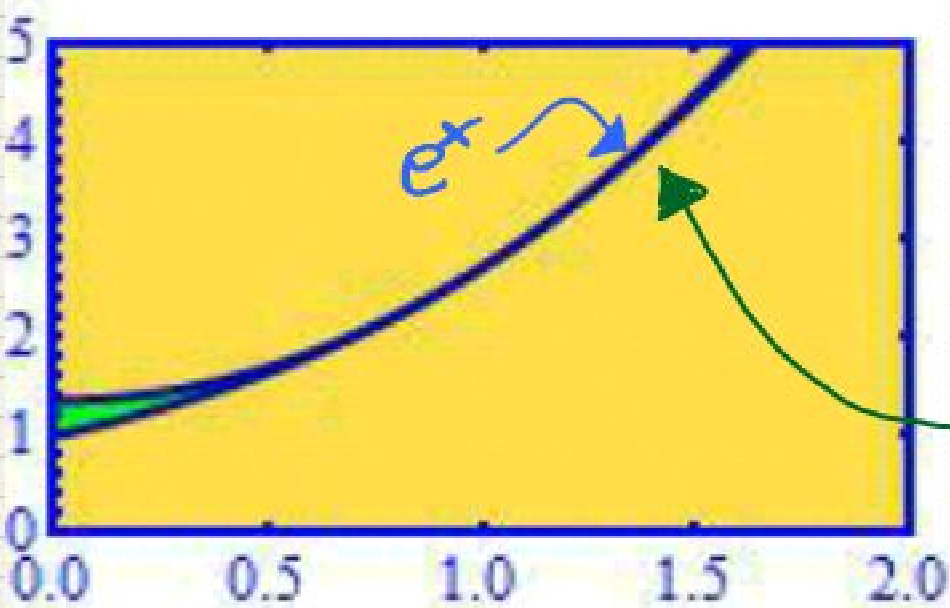
e^1



FIRST ORDER EXPANSION

$f(x_0) + f'(x_0)(x-x_0)$

$e^1 + e^1(x-1)$



SECOND ORDER EXPANSION

$f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2}(x-x_0)^2$

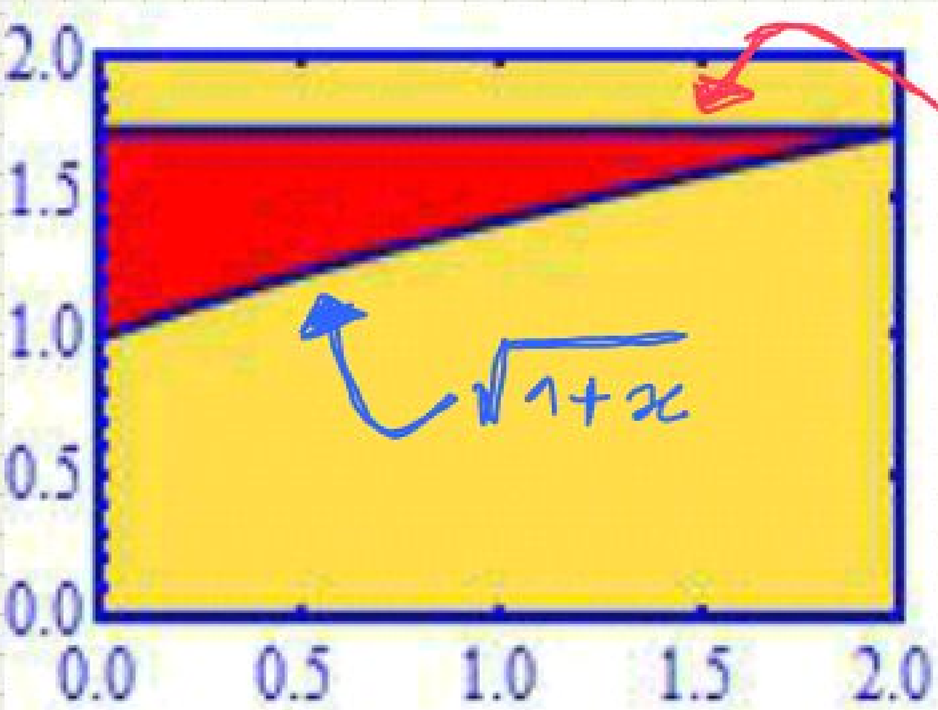
$e^1 + e^1(x-1) + \frac{e^1}{2}(x-1)^2$

AGORA DESENVOLVEMOS $\sqrt{1+x}$ EM $x_0=2$

$$f'(x) = \frac{1}{2\sqrt{1+x}} = \frac{1}{2}(1+x)^{-\frac{1}{2}} \qquad f'(2) = \frac{1}{2\sqrt{3}}$$

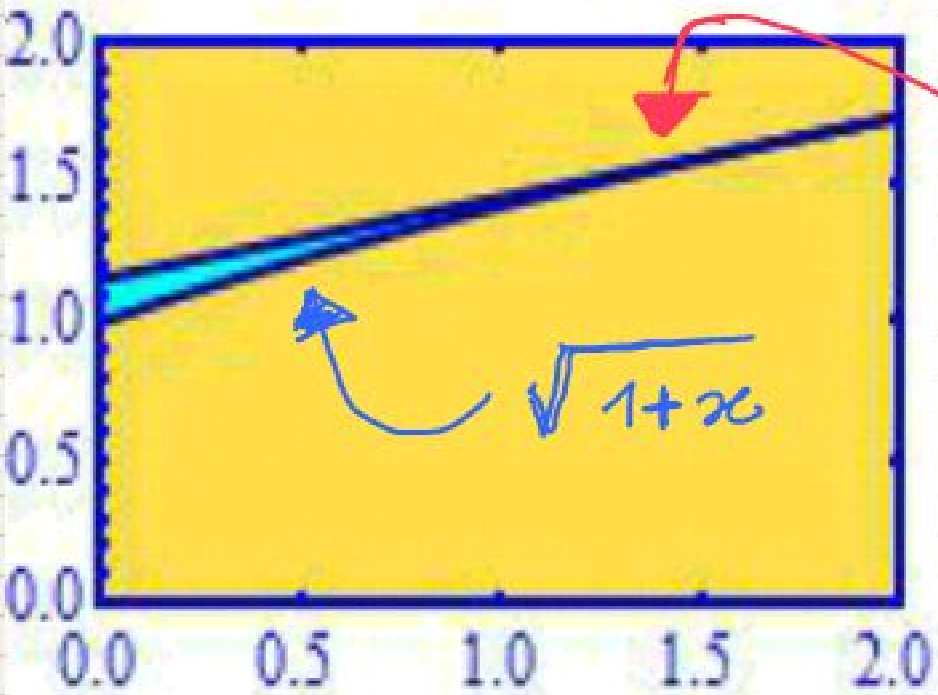
$$f''(x) = -\frac{1}{4}(1+x)^{-3/2} \qquad f''(2) = -\frac{1}{4\sqrt{27}} = -\frac{1}{12\sqrt{3}}$$

$$\boxed{\sqrt{1+x} = \sqrt{3} + \frac{1}{2\sqrt{3}}(x-2) - \frac{1}{24\sqrt{3}}(x-2)^2 + \dots}$$



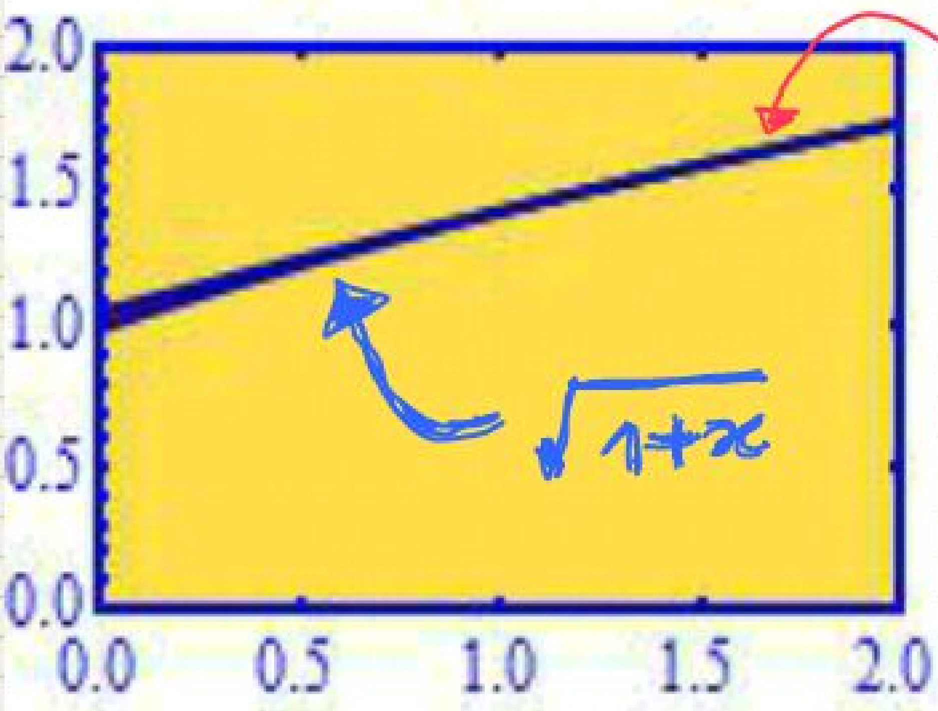
ZERO ORDER EXPANSION

$$\boxed{\sqrt{3}}$$



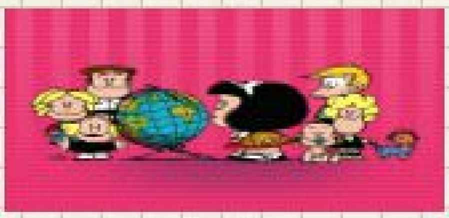
FIRST ORDER EXPANSION

$$\boxed{\sqrt{3} + \frac{1}{2\sqrt{3}}(x-2)}$$



SECOND ORDER EXPANSION

$$\boxed{\sqrt{3} + \frac{1}{2\sqrt{3}}(x-2) - \frac{1}{24\sqrt{3}}(x-2)^2}$$



Usando Taylor para calcular limites

$$\lim_{x\to 0} \frac{e^x-1}{x} = \frac{1-1}{0} = \frac{0}{0} \quad ?$$

O PONTO DE INTERESSE É $x=0$

ENTÃO DESENVOLVEMOS e^x EM $x_0=0$: $e^x = e^0 + e^0(x-0) + \frac{e^0}{2}(x-0)^2$

$= 1 + x + \frac{x^2}{2}$

$$\lim_{x\to 0} \frac{e^x-1}{x} = \lim_{x\to 0} \frac{\cancel{1}+x+\frac{x^2}{2}-\cancel{1}}{x}$$

$$= \lim_{x\to 0} \frac{x+\frac{x^2}{2}}{x}$$

$$= \lim_{x\to 0} \left(1+\frac{x}{2}\right) = \boxed{1}$$

RESPOSTA

$$\lim_{x\to x_0} \frac{f(x)}{g(x)} = \lim_{x\to x_0} \frac{f(x_0) + f'(x_0)(x-x_0) + f''(x_0)(x-x_0)^2/2 + \dots}{g(x_0) + g'(x_0)(x-x_0) + g''(x_0)(x-x_0)^2/2 + \dots}$$

$$\lim_{x\to 1} \frac{(e^{x-1}-1)^2}{-x^3+5x^2-7x+3} = \frac{(1-1)^2}{-1+5-7+3} = \frac{0}{0}$$

$$\frac{2(e^{x-1}-1)\,e^{x-1}}{-3x^2+10x-7} = \frac{2(1-1)1}{-3+10-7} = \frac{0}{0}$$

$$\frac{2e^{x-1}e^{x-1} + 2(e^{x-1}-1)e^{x-1}}{-6x+10} = \frac{2+\cancel{2(1-1)}}{4} = \boxed{\frac{1}{2}}$$

$$\lim_{x\to 1} \frac{(e^{x-1}-1)^2}{(x-1)^{3/2}} = \frac{(1-1)^2}{(1-1)^{3/2}} = \frac{0}{0}$$

$$\frac{2(e^{x-1}-1)\,e^{x-1}}{\frac{3}{2}(x-1)^{\frac{1}{2}}} = \frac{2(1-1)}{\frac{3}{2}\sqrt{1-1}} = \frac{0}{0}$$

$$\frac{2e^{x-1}e^{x-1} + 2(e^{x-1}-1)e^{x-1}}{\frac{3}{4}(x-1)^{-\frac{1}{2}}} = \frac{2+\cancel{2(1-1)}}{\frac{3}{4}\,\frac{1}{\sqrt{1-1}}} = \frac{8}{3}\sqrt{0} = \boxed{0}$$

$$\lim_{x\to 0} \frac{(e^x-1)^3}{3x^3-x^4} = \frac{(1-1)^3}{0-0} = \frac{0}{0} \rightarrow \frac{3(e^x-1)^2e^x}{9x^2-4x^3} = \frac{0}{0}$$

$$\frac{3(e^x-1)^2e^x}{9x^2-4x^3} = \frac{3e^{3x}-6e^{2x}+3e^x}{9x^2-4x^3} \rightarrow \frac{9e^{3x}-12e^{2x}+3e^x}{18x-12x^2} = \frac{0}{0}$$

$$\rightarrow \frac{27e^{3x}-24e^{2x}+3e^x}{18-24x} = \frac{27-24+3}{18} = \boxed{\frac{1}{3}}$$

$$\lim_{x\to 0} \frac{\operatorname{sen}(x^2)}{x^2} = \lim_{x\to 0} \frac{\cos(x^2)\cdot \cancel{2x}}{\cancel{2x}} = 1$$

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \lim_{x\to 0} \frac{\sin x}{2x} = \lim_{x\to 0} \frac{\cos x}{2} = \frac{1}{2}$$

$$\lim_{x\to 0} \frac{e^{\sqrt{x}}-1}{x^{\alpha}}$$

PARA QUAL VALOR DE α O LIMITE É FINITO E DIFERENTE DE ZERO ?

$$\frac{\frac{1}{2\sqrt{x}}\, e^{\sqrt{x}}}{\alpha x^{\alpha-1}} = \frac{e^{\sqrt{x}}}{2\alpha x^{\alpha-\frac{1}{2}}}$$

RESPOSTA $\alpha = \frac{1}{2}$

$$\lim_{x\to 0} \frac{e^{\sqrt{x}}-1}{\sqrt{x}} = \lim_{x\to 0} \frac{\cancel{\frac{1}{2\sqrt{x}}}\, e^{\sqrt{x}}}{\cancel{\frac{1}{2\sqrt{x}}}} = 1$$

SE $\lim_{x\to 0} f(x) = 0$

$$\lim_{x\to 0} \frac{e^{f(x)}-1}{f(x)} = 1$$

$$\lim_{x\to 0} \frac{\operatorname{sen} f(x)}{f(x)} = 1$$

$$\lim_{x\to 0} \frac{1-\cos f(x)}{f(x)^2} = \frac{1}{2}$$

$$\lim_{x\to 0} \frac{\left[e^{f(x)}-1\right]^2}{1-\cos f(x)} = \lim_{x\to 0} \frac{2\left[e^{f(x)}-1\right] e^{f(x)}\, \cancel{f'(x)}}{\operatorname{sen} f(x)\, \cancel{f'(x)}} =$$

$$= \lim_{x\to 0} \frac{2e^{2f(x)} - 2e^{f(x)}}{\operatorname{sen} f(x)}$$

$$\lim_{x\to 0} \frac{\left[4e^{2f(x)} - 2e^{f(x)}\right]\cancel{f'(x)}}{\cos f(x)\; \cancel{f'(x)}} = 2$$

$e^{f(x)} - 1$ SE COMPORTA COMO $f(x)$

$1 - \cos f(x)$ SE COMPORTA COMO $f^2(x)/2$

LOGO $\dfrac{\left[e^{f(x)}-1\right]^2}{1-\cos f(x)} \longrightarrow \dfrac{f(x)^2}{\frac{f(x)^2}{2}} = 2$

$$\lim_{x\to 0} \frac{\left(e^{f(x)}-1\right)^2 \operatorname{sen}\left[f^2(x)\right] \operatorname{sen} f(x)}{\left[1-\cos f(x)\right]^2\; f(x)} \rightarrow \frac{f^2\, f^2\, f}{\left(\frac{f^2}{2}\right)^2 f} \rightarrow 4$$