Integração

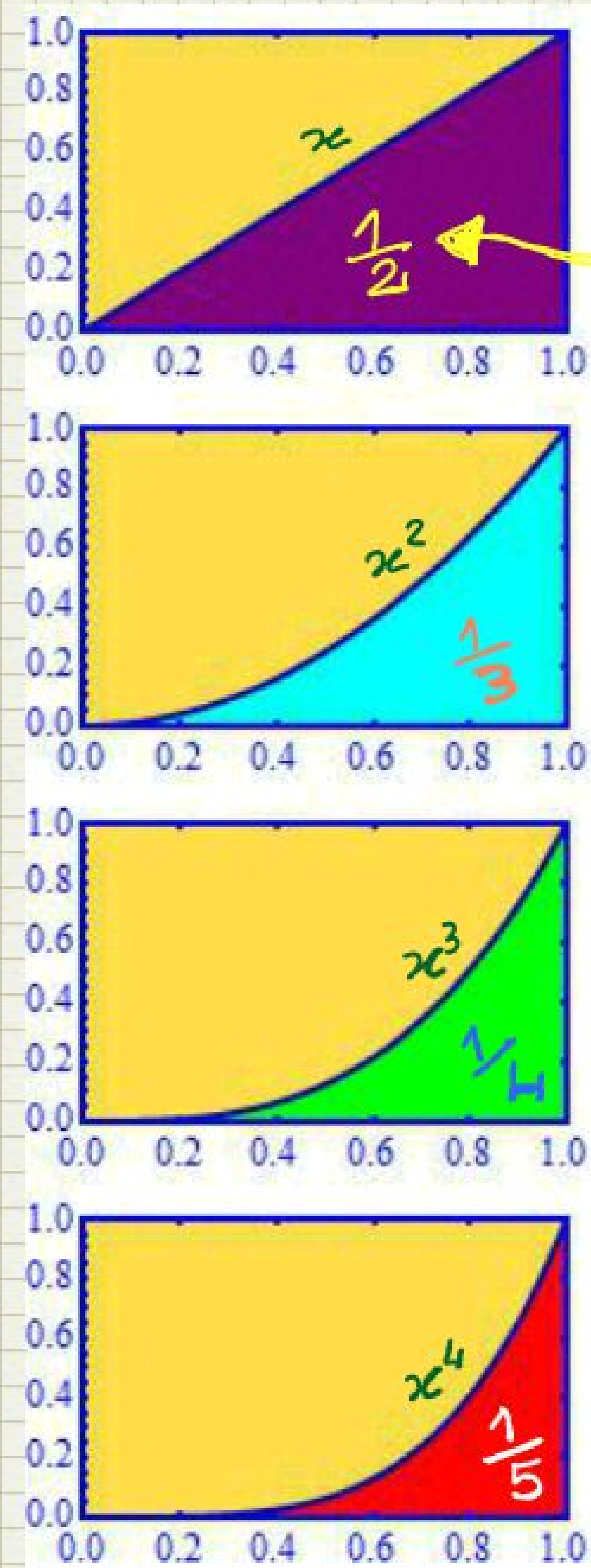
$$(x^n)' \xrightarrow{D} n x^{n-1}$$

INTEGRAÇÃO

$$I[x^n] = \frac{x^{n+1}}{n}$$

EXEMPLOS $\int_0^1 dx\, x = \left[\frac{x^2}{2}\right]_0^1 = \frac{1^2}{2} - \frac{0^2}{2} = \frac{1}{2}$

O QUE REPRESENTA?



$$\int_0^1 dx\, x = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}$$

ÁREA DO TRIÂNGULO

$$\int_0^1 dx\, x^2 = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}$$

$$\int_0^1 dx\, x^3 = \left[\frac{x^4}{4}\right]_0^1 = \frac{1}{4}$$

$$\int_0^1 dx\, x^4 = \left[\frac{x^5}{5}\right]_0^1 = \frac{1}{5}$$

CALCULAMOS AGORA A ÁREA DA FUNÇÃO xe^x

$$(xe^x)' = xe^x + e^x$$

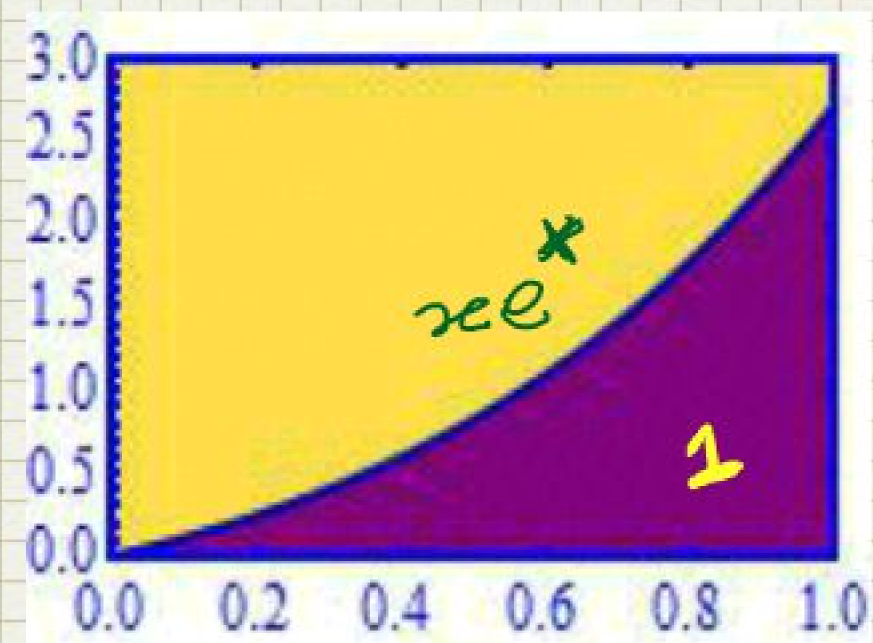
$$xe^x = (xe^x)' + e^x$$

$$\int dx\, xe^x = \int dx\, (xe^x)' + \int dx\, e^x$$

$$\int dx\, x e^x = x e^x - e^x$$

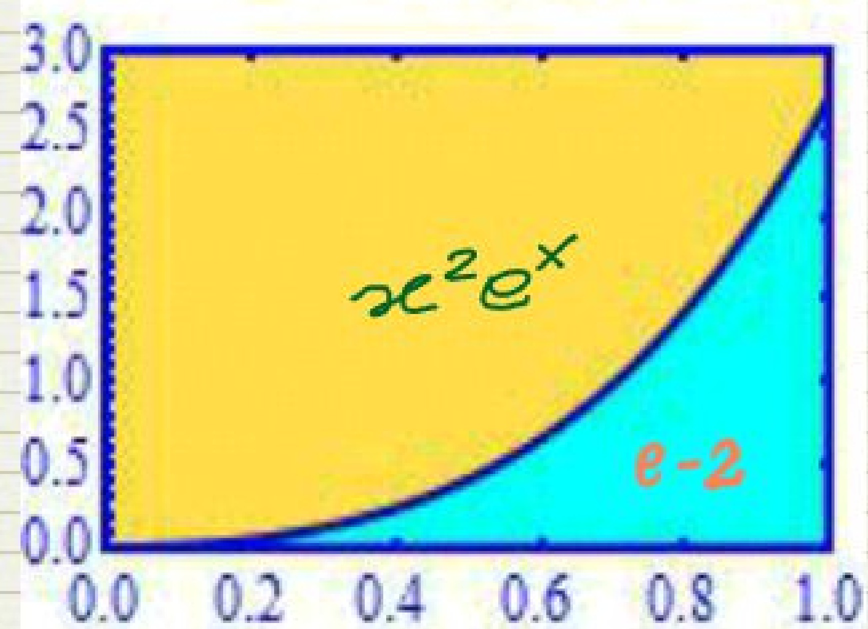
CONTROLE

$$(xe^x - e^x)' = x'e^x + x(e^x)' - (e^x)'$$
$$= \cancel{e^x} + \underline{xe^x} \cancel{- e^x} \checkmark$$



$$I[xe^x] = (x-1)e^x$$

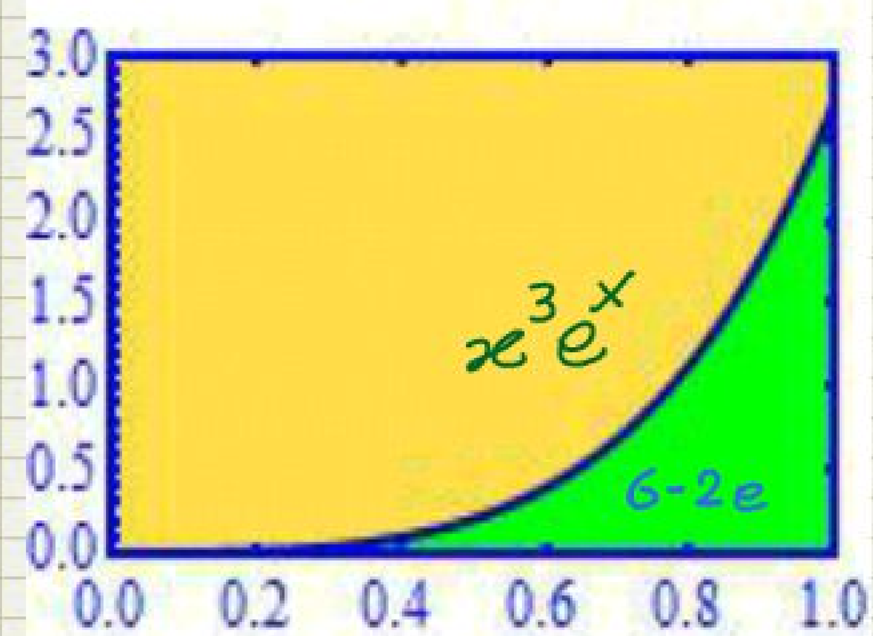
$$\left[(x-1)e^x\right]_0^1 = 1$$



$$(x^2e^x)' = 2xe^x + x^2e^x$$
$$x^2e^x = (x^2e^x)' - 2xe^x$$
$$I[x^2e^x] = x^2e^x - 2(x-1)e^x$$
$$= (x^2 - 2x + 2)e^x$$

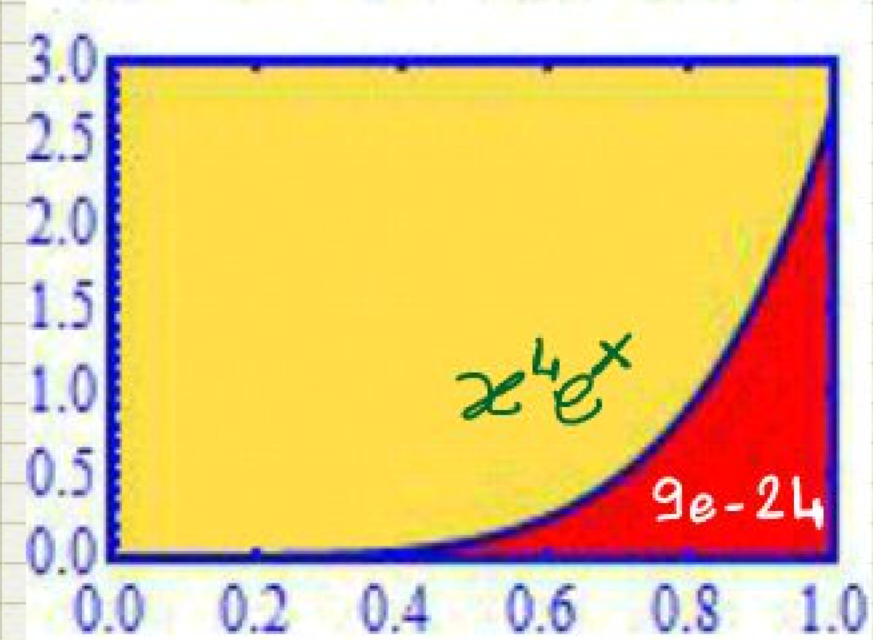
$$\left[(x^2 - 2x + 2)e^x\right]_0^1 = e - 2$$

$$x^2 \to -(x^2)' = -2x \to -(-2x)' = 2$$



$$I[x^3e^x] = (x^3 - 3x^2 + 6x - 6)e^x$$

$$\left[(x^3 - 3x^2 + 6x - 6)e^x\right]_0^1 = -2e + 6$$



$$I[x^4] = (x^4 - 4x^3 + 12x^2 - 24x + 24)e^x$$

ÁREA $9e - 24$

$e \sim 2.72$

ÁREAS: 1, 0.72, 0.56, 0.46

Integral approximations

1^a POSSIBILIDADE : USAR TAYLOR $\qquad \int_{x_1}^{x_2} dx\, f(x)$

FIXANDO $x_0 = (x_2 - x_1)/2$

EXEMPLO

$$\int_0^2 dx\; e^x \qquad x_0 = 1 \qquad e^x = e + e(x-1) + e(x-1)^2/2$$

$$\hookrightarrow e^2 - 1 \sim 6.39 \qquad = \cancel{e} + \cancel{ex} - \cancel{e} + \frac{e}{2}x^2 - \cancel{ex} + \frac{e}{2}$$

$$= \frac{e}{2}(x^2+1)$$

$$\int_0^2 dx\; e^x \approx \int_0^2 dx\; \frac{e}{2}(x^2+1) = \frac{e}{2}\left[\frac{x^3}{3} + x\right]_0^2 = \frac{e}{2}\left(\frac{8}{3}+2\right) = \frac{e}{2}\,\frac{14}{3} = \frac{7}{3}e$$

SECOND ORDER APPROXIMATION

THIRD ORDER APPROXIMATION $\qquad e^x \approx \frac{e}{2}(x^2+1) + \frac{e}{2}\frac{(x-1)^3}{3!}$

$$e^x \approx \frac{e}{2}(x^2+1) + \frac{e}{12}(x^3 - 3x^2 + 3x - 1) = e\left[\frac{x^3}{12} + \frac{x^2}{4} + \frac{x}{4} + \frac{5}{12}\right]$$

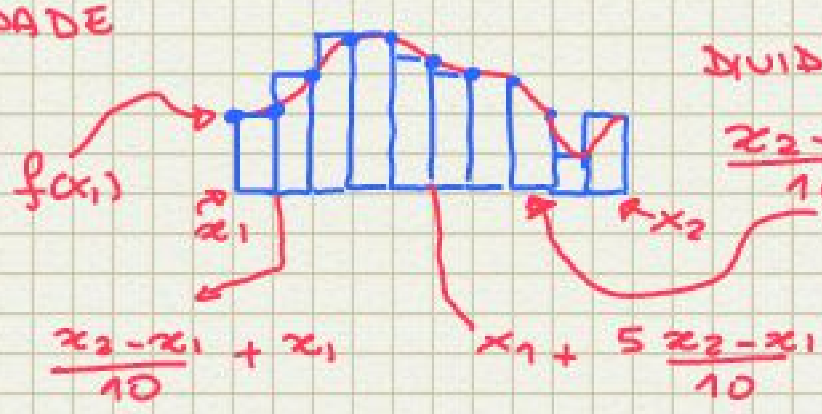
$$\frac{e}{12}\left[x^3 + 3x^2 + 3x + 5\right] \rightarrow \frac{e}{12}\left[\frac{x^4}{4} + x^3 + \frac{3}{2}x^2 + 5x\right]_0^2 = \frac{e}{\cancel{12}_{3}}\left[4 + 8 + 6 + 10\right]^{\cancel{28}\,7}$$

RESULTADO e^2 $\qquad$ APPROXIMAÇÕES $\qquad$ ORDEM 1: $2e \sim 5.44$

2: $\frac{7}{3}e \sim 6.34$

$$\int_0^2 dx\, \underbrace{(x-1)}_{y}{}^3 = \int_{-1}^{1} dy\; y^3 = \left[\frac{y^4}{4}\right]_{-1}^{1} = 0!$$

2^a POSSIBILIDADE



DIVIDIMOS O INTERVALO EM 10

$\frac{x_2 - x_1}{10}$ base

APPROXIMAÇÃO SERIA $\quad f\left(x_1 + \frac{x_2 - x_1}{10}\right)\frac{x_2 - x_1}{10} + f\left(x_1 + 2\,\frac{x_2 - x_1}{10}\right)\frac{x_2 - x_1}{10}$

$$+ \cdots + f\left(x_1 + 10\,\frac{x_2 - x_1}{10}\right)\frac{x_2 - x_1}{10}$$

GENERALIZANDO

$$\boxed{\frac{x_2 - x_1}{N}\sum_{i=1}^{N} f\left(x_1 + i\,\frac{x_2 - x_1}{N}\right)}$$

ALGUNS EXEMPLOS A INTEGRAÇÃO É FEITA ENTRE 0 E 1

$f(x)$	ANALÍTICO	T1	T2	N10	N20	N50	N100
xe^x	1	0.824	0.996	1.140	1.069	1.027	1.014
x^2e^x	$e-2$ (0.718)	0.412	0.704	0.861	0.788	0.746	0.732
x^3e^x	$6-2e$ (0.563)	0.206	0.524	0.708	0.634	0.591	0.577