$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1} = \begin{pmatrix} \tilde A & \tilde B \\ \tilde C & \tilde D \end{pmatrix}$$

$$\tilde A = (A - BD^{-1}C)^{-1} \qquad \tilde A = -\tilde B D B^{-1}$$
$$\tilde B = (C - DB^{-1}A)^{-1} \qquad \tilde B = -\tilde A B D^{-1}$$
$$\tilde C = (B - AC^{-1}D)^{-1} \quad \text{ou} \quad \tilde C = -\tilde D C A^{-1}$$
$$\tilde D = (D - CA^{-1}B)^{-1} \qquad \tilde D = -\tilde C A C^{-1}$$

por exemplo se $A = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}$ $\cancel{A^{-1}}$ $\rightsquigarrow \begin{bmatrix} 1 & \alpha \\ 2 & 1 \end{bmatrix}$ e depois fazer $\alpha \to 1/2$

$$\begin{pmatrix} 1 & \alpha & 0 & 1 \\ 2 & 1 & 1 & 1 \\ \beta & 1 & 0 & 1 \\ 1 & 2 & 1 & 1 \end{pmatrix} \qquad A^{-1} = \frac{1}{1-2\alpha}\begin{pmatrix} 1 & -\alpha \\ -2 & 1 \end{pmatrix} \qquad B^{-1} = D^{-1} = \begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix}$$

$$C^{-1} = \frac{1}{2\beta-1}\begin{pmatrix} 2 & -1 \\ -1 & \beta \end{pmatrix}$$

$$\tilde A = (A - C)^{-1} = \begin{pmatrix} 1-\beta & \alpha-1 \\ 1 & -1 \end{pmatrix}^{-1} = \boxed{\frac{1}{\alpha-\beta}\begin{pmatrix} 1 & \alpha-1 \\ 1 & \beta-1 \end{pmatrix} = \tilde A} \qquad \boxed{\tilde B = -\tilde A}$$

$$\tilde C = \left[\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} - \begin{pmatrix} 1 & \alpha \\ 2 & 1 \end{pmatrix}\frac{1}{2\beta-1}\begin{pmatrix} 2 & -1 \\ -1 & \beta \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}\right]^{-1}$$

$$= (2\beta-1)\left[\begin{pmatrix} 0 & 2\beta-1 \\ 2\beta-1 & 2\beta-1 \end{pmatrix} - \begin{pmatrix} 2-\alpha & \alpha\beta-1 \\ 3 & \beta-2 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}\right]^{-1}$$

$$= (2\beta-1)\left[\begin{pmatrix} 0 & 2\beta-1 \\ 2\beta-1 & 2\beta-1 \end{pmatrix} - \begin{pmatrix} \alpha\beta-1 & 1-\alpha+\alpha\beta \\ \beta-2 & 1+\beta \end{pmatrix}\right]^{-1}$$

$$= (2\beta-1)\begin{pmatrix} 1-\alpha\beta & 2\beta-2+\alpha-\alpha\beta \\ \beta+1 & \beta-2 \end{pmatrix}^{-1}$$

$$= (2\beta-1)\begin{pmatrix} 1-\alpha\beta & (2-\alpha)(\beta-1) \\ \beta+1 & \beta-2 \end{pmatrix}^{-1}$$

$$= \cancel{(2\beta-1)}\frac{1}{(1-\alpha\beta)(\beta-2) - (\beta^2-1)(2-\alpha)}\begin{bmatrix} \beta-2 & (\alpha-2)(\beta-1) \\ -(\beta+1) & 1-\alpha\beta \end{bmatrix}$$

$$\beta - \cancel{2} - \cancel{\alpha\beta^2} + 2\alpha\beta - 2\beta^2 + \cancel{\alpha\beta^2} + \cancel{2} - \alpha$$

$$\beta(1-2\beta) + \alpha(2\beta-1)$$

$$\cancel{(2\beta-1)}(\alpha-\beta)$$

$$\boxed{\tilde C = \frac{1}{\alpha-\beta}\begin{bmatrix} \beta-2 & (\alpha-2)(\beta-1) \\ -(\beta+1) & 1-\alpha\beta \end{bmatrix}}$$

$$\tilde D = -\tilde C A C^{-1} = \frac{1}{\beta-\alpha}\begin{pmatrix} \beta-2 & (\alpha-2)(\beta-1) \\ -(1+\beta) & 1-\alpha\beta \end{pmatrix}\begin{pmatrix} 1 & \alpha \\ 2 & 1 \end{pmatrix}\frac{1}{2\beta-1}\begin{pmatrix} 2 & -1 \\ -1 & \beta \end{pmatrix}$$

$$= \frac{1}{(\beta-\alpha)(2\beta-1)}\begin{bmatrix} \beta-2 & (\alpha-2)(\beta-1) \\ -(1+\beta) & 1-\alpha\beta \end{bmatrix}\begin{bmatrix} 2-\alpha & \alpha\beta-1 \\ 3 & \beta-2 \end{bmatrix}$$

$$= \frac{1}{(\beta-\alpha)(2\beta-1)}\begin{bmatrix} (\alpha-2)(2\beta-1) & (\beta-2)(\alpha-1)(2\beta-1) \\ (1-2\beta)(\alpha+1) & (1-\alpha\beta)(2\beta-1) \end{bmatrix}$$

$$\boxed{\tilde D = \frac{1}{\alpha-\beta}\begin{bmatrix} 2-\alpha & (2-\beta)(\alpha-1) \\ \alpha+1 & \alpha\beta-1 \end{bmatrix}}$$

$$\frac{1}{\alpha-\beta}\begin{bmatrix} 1 & \alpha-1 & -1 & 1-\alpha \\ 1 & \beta-1 & -1 & 1-\beta \\ \beta-2 & (\alpha-2)(\beta-1) & 2-\alpha & (2-\beta)(\alpha-1) \\ -(1+\beta) & 1-\alpha\beta & \alpha+1 & \alpha\beta-1 \end{bmatrix}$$

controlar por exemplo

$\beta = 0 \quad \alpha = 1$