Inversa de uma matriz quadrada di dimensão 2n

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \qquad M^{-1} = \begin{bmatrix} \tilde{A} & \tilde{B} \\ \tilde{C} & \tilde{D} \end{bmatrix}$$

COND. $\tilde{A}A + \tilde{B}C = \mathbb{1}$, $\tilde{A}B + \tilde{B}D = 0$

$\tilde{C}B + \tilde{D}D = \mathbb{1}$, $\tilde{C}A + \tilde{D}C = 0$

$$\begin{cases} \tilde{A} = -\tilde{B}DB^{-1} & \tilde{B} = -\tilde{A}BD^{-1} \\ \tilde{C} = -\tilde{D}CA^{-1} & \tilde{D} = -\tilde{C}AC^{-1} \end{cases}$$

$$\boxed{\begin{aligned} \tilde{A} &= (A - BD^{-1}C)^{-1} \\ \tilde{B} &= (C - DB^{-1}A)^{-1} \\ \tilde{C} &= (B - AC^{-1}D)^{-1} \\ \tilde{D} &= (D - CA^{-1}B)^{-1} \end{aligned}}$$

CASO LIMITE a, b, c, d números

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad M^{-1} = \begin{pmatrix} \tilde{a} & \tilde{b} \\ \tilde{c} & \tilde{d} \end{pmatrix}$$

$$\tilde{a} = \left(a - \frac{bc}{d}\right)^{-1} = \frac{d}{\det M} \qquad \tilde{b} = \left(c - \frac{da}{b}\right)^{-1} = -\frac{b}{\det M}$$

$$\tilde{c} = \left(b - \frac{ad}{c}\right)^{-1} = -\frac{c}{\det M} \qquad \tilde{d} = \left(d - \frac{cb}{a}\right) = \frac{a}{\det M}$$

$$M^{-1} = \frac{1}{\det M} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \quad !!!$$

$$M = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 2 & 1 & 1 \end{bmatrix}$$

$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ $\quad B = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ $\quad \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}^{-1} = -\frac{1}{3}\begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix}$

$C = \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$ $\quad D = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ $\quad \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} = -\begin{pmatrix} 1 & -1 \\ -1 & 0 \end{pmatrix}$

$A^{-1} = \frac{1}{3}\begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}$ $\quad B^{-1} = D^{-1} = \begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix}$ $\quad C^{-1} = \begin{pmatrix} -2 & 1 \\ 1 & 0 \end{pmatrix}$ $\quad \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} = -\begin{pmatrix} 2 & -1 \\ -1 & 0 \end{pmatrix}$

$$\tilde{A} = \left[\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} - \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix}}\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} \right]^{-1}$$

$$\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} \longrightarrow \boxed{\tilde{A} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^{-1} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}}$$

$$\tilde{B} = \left[\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} - \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix}}\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \right]^{-1}$$

$$\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}}$$

$$\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \longrightarrow \boxed{\tilde{B} = \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}^{-1} = \frac{1}{2}\begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}}$$

$$\tilde{C} = \left[\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} - \underbrace{\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} -2 & 1 \\ 1 & 0 \end{pmatrix}}\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \right]^{-1}$$

$$\begin{pmatrix} 0 & 1 \\ -3 & 2 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix} \longrightarrow \boxed{\tilde{C} = \begin{pmatrix} -1 & 0 \\ -1 & 2 \end{pmatrix}^{-1} = \frac{1}{2}\begin{pmatrix} -2 & 0 \\ -1 & 1 \end{pmatrix}}$$

$$\tilde{D} = \left[\begin{pmatrix}0&1\\1&1\end{pmatrix} - \underbrace{\begin{pmatrix}0&1\\1&2\end{pmatrix}\frac{1}{3}\begin{pmatrix}-1&2\\2&-1\end{pmatrix}}_{\frac{1}{3}\begin{pmatrix}2&-1\\3&0\end{pmatrix}}\begin{pmatrix}0&1\\1&1\end{pmatrix}\right]^{-1}$$

$$\underbrace{\frac{1}{3}\begin{pmatrix}2&-1\\3&0\end{pmatrix}\begin{pmatrix}0&1\\1&1\end{pmatrix}}$$

$$\begin{pmatrix}0&1\\1&1\end{pmatrix} - \frac{1}{3}\begin{pmatrix}-1&1\\0&3\end{pmatrix} \longrightarrow \boxed{\tilde{D} = \left(\frac{1}{3}\right)^{-1}\begin{pmatrix}1&2\\3&0\end{pmatrix}^{-1} = \frac{1}{2}\begin{pmatrix}0&2\\3&-1\end{pmatrix}}$$

$$\boxed{M^{-1} = \frac{1}{2}\begin{bmatrix}1&1&-1&-1\\1&-1&-1&1\\-2&0&0&2\\-1&1&3&-1\end{bmatrix}} \xleftarrow{\alpha=2} \quad M = \begin{bmatrix}1&\frac{1}{2}&0&1\\2&1&1&1\\0&1&0&1\\1&2&1&1\end{bmatrix} \qquad \cancel{A^{-1}}$$

$$\boxed{\begin{array}{lcl}
\tilde{A} = [A - BD^{-1}C]^{-1} & & \tilde{A} = -\tilde{B}DB^{-1} \\
\tilde{B} = [C - DB^{-1}A]^{-1} & \text{ou} & \tilde{B} = -\tilde{A}BD^{-1} \\
\tilde{C} = [B - AC^{-1}D]^{-1} & & \tilde{C} = -\tilde{D}CA^{-1} \\
\tilde{D} = [D - CA^{-1}B] & & \tilde{D} = -\tilde{C}AC^{-1}
\end{array}}$$

TEMOS QUE ESCOLHER !!!

NÃO PODEMOS USAR POR EXEMPLO $\tilde{D}$ (1ª col) E $\tilde{C}$ (2ª col)

$$\tilde{A} = \left[\begin{pmatrix}1&1/2\\2&1\end{pmatrix} - \underbrace{\begin{pmatrix}0&1\\1&1\end{pmatrix}\begin{pmatrix}-1&1\\1&0\end{pmatrix}}_{\mathbb{1}}\begin{pmatrix}0&1\\1&2\end{pmatrix}\right]^{-1} = \begin{pmatrix}1&-1/2\\1&-1\end{pmatrix} = \begin{pmatrix}2&-1\\2&-2\end{pmatrix}$$

$$\tilde{B} = \begin{pmatrix}-2&1\\-2&2\end{pmatrix}\begin{pmatrix}0&1\\1&1\end{pmatrix}\begin{pmatrix}-1&1\\1&0\end{pmatrix} = \begin{pmatrix}-2&1\\-2&2\end{pmatrix}$$

$$\tilde{C} = \left[\begin{pmatrix}0&1\\1&1\end{pmatrix} - \underbrace{\begin{pmatrix}1&1/2\\2&1\end{pmatrix}\begin{pmatrix}-2&1\\1&0\end{pmatrix}}_{\begin{pmatrix}-3/2&1\\-3&2\end{pmatrix}}\begin{pmatrix}0&1\\1&1\end{pmatrix}\right]^{-1} = \left[\underbrace{\begin{pmatrix}0&1\\1&1\end{pmatrix} - \begin{pmatrix}1&-1/2\\2&-1\end{pmatrix}}_{\begin{pmatrix}-1&3/2\\-1&2\end{pmatrix}}\right]^{-1} = \begin{pmatrix}-4&3\\-2&2\end{pmatrix}$$

$$\tilde{D} = \begin{pmatrix}4&-3\\2&-2\end{pmatrix}\begin{pmatrix}-3/2&1\\-3&2\end{pmatrix} = \begin{pmatrix}3&-2\\3&-2\end{pmatrix}$$

$$\boxed{M^{-1} = \begin{pmatrix}2&-1&-2&1\\2&-2&-2&2\\-4&3&3&-2\\-2&2&3&-2\end{pmatrix}}$$

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$$\begin{bmatrix}1&\alpha&0&1\\2&1&1&1\\0&1&0&1\\1&2&1&1\end{bmatrix}^{-1} = \frac{1}{\alpha}\begin{bmatrix}1&\alpha-1&-1&1-\alpha\\1&-1&-1&1\\-2&2-\alpha&2-\alpha&2(\alpha-1)\\-1&1&1+\alpha&-1\end{bmatrix}$$

PARA $\alpha = 0$ $\cancel{M^{-1}}$

MATRIZ 3x3

$$M = \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 2 & 1 & 1 \\ \hline 1 & 1 & 1 \end{array}\right) \qquad A^{-1} = \begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix} \qquad D^{-1} = 1$$

$$\tilde{A} = \left[\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}\right]^{-1} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^{-1} = \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\tilde{D} = \left[1 - (1\ 1)\begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right]^{-1} = \left[1 - (1\ 1)\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right]^{-1} = 1$$

$$\begin{pmatrix} 0 & 1 & x \\ 1 & -1 & x \\ x & x & 1 \end{pmatrix}$$

$$\tilde{B} = -\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\tilde{C} = -(1\ 1)\begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix} = (-1\ \ 0)$$

$$\boxed{M^{-1} = \begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & 0 & 1 \end{pmatrix}}$$