Forma quadraticas

$$Ax^2 + 2Bxy + C^2y^2 + \dots$$

$$(x\ y)\begin{pmatrix} A & B \\ B & C \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = Ax^2 + 2Bxy + C^2$$

AUTOVALORES: $(A-\lambda)(C-\lambda) - B^2 = 0$

$$\lambda^2 - (A+C)\lambda + AC - B^2 = 0 \Rightarrow \lambda_{1,2} = \frac{A+C \pm \sqrt{(A+C)^2 - 4AC + B^2}}{2} = \frac{A+C \pm \sqrt{(A-C)^2 + B^2}}{2}$$

AUTOVALORES REAIS E DISTINTOS!

$$\begin{pmatrix} A & B \\ B & C \end{pmatrix} = S\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}S^{-1}$$

$$\begin{pmatrix} A & B \\ B & C \end{pmatrix}\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

APLICAMOS A TRANSPOSTA

$$\begin{pmatrix} \alpha & \gamma \\ \beta & \delta \end{pmatrix}\begin{pmatrix} A & B \\ B & C \end{pmatrix} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}\begin{pmatrix} \alpha & \gamma \\ \beta & \delta \end{pmatrix}$$

$$S^t M = D \underset{\substack{\downarrow \\ S^{-1}}}{S^t}$$

$$\Rightarrow (x\ y)\begin{pmatrix} A & B \\ B & C \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} =$$

$$\underbrace{(x\ y)\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}}\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}\underbrace{\begin{pmatrix} \alpha & \gamma \\ \beta & \delta \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}} =$$

$$(X\ Y)\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix}$$

$\lambda_1\lambda_2 > 0$ ELIPSE

$\lambda_1\lambda_2 = 0$ PARÁBOLA

$\lambda_1\lambda_2 < 0$ HIPÉRBOLE

EX 1 $3x^2 + 10xy + 3y^2 - 2x - 14y - 13 = 0$

$$\begin{pmatrix} A & B \\ B & C \end{pmatrix} = \begin{pmatrix} 3 & 5 \\ 5 & 3 \end{pmatrix} \qquad (3-\lambda)^2 - 25 = 0 \qquad \lambda^2 - 6\lambda - 16 = 0$$

$$\lambda_{1,2} = 3 \pm \sqrt{9+16} < \begin{matrix} -2 \\ 8 \end{matrix}$$

HIPERBOLE

$$\begin{pmatrix} 3 & 5 \\ 5 & 3 \end{pmatrix}\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}\begin{pmatrix} -2 & 0 \\ 0 & 8 \end{pmatrix}$$

$3\alpha + 5\gamma = -2\alpha \qquad 3\beta + 5\delta = 8\beta$

$5\alpha + 3\gamma = -2\gamma \qquad 5\beta + 3\delta = 8\delta$

$\boxed{\alpha = -\gamma} \qquad \boxed{\beta = \delta}$

$\boxed{\tan\theta} = \frac{\gamma}{\alpha}, \frac{\delta}{\beta}$

$\boxed{-1 \quad 1}$

$\theta, \frac{\pi}{2}+\theta, \pi+\theta, \frac{3}{2}\pi+\theta$

$\tan\left(\frac{\pi}{2}+\theta\right) = -\frac{1}{\tan\theta}, \tan(\pi+\theta) = \tan\theta, \tan\left(\frac{3}{2}\pi+\theta\right) = -\frac{1}{\tan\theta}$

EX. 2 $25x^2 - 14xy + 25y^2 + 64x - 64y - 224 = 0$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 25 & -7 \\ -7 & 25 \end{pmatrix} \qquad (25-\lambda)^2 = 49 \qquad 25 - \lambda = \pm 7 \Rightarrow \lambda < \begin{matrix} 18 \\ 32 \end{matrix}$$

ELIPSE

$\tan\theta$: 1, -1

$$\begin{pmatrix} 25 & -7 \\ -7 & 25 \end{pmatrix}\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}\begin{pmatrix} 18 & 0 \\ 0 & 32 \end{pmatrix}$$

$25\alpha - 7\gamma = 18\gamma \qquad 25\beta - 7\delta = 32\beta$

$-7\gamma + 25\alpha = 18\gamma \qquad -7\beta + 25\delta = 32\delta$

EX. 3 $\quad 4xy + 3y^2 + 16x + 12y - 36 = 0$

$$\begin{pmatrix} A & B \\ B & C \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 2 & 3 \end{pmatrix} \qquad \begin{matrix} (3-\lambda)(-\lambda) - 4 = 0 \\ \lambda^2 - 3\lambda - 4 = 0 \end{matrix} \Rightarrow \lambda_{1,2} = \frac{3 \pm \sqrt{9+16}}{2} \begin{cases} -1 \\ 4 \end{cases}$$

HIPÉRBOLE

$$\begin{pmatrix} 0 & 2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 4 \end{pmatrix}$$

$$\begin{matrix} 2\gamma = -\alpha \\ 2\alpha + 3\gamma = -\gamma \end{matrix} \qquad \begin{matrix} 2\delta = 4\beta \\ 2\beta + 3\delta = 4\delta \end{matrix} \qquad \gamma = -\frac{\alpha}{2} \nearrow -\frac{1}{2} \qquad \delta = 2\beta \nearrow 2$$

$$\boxed{\tan\theta = -\frac{1}{2}, 2}$$

EX. 4 $\quad 9x^2 - 24xy + 16y^2 - 20x + 110y - 50 = 0$

PARÁBOLA

$$\begin{pmatrix} A & B \\ B & C \end{pmatrix} = \begin{pmatrix} 9 & -12 \\ -12 & 16 \end{pmatrix} \qquad \begin{matrix} (9-\lambda)(16-\lambda) - 144 = 0 \\ \lambda^2 - 25\lambda + \cancel{144} - \cancel{144} = 0 \end{matrix} \Rightarrow \lambda = 0 \quad \lambda = 25$$

$$\begin{pmatrix} 9 & -12 \\ -12 & 16 \end{pmatrix}\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 25 \end{pmatrix}$$

$$\boxed{\tan\theta : \frac{3}{4}, -\frac{4}{3}}$$

$$\begin{matrix} 9\alpha - 12\gamma = 0 \\ -12\alpha + 16\gamma = 0 \end{matrix} \qquad \begin{matrix} 9\beta - 12\delta = 25\beta \\ -12\beta + 16\delta = 25\delta \end{matrix} \Rightarrow \gamma = \frac{3}{4}\alpha \qquad \delta = -\frac{4}{3}\beta$$

EX. 5 $\quad \frac{x}{2} - \frac{x}{\sqrt{3}} + \frac{7x^2}{48} - \frac{y}{3} - \frac{\sqrt{3}}{2}y - \frac{5xy}{24\sqrt{3}} + \frac{31}{144}y^2 = -1$

$$\begin{pmatrix} A & B \\ B & C \end{pmatrix} = \begin{pmatrix} \frac{7}{48} & -\frac{5}{48\sqrt{3}} \\ -\frac{5}{48\sqrt{3}} & \frac{31}{144} \end{pmatrix} \rightarrow \begin{pmatrix} 21 & -5\sqrt{3} \\ -5\sqrt{3} & 31 \end{pmatrix} \rightarrow \begin{matrix} (21-\lambda)(31-\lambda) - 75 = 0 \\ \lambda^2 - 52\lambda + 576 = 0 \end{matrix}$$

$$\hookrightarrow \lambda = 26 \pm 10 \begin{cases} 16 \\ 36 \end{cases}$$

ELIPSE

$$\begin{pmatrix} 21 & -5\sqrt{3} \\ -5\sqrt{3} & 31 \end{pmatrix}\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}\begin{pmatrix} 16 & 0 \\ 0 & 36 \end{pmatrix}$$

$$\begin{matrix} 21\alpha - 5\sqrt{3}\gamma = 16\alpha \\ -5\sqrt{3}\alpha + 31\gamma = 16\gamma \end{matrix} \qquad \begin{matrix} 21\beta - 5\sqrt{3}\delta = 36\beta \\ -5\sqrt{3}\beta + 31\delta = 36\delta \end{matrix} \qquad \begin{matrix} \alpha = \sqrt{3}\gamma \\ 3\gamma = \sqrt{3}\alpha \\ \frac{\gamma}{\alpha} = \frac{1}{\sqrt{3}} \end{matrix} \qquad \begin{matrix} -\sqrt{3}\delta = 3\beta \\ -\sqrt{3}\beta = \delta \\ \frac{\delta}{\beta} = -\sqrt{3} \end{matrix}$$

$$\boxed{\tan\theta : \frac{1}{\sqrt{3}}, -\sqrt{3}}$$