Aplicando a técnica de escalonamento

4 vetores em $\mathbb{R}^3$ serão sempre L.D. (Linearmente Dependentes)

dados $\begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}2\\1\\2\end{pmatrix}, \begin{pmatrix}3\\0\\1\end{pmatrix}, \begin{pmatrix}4\\1\\1\end{pmatrix}$ encontrar a combinação linear que leva ao vetor nulo

$$\alpha\begin{pmatrix}1\\1\\0\end{pmatrix} + \beta\begin{pmatrix}2\\1\\2\end{pmatrix} + \gamma\begin{pmatrix}3\\0\\1\end{pmatrix} + \delta\begin{pmatrix}4\\1\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix} \Rightarrow \underbrace{\begin{pmatrix}1&2&3&4\\0&1&1&0\\0&2&1&1\end{pmatrix}}_{3\times4}\underbrace{\begin{pmatrix}\alpha\\\beta\\\gamma\\\delta\end{pmatrix}}_{4\times1} = \underbrace{\begin{pmatrix}0\\0\\0\end{pmatrix}}_{3\times1}$$

ESCALONAMENTO

$$\begin{matrix}1&2&3&4\\1&1&0&1\\0&2&1&1\end{matrix} \Rightarrow \begin{matrix}1&2&3&4\\\boxed{0\;1\;3\;3}\\0&2&1&1\end{matrix} \Rightarrow \begin{matrix}1&2&3&4\\0&1&3&3\\\boxed{0\;0\;5\;5}\end{matrix} \quad \begin{matrix}\alpha & & 0\\\beta & = & 0\\\gamma & & 0\\\delta & & \end{matrix}$$

$\boxed{\text{solução} \quad \delta\{-1, 0, -1, 1\}}$

$\alpha + 2\beta + 3\gamma + 4\delta = 0 \rightarrow \alpha = -\delta$

$\beta + 3\gamma + 3\delta = 0 \rightarrow \beta = 0$

$5\gamma + 5\delta = 0 \rightarrow \gamma = -\delta$

$$-1\cdot\begin{pmatrix}1\\1\\0\end{pmatrix} + 0\cdot\begin{pmatrix}2\\1\\2\end{pmatrix} - 1\cdot\begin{pmatrix}3\\0\\1\end{pmatrix} + 1\cdot\begin{pmatrix}4\\1\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix} \checkmark$$

Determinar se os vetores $\begin{pmatrix}1\\1\\0\\1\end{pmatrix}, \begin{pmatrix}2\\1\\2\\1\end{pmatrix}, \begin{pmatrix}3\\0\\1\\1\end{pmatrix}$ são L.D. ou L.I., se forem L.D. calcular a combinação linear que leva ao vetor nulo

$$\begin{matrix}1&2&3\\1&1&0\\0&2&1\\1&1&1\end{matrix} \Rightarrow \begin{matrix}1&2&3\\\boxed{0\;1\;3}\\0&2&1\\1&1&1\end{matrix} \Rightarrow \begin{matrix}1&2&3\\0&1&3\\\boxed{0\;0\;5}\\1&1&1\end{matrix} \quad \begin{matrix}\rightarrow \alpha = 0\\\rightarrow \beta = 0\\\rightarrow \gamma = 0\\ \end{matrix}$$

$\boxed{\text{vetores L.I.}}$

Repetir o exercício anterior com os vetores $\begin{pmatrix}1\\1\\0\\2\end{pmatrix}, \begin{pmatrix}2\\1\\1\\3\end{pmatrix}, \begin{pmatrix}3\\0\\3\\3\end{pmatrix}$

$$\begin{matrix}1&2&3\\1&1&0\\0&1&3\\2&3&3\end{matrix} \Rightarrow \begin{matrix}1&2&3\\\boxed{0\;1\;3}\\0&1&3\\2&3&3\end{matrix} \Rightarrow \begin{matrix}1&2&3\\0&1&3\\\boxed{0\;0\;0}\\2&3&3\end{matrix} \Rightarrow \begin{matrix}1&2&3\\0&1&3\\0&0&0\\\boxed{0\;1\;3}\end{matrix} \rightarrow 0\;0\;0$$

DUAS LINHAS NULA GARANTEM A LINEAR DEPENDÊNCIA

4 EQUAÇÕES E 3 PARAMETROS α, β, γ

PRECISAMOS DE 2 EQUAÇÕES E 3 PARAMETROS

$\boxed{\text{REGRA} \quad n\times m \quad m\times 1 \quad n-m+1 \text{ LINHAS NULAS}}$

solução $\begin{matrix}\alpha + 2\beta + 3\gamma = 0\\\beta + 3\gamma = 0\end{matrix} \Rightarrow \boxed{\gamma = \{3, -3, 1\}}$

Exemplos de vetores L.D. em $\mathbb{R}^3$

1) $\left\{\begin{pmatrix}1\\1\\2\end{pmatrix},\begin{pmatrix}2\\2\\5\end{pmatrix},\begin{pmatrix}3\\3\\7\end{pmatrix}\right\}$ 2) $\left\{\begin{pmatrix}1\\1\\2\end{pmatrix},\begin{pmatrix}2\\2\\5\end{pmatrix},\begin{pmatrix}3\\3\\8\end{pmatrix}\right\}$ encontrar a combinação linear que leva o vetor nulo

Melhor resolver usando a matriz $\begin{matrix}1&2&3\\1&2&3\\2&5&a\end{matrix} \Rightarrow \begin{matrix}1&2&3\\0&0&0\\0&1&a-6\end{matrix} \Rightarrow \begin{matrix}\alpha+2\beta+3\gamma=0\\ \beta+(a-6)\gamma=0\end{matrix}$

$\beta=(6-a)\gamma$, $\alpha+2(6-a)\gamma+3\gamma=0 \Rightarrow \boxed{\gamma\{2a-15,6-a,1\}}$

$a=7$ $\gamma\{-1,-1,1\}$ $\begin{pmatrix}-1-2+3\\-1-2+3\\-2-5+7\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$

$a=8$ $\gamma\{1,-2,1\}$ $\begin{pmatrix}1-4+3\\1-4+3\\2-10+8\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$

DADO O SISTEMA

$$\begin{aligned}x+2y+3z&=2\\2x+y+z&=2\\3x+3y+4z&=b\end{aligned}$$

DETERMINAR PARA QUAL VALOR DE b O SISTEMA NÃO TEM SOLUÇÃO

$\begin{matrix}1&2&3\\+&+&+\\2&1&1\\=&=&=\\3&3&4\end{matrix}$ A MATRIZ NÃO TEM INVERSA

$$\left(\begin{array}{ccc|c}1&2&3&2\\2&1&1&2\\3&3&4&b\end{array}\right) \Rightarrow \left(\begin{array}{ccc|c}1&2&3&2\\0&3&5&2\\3&3&4&b\end{array}\right)$$

$$\Downarrow$$

$$\left(\begin{array}{ccc|c}1&2&3&2\\0&3&5&2\\0&0&0&4-b\end{array}\right) \Leftarrow \left(\begin{array}{ccc|c}1&2&3&2\\0&3&5&2\\0&3&5&6-b\end{array}\right)$$

O SISTEMA TERÁ SOLUÇÃO SOMENTE QUANDO b FOR IGUAL A 4

PARA $b=4$ A SOLUÇÃO SERÁ DADA POR

$$\begin{cases}x+2y+3z=2 \longrightarrow x=\dfrac{2+z}{3}\\ 3y+5z=2 \longrightarrow y=\dfrac{2-5z}{3}\end{cases}$$

SOLUÇÃO $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}2/3\\2/3\\0\end{pmatrix}+z\begin{pmatrix}1/3\\-5/3\\1\end{pmatrix}$

CONTROLE!

$$\begin{pmatrix}1&2&3\\2&1&1\\3&3&4\end{pmatrix}\begin{pmatrix}2/3\\2/3\\0\end{pmatrix}=\begin{pmatrix}2\\2\\4\end{pmatrix} \quad ✓$$

$$\begin{pmatrix}1&2&3\\2&1&1\\3&3&4\end{pmatrix}\begin{pmatrix}1/3\\-5/3\\1\end{pmatrix}=\begin{pmatrix}1/3-10/3+3\\2/3-5/3+1\\1-5+4\end{pmatrix} \quad ✓$$