Processo de ortogonalização de GRAM-SCHMIDT

$\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \ldots\} \rightarrow \{\vec{u}_1, \vec{u}_2, \vec{u}_3, \ldots\}$ ortogonal set of vectors

$$\vec{u}_1 = \vec{v}_1$$
$$\vec{u}_2 = \vec{v}_2 + \alpha \vec{u}_1$$
$$\vec{u}_3 = \vec{v}_3 + \beta \vec{u}_1 + \gamma \vec{u}_2$$

$$\vec{u}_1 \cdot \vec{u}_2 = 0 \rightarrow \alpha = -\frac{\vec{u}_1 \cdot \vec{v}_2}{|\vec{u}_1|^2}$$
$$\vec{u}_1 \cdot \vec{u}_3 = 0 \rightarrow \beta = -\frac{\vec{u}_1 \cdot \vec{v}_3}{|\vec{u}_1|^2}$$
$$\vec{u}_2 \cdot \vec{u}_3 = 0 \rightarrow \gamma = -\frac{\vec{u}_2 \cdot \vec{v}_3}{|\vec{u}_2|^2}$$

$$\boxed{\vec{u}_i = \vec{v}_i - \sum_{2=k}^{i} \frac{\vec{u}_{k-1} \cdot \vec{v}_i}{|\vec{u}_{k-1}|^2} \vec{u}_{k-1}}$$

$i = 2, 3, \ldots \qquad \vec{u}_1 = \vec{v}_1$

Ex: span: $\left\{ \begin{bmatrix} 1\\2\\3\\0 \end{bmatrix}, \begin{bmatrix} 2\\3\\4\\0 \end{bmatrix}, \begin{bmatrix} 3\\4\\5\\0 \end{bmatrix}, \begin{bmatrix} 0\\3\\4\\5 \end{bmatrix} \right\} \rightarrow$ Base ortonormal !!!

$$\vec{u}_1 = \begin{bmatrix} 1\\2\\3\\0 \end{bmatrix}; \quad \vec{u}_2 = \begin{bmatrix} 2\\3\\4\\0 \end{bmatrix} - \frac{\cancel{20}\ 10}{\cancel{14}\ 7} \begin{bmatrix} 1\\2\\3\\0 \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 4\\1\\-2\\0 \end{bmatrix} \rightarrow \begin{bmatrix} 4\\1\\-2\\0 \end{bmatrix}$$

$$\vec{u}_3 = \begin{bmatrix} 3\\4\\5\\0 \end{bmatrix} - \frac{\overset{13}{\cancel{26}}}{} \frac{3+8+15}{\cancel{14}\ 7} \begin{bmatrix} 1\\2\\3\\0 \end{bmatrix} - \frac{\overset{6}{12+4-10}}{21} \begin{bmatrix} 4\\1\\-2\\0 \end{bmatrix} = \frac{1}{21} \begin{bmatrix} 3\times21 - 3\times13 - 24 \\ 4\times21 - 6\times13 - 6 \\ 5\times21 - 9\times13 + 12 \\ 0 \end{bmatrix} = \begin{bmatrix} 0\\0\\0\\0 \end{bmatrix}$$

$\vec{v}_3 = a\vec{v}_1 + b\vec{v}_2 \Rightarrow \vec{u}_3 = \vec{0}$!

$$\vec{u}_4 = \begin{bmatrix} 0\\3\\4\\5 \end{bmatrix} - \frac{\overset{9\ \cancel{18}}{2\times3+3\times4}}{\cancel{14}\ 7} \begin{bmatrix} 1\\2\\3\\0 \end{bmatrix} - \frac{\overset{-5}{3-8}}{21} \begin{bmatrix} 4\\1\\-2\\0 \end{bmatrix} = \frac{1}{21} \begin{bmatrix} 0 - 27 + 20 \\ 63 - 54 + 5 \\ 84 - 81 - 10 \\ 105 \end{bmatrix} = \frac{1}{21} \begin{bmatrix} -7\\14\\-7\\105 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} -1\\2\\-1\\15 \end{bmatrix} \rightarrow \begin{bmatrix} -1\\2\\-1\\15 \end{bmatrix}$$

Base ortogonal $\rightarrow \left\{ \begin{bmatrix} 1\\2\\3\\0 \end{bmatrix}, \begin{bmatrix} 4\\1\\-2\\0 \end{bmatrix}, \begin{bmatrix} -1\\2\\-1\\15 \end{bmatrix} \right\}$

ortonormal $\quad \frac{1}{14} \quad \frac{1}{21} \quad \frac{1}{231}$

$\{(1,1), (0,1)\}$

$\vec{u}_1 = (1,1) \qquad \vec{u}_2 = (0,1) - \frac{(1,1)\cdot(0,1)}{2}(1,1) = \left(-\frac{1}{2}, \frac{1}{2}\right) \rightarrow (1,-1)$

$\{(1,2), (1,-1)\}$

$\vec{u}_1 = (1,2) \qquad \vec{u}_2 = (1,-1) - \frac{(1,2)\cdot(1,-1)}{5}(1,2) = \left(\frac{6}{5}, -\frac{3}{5}\right) \rightarrow (2,-1)$