$$i x_0' + j y_0' + k z_0' = e^{\frac{I\theta}{2}} (i x_0 + j y_0 + k z_0)\, e^{-\frac{I\theta}{2}}$$

$$I = i\alpha + j\beta + k\gamma$$

(α, β, γ) $\quad \alpha^2+\beta^2+\gamma^2=1$

↳ VERSOR DE ROTAÇÃO

Rotações no espaço

$i^2=j^2=k^2=-1$

$ij=-ji=k \quad jk=-kj=i \quad ki=-ik=j$

REP. MATRICIAL $\quad i=\begin{pmatrix}0&1\\-1&0\end{pmatrix} \quad j=\begin{pmatrix}0&i\\i&0\end{pmatrix} \quad k=\begin{pmatrix}i&0\\0&-i\end{pmatrix}$

$$i x_0 + j y_0 + k z_0 \Rightarrow \begin{pmatrix} i z_0 & x_0 + i y_0 \\ -x_0 + i y_0 & -i z_0\end{pmatrix} := V_0$$

$$e^{I\theta/2} \Rightarrow \begin{pmatrix}\cos\theta/2 + i\gamma\sin\theta/2 & \alpha\sin\theta/2 + i\beta\sin\theta/2 \\ -\alpha\sin\frac{\theta}{2} + i\beta\sin\frac{\theta}{2} & \cos\frac{\theta}{2} - i\gamma\sin\frac{\theta}{2}\end{pmatrix} := R(\theta)$$

$$\bullet\ V_0' = R(\theta)\, V_0\, R(-\theta)\ \bullet$$

$(x_0,y_0,z_0)=(1,0,0) \quad \theta=\pi \quad (\alpha,\beta,\gamma)=(1,1,1)/\sqrt3$

$$V_0' = \frac{1}{\sqrt3}\begin{pmatrix} i & 1+i\\ -1+i & -i\end{pmatrix}\begin{pmatrix}0&1\\-1&0\end{pmatrix}\frac{1}{\sqrt3}\begin{pmatrix}-i & -1-i\\ 1-i & i\end{pmatrix} = \frac13 \begin{pmatrix} i & 1+i\\ -1+i & -i\end{pmatrix}\begin{pmatrix}1-i & i\\ i & 1+i\end{pmatrix}$$

$$= \frac13\begin{pmatrix} i+\cancel{1}+i-\cancel{1} & -1+2i\\ 1+2i & -i-\cancel{1}-i+\cancel{1}\end{pmatrix} \Rightarrow \boxed{\frac13\,(-1,2,2)}$$

$\theta=\pi \quad (\alpha,\beta,\gamma)=(1,1,1)/\sqrt3 \quad (x_0,y_0,z_0)=(1,1,0) \longrightarrow \frac13(1,1,4)$ (a)

$(1,1,1/2)\longrightarrow \frac13(2,2,7/2)$ (b)

$(1,1,2/3)\longrightarrow \frac13(\frac73,\frac73,\frac{10}{3})$ (c)

(a)

$$V_0' = \frac13 \begin{pmatrix} i & 1+i\\ -1+i & -i\end{pmatrix}\begin{pmatrix}0 & 1+i\\ -1+i & 0\end{pmatrix}\begin{pmatrix}-i&-1-i\\1-i&i\end{pmatrix} = \frac13\begin{pmatrix} i & 1+i\\ -1+i & -i\end{pmatrix}\begin{pmatrix}2 & -1+i\\ 1+i & 2\end{pmatrix}$$

$$=\frac13\begin{pmatrix}4i & -i-1+2i+2\\ -2+2i-i+1 & -4i\end{pmatrix} = \frac13\begin{pmatrix}4i & 1+i\\ -1+i & -4i\end{pmatrix} \Rightarrow \frac13(1,1,4)\ ✓$$

(b)

$$V_0' = \frac13\begin{pmatrix} i & 1+i\\ -1+i & -i\end{pmatrix}\begin{pmatrix} i/2 & 1+i\\ -1+i & -i/2\end{pmatrix}\begin{pmatrix}-i & -1-i\\ 1-i & i\end{pmatrix} = \frac13\begin{pmatrix} i & 1+i\\ -1+i & -i\end{pmatrix}\begin{pmatrix}2+\frac12 & \frac{i}{2}(1+i)\\ \frac{i}{2}(1-i) & 2+\frac12\end{pmatrix}$$

$$=\frac16\begin{pmatrix} i & 1+i\\ -1+i & -i\end{pmatrix}\begin{pmatrix}5 & i(1+i)\\ i(1-i) & 5\end{pmatrix} = \frac16\begin{pmatrix}7i & 4(1+i)\\ -4(1-i) & -7i\end{pmatrix} \Rightarrow \frac16(4,4,7)\ ✓$$

(c)

$$V_0' = \frac13\begin{pmatrix} i & 1+i\\ -1+i & -i\end{pmatrix}\begin{pmatrix} 2i/3 & 1+i\\ -1+i & -2i/3\end{pmatrix}\begin{pmatrix}-i & -1-i\\ 1-i & i\end{pmatrix} = \frac13\begin{pmatrix} i & 1+i\\ -1+i & -i\end{pmatrix}\begin{pmatrix}2+\frac23 & \frac{1}{3}i(1+i)\\ \frac{1}{3}i(1-i) & 2+\frac23\end{pmatrix}$$

$$=\frac19\begin{pmatrix} i & 1+i\\ -1+i & -i\end{pmatrix}\begin{pmatrix}8 & i(1+i)\\ i(1-i) & 8\end{pmatrix} = \frac19\begin{pmatrix}10i & 7(1+i)\\ -7(1-i) & -10i\end{pmatrix} \Rightarrow \frac19(7,7,10)\ ✓$$

$$\left(1,1,\frac{n}{n+1}\right) \Rightarrow \frac{1}{3(n+1)}\left(2(n+1)+n-1,\ 2(n+1)+n-1,\ 2(n+1)+n+2\right) = \left(\frac{3n+1}{3n+3}, \frac{3n+1}{3n+3}, \frac{3n+4}{3n+3}\right)$$