Rotações no plano

$$x_0' + i\,y_0' = e^{i\theta}(x_0 + i\,y_0)$$

$e^{i\theta} = \cos\theta + i\,\operatorname{sen}\theta$

$$x_0' + i\,y_0' = \cos\theta\,x_0 - \operatorname{sen}\theta\,y_0 + i\,(\cos\theta\,y_0 + \operatorname{sen}\theta\,x_0)$$

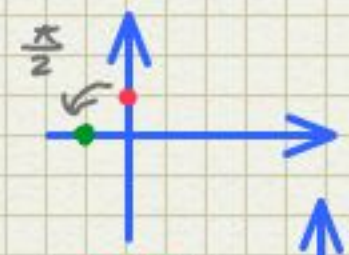
$$x_0' = \cos\theta\,x_0 - \operatorname{sen}\theta\,y_0$$
$$y_0' = \cos\theta\,y_0 + \operatorname{sen}\theta\,x_0$$

$$\begin{pmatrix} x_0' \\ y_0' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

EXEMPLOS:

$(x_0, y_0) = (0,1)$ $\quad \theta = \frac{\pi}{2}$

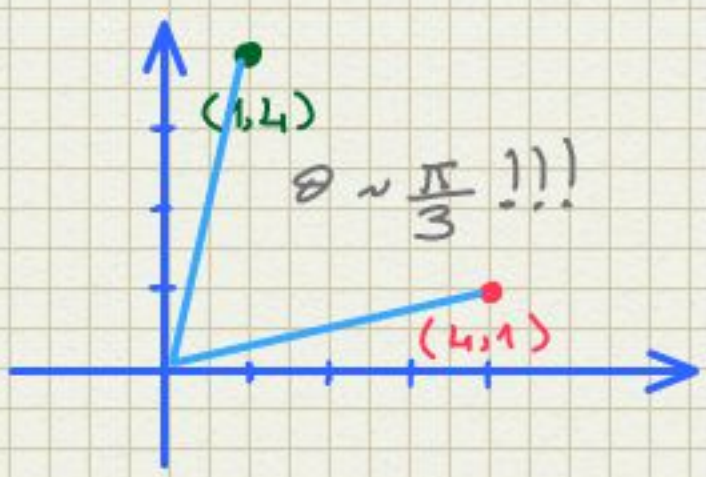
$$\begin{pmatrix} x_0' \\ y_0' \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$



$(1,1)$ $\quad \theta = \frac{\pi}{6}$

$$\begin{pmatrix} x_0' \\ y_0' \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}-1}{2} \\ \frac{\sqrt{3}+1}{2} \end{pmatrix} \approx \begin{pmatrix} 0.35 \\ 1.35 \end{pmatrix}$$

IMPORTANTE OBSERVAR QUE $x_0'^2 + y_0'^2 = x_0^2 + y_0^2$!



$\theta \sim \frac{\pi}{3}$!!!

$$\begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

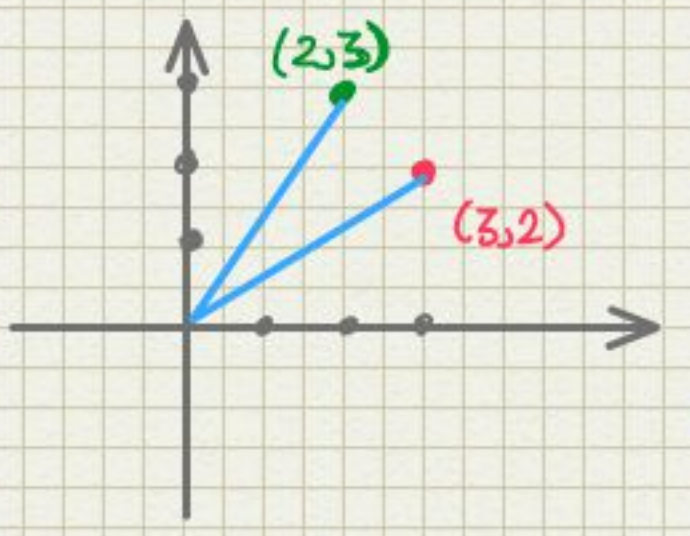
$$1 = 4\cos\theta - \operatorname{sen}\theta$$
$$4 = 4\operatorname{sen}\theta + \cos\theta$$

$\Rightarrow 1 = \operatorname{sen}\theta + \frac{1}{4}\cos\theta$

$\Rightarrow 2 = \frac{17}{4}\cos\theta$

$$\cos\theta = \frac{8}{17}$$

$\cos\theta = \frac{1}{2} \Rightarrow \theta \sim \frac{\pi}{3}$



$$2 = 3\cos\theta - 2\operatorname{sen}\theta$$
$$3 = 3\operatorname{sen}\theta + 2\cos\theta$$

$\Rightarrow 4 = \frac{13}{3}\cos\theta$

$$2 = 2\operatorname{sen}\theta + \frac{4}{3}\cos\theta$$

$$\cos\theta = \frac{12}{13}$$

$\cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta \sim \frac{\pi}{6}$

$(1, \sqrt{3}) \longrightarrow (0,2)$ (a)

$(1, \sqrt{3}) \longrightarrow (-1, \sqrt{3})$ (b)

$(2\sqrt{3}, 2) \longrightarrow (2, 2\sqrt{3})$ (c)

$$x_0\,x_0' = (x_0\cos\theta - y_0\operatorname{sen}\theta)\,x_0$$
$$y_0\,y_0' = (y_0\cos\theta + x_0\operatorname{sen}\theta)\,y_0$$

$$\frac{x_0 x_0' + y_0 y_0'}{x_0^2 + y_0^2} = \cos\theta$$

(a) $\pi/6$
(b) $\pi/3$
(c) $\pi/6$

(a) $\cos\theta = 2\sqrt{3}/4 = \sqrt{3}/2$ (b) $\cos\theta = (-1+3)/4 = 1/2$ (c) $\cos\theta = 8\sqrt{3}/16 = \sqrt{3}/2$