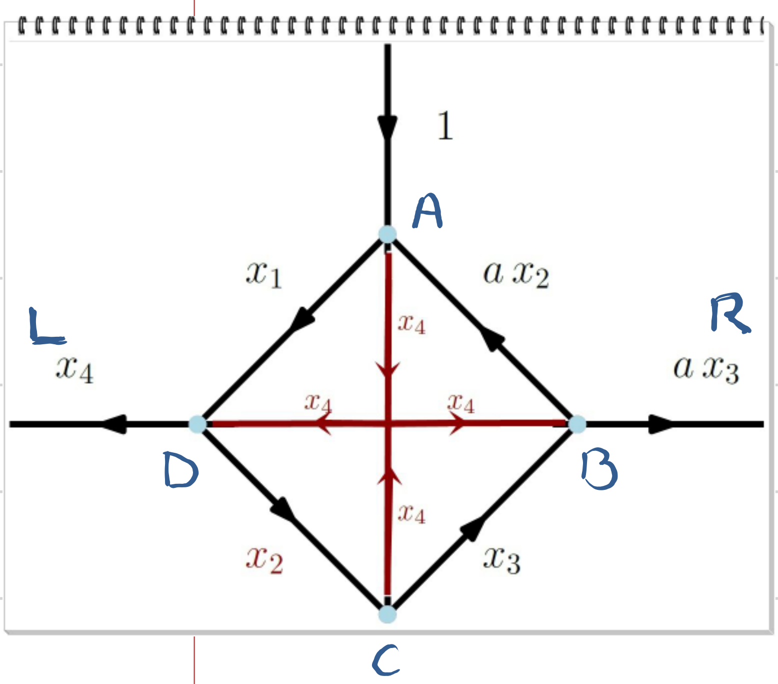
Teste sobre redes, resolução através da decomposição em blocos



A: $1 + a x_2 = x_1 + x_4$

B: $x_3 + x_4 = a x_2 + x_3$

C: $x_2 = x_3 + x_4$

D: $x_1 + x_4 = x_2 + x_4$

LR: $1 = a x_3 + x_4$

MAXIMIZAR E MINIMIZAR AS SAÍDAS

PRECISAMOS RESOLVER O SEGUINTE SISTEMA MATRICIAL

$$\begin{pmatrix} 1 & -a & 0 & 1 \\ 0 & a & a-1 & -1 \\ 0 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & -a \\ 0 & a \end{pmatrix} \quad B = \begin{pmatrix} 0 & 1 \\ a-1 & -1 \end{pmatrix} \quad C = \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix} \quad D = \begin{pmatrix} -1 & -1 \\ 0 & 0 \end{pmatrix}$$

$$A^{-1} = \frac{1}{a}\begin{pmatrix} a & a \\ 0 & 1 \end{pmatrix} \quad B^{-1} = \frac{1}{1-a}\begin{pmatrix} -1 & -1 \\ 1-a & 0 \end{pmatrix} \quad C^{-1} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \quad \cancel{D^{-1}}$$

USAREMOS $\tilde{A} = -C^{-1} D \tilde{C}$ $\quad \tilde{B} = (C - D B^{-1} A)^{-1}$

$\tilde{C} = (B - A C^{-1} D)^{-1}$ $\quad \tilde{D} = -B^{-1} A \tilde{B}$

$$\tilde{C} = \left[\begin{pmatrix} 0 & 1 \\ a-1 & -1 \end{pmatrix} - \begin{pmatrix} 1 & -a \\ 0 & a \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ 0 & 0 \end{pmatrix} \right]^{-1}$$

$$= \left[\begin{pmatrix} 0 & 1 \\ a-1 & -1 \end{pmatrix} - \begin{pmatrix} 1 & -a \\ 0 & a \end{pmatrix} \begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix} \right]^{-1}$$

$$= \left[\begin{pmatrix} 0 & 1 \\ a-1 & -1 \end{pmatrix} + \begin{pmatrix} 1-a & 1-a \\ a & a \end{pmatrix} \right]^{-1}$$

$$= \begin{pmatrix} 1-a & 2-a \\ 2a-1 & a-1 \end{pmatrix}^{-1} = \frac{1}{a^2 - 3a + 1} \begin{pmatrix} a-1 & a-2 \\ 1-2a & 1-a \end{pmatrix}$$

$$\tilde{B} = \left[\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix} - \begin{pmatrix} -1 & -1 \\ 0 & 0 \end{pmatrix} \frac{1}{1-a} \begin{pmatrix} -1 & -1 \\ 1-a & 0 \end{pmatrix} \begin{pmatrix} 1 & -a \\ 0 & a \end{pmatrix} \right]^{-1}$$

$$= \left\{ \frac{1}{1-a} \left[\begin{pmatrix} 0 & 1-a \\ 1-a & a-1 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ 1-a & 0 \end{pmatrix} \begin{pmatrix} 1 & -a \\ 0 & a \end{pmatrix} \right] \right\}^{-1}$$

$$= (1-a) \left[\begin{pmatrix} 0 & 1-a \\ 1-a & a-1 \end{pmatrix} + \begin{pmatrix} -a & -1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & -a \\ 0 & a \end{pmatrix} \right]^{-1}$$

$$= (1-a) \left[\begin{pmatrix} 0 & 1-a \\ 1-a & a-1 \end{pmatrix} + \begin{pmatrix} -a & a(a-1) \\ 0 & 0 \end{pmatrix} \right]^{-1}$$

$$= (1-a) \begin{pmatrix} -a & (1-a)^2 \\ 1-a & a-1 \end{pmatrix}^{-1}$$

$$= \frac{1-a}{a(1-a) - (1-a)^3} \begin{pmatrix} a-1 & -(1-a)^2 \\ -(1-a) & -a \end{pmatrix} = \frac{1}{a^2 - 3a + 1} \begin{pmatrix} 1-a & (1-a) \\ 1-a & a \end{pmatrix}$$

$$\tilde{A} = -\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} -1 & -1 \\ 0 & 0 \end{pmatrix}\frac{1}{a^2-3a+1}\begin{pmatrix} a-1 & a-2 \\ 1-2a & 1-a \end{pmatrix}$$
$$= \frac{1}{a^2-3a+1}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} a-1 & a-2 \\ 1-2a & 1-a \end{pmatrix}$$
$$= \frac{1}{a^2-3a+1}\begin{pmatrix} -a & -1 \\ -a & -1 \end{pmatrix}$$

$$\tilde{D} = -\frac{1}{1-a}\begin{pmatrix} -1 & -1 \\ 1-a & 0 \end{pmatrix}\begin{pmatrix} 1 & -a \\ 0 & a \end{pmatrix}\frac{1}{a^2-3a+1}\begin{pmatrix} 1-a & (1-a)^2 \\ 1-a & a \end{pmatrix}$$
$$= \frac{1}{(1-a)(a^2-3a+1)}\begin{pmatrix} 1 & 1 \\ a-1 & 0 \end{pmatrix}\begin{pmatrix} 1 & -a \\ 0 & a \end{pmatrix}\begin{pmatrix} 1-a & (1-a)^2 \\ 1-a & a \end{pmatrix}$$
$$= \frac{1}{(1-a)(a^2-3a+1)}\begin{pmatrix} 1 & 1 \\ a-1 & 0 \end{pmatrix}\begin{pmatrix} (1-a)^2 & 1-2a \\ a(1-a) & a^2 \end{pmatrix}$$
$$= \frac{1}{(1-a)(a^2-3a+1)}\begin{pmatrix} (1-a)^2+a(1-a) & (1-a)^2 \\ (a-1)(1-a)^2 & (a-1)(1-2a) \end{pmatrix}$$
$$= \frac{1}{a^2-3a+1}\begin{pmatrix} 1 & 1-a \\ -(1-a)^2 & 2a-1 \end{pmatrix}$$

RESOLUSÃO

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \frac{1}{a^2-3a+1}\begin{pmatrix} -a & -1 & 1-a & (1-a)^2 \\ -a & -1 & 1-a & a \\ a-1 & a-2 & 1 & 1-a \\ 1-2a & 1-a & -(1-a)^2 & 2a-1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$x_1 = x_2 = -a/(a^2-3a+1)$

$x_3 = (a-1)/(a^2-3a+1)$ $\qquad$ $x_4 = (1-2a)/(a^2-3a+1)$

$a^2-3a+1=0 \Rightarrow a_{1,2} = (3 \pm \sqrt{5})/2$ $\qquad$ $+ \quad \underset{a_1}{\frac{3-\sqrt{5}}{2}} \quad - \quad \underset{a_2}{\frac{3+\sqrt{5}}{2}} \quad +$

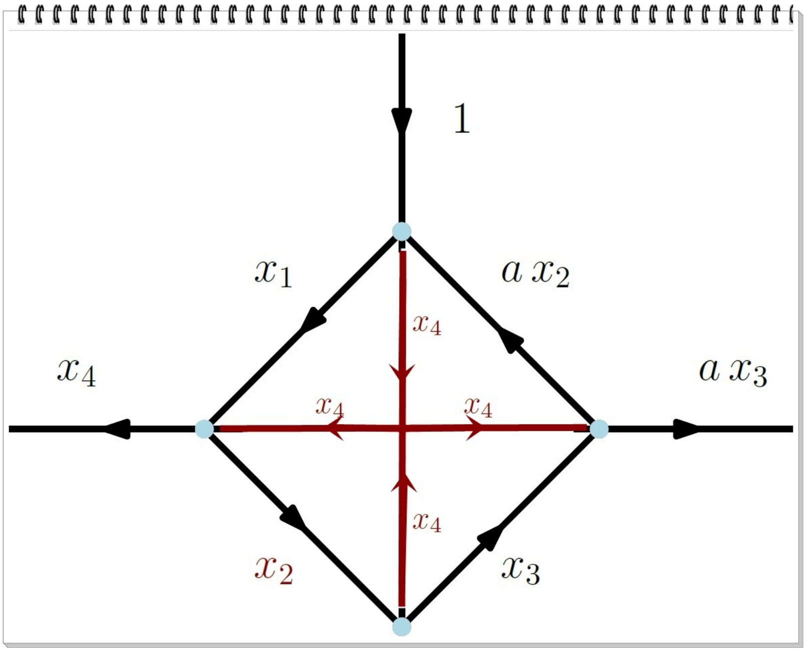
Point B $\quad x_3 + x_4 = a x_2 + a x_3$ $\qquad x_4 = 0 \qquad x_2 = x_3 \qquad a x_2 = (1-a) x_3$

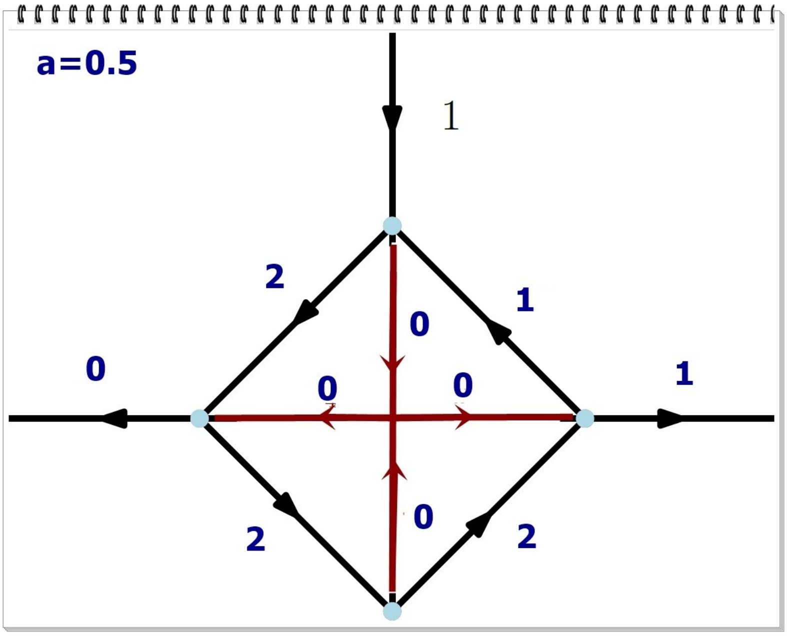
Point C $\quad x_2 = x_3 + x_4$ $\qquad x_3 = 0 \qquad x_2 = x_4 \qquad a x_2 = x_4$

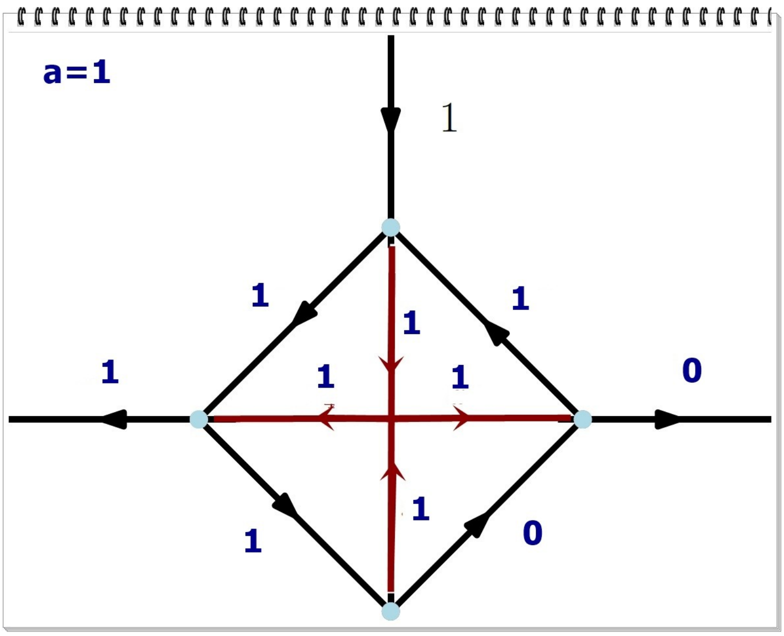
$x_4 = 0 \quad a = 1/2$ $\qquad$ Min $a x_3 \equiv$ Max x_4 $\qquad a = 1$

$x_3 = 0 \quad a = 1$ $\qquad$ Max $a x_3 \equiv$ Min x_4 $\qquad a = 1/2$

TEREMOS SOLUSÕES PARA $0.5 \leq a \leq 1.0$


1
x_1
$a\,x_2$
x_4
x_4
$a\,x_3$
x_4
x_4
x_4
x_2
x_3


a=0.5
1
2
1
0
0
0
0
1
0
2
2


a=1
1
1
1
1
1
1
1
0
1
1
0