

# Inversa de uma matriz quadrada de dimensão $2n$

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad M^{-1} = \begin{bmatrix} \tilde{A} & \tilde{B} \\ \tilde{C} & \tilde{D} \end{bmatrix}$$

COND.  $\rightarrow$

$$\begin{cases} \tilde{A}A + \tilde{B}C = \mathbb{1} & \tilde{A}B + \tilde{B}D = 0 \\ \tilde{C}B + \tilde{D}D = 0 & \tilde{C}A + \tilde{D}C = \mathbb{1} \end{cases}$$

$$\begin{cases} \tilde{A} = -\tilde{B}DB^{-1} & \tilde{B} = -\tilde{A}BD^{-1} \\ \tilde{C} = -\tilde{D}CA^{-1} & \tilde{D} = -\tilde{C}AC^{-1} \end{cases}$$

$$\begin{aligned} \tilde{A} &= (A - BD^{-1}C)^{-1} \\ \tilde{B} &= (C - DB^{-1}A)^{-1} \\ \tilde{C} &= (B - AC^{-1}D)^{-1} \\ \tilde{D} &= (D - CA^{-1}B)^{-1} \end{aligned}$$

CASO LIMITE  $a, b, c, d$  números

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad M^{-1} = \begin{pmatrix} \tilde{a} & \tilde{b} \\ \tilde{c} & \tilde{d} \end{pmatrix}$$

$$\tilde{a} = \left(a - \frac{bc}{d}\right)^{-1} = \frac{d}{\det M} \quad \tilde{b} = \left(c - \frac{da}{b}\right)^{-1} = -\frac{b}{\det M}$$

$$\tilde{c} = \left(b - \frac{ad}{c}\right)^{-1} = -\frac{c}{\det M} \quad \tilde{d} = \left(d - \frac{cb}{a}\right)^{-1} = \frac{a}{\det M}$$

$$M^{-1} = \frac{1}{\det M} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} !!!$$

$$M = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 1 & 2 & 1 & 1 \end{bmatrix}$$

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}^{-1} = -\frac{1}{3} \begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$$

$$D = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}^{-1} = -\begin{pmatrix} 1 & -1 \\ -1 & 0 \end{pmatrix}$$

$$A^{-1} = \frac{1}{3} \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}$$

$$B^{-1} = D^{-1} = \begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix}$$

$$C^{-1} = \begin{pmatrix} -2 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}^{-1} = -\begin{pmatrix} 2 & -1 \\ -1 & 0 \end{pmatrix}$$

$$\tilde{A} = \left[ \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} \right]^{-1}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} \rightarrow \tilde{A} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^{-1} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\tilde{A} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^{-1} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\tilde{B} = \left[ \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \right]^{-1}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \rightarrow \tilde{B} = \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}^{-1} = \frac{1}{2} \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$\tilde{B} = \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}^{-1} = \frac{1}{2} \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$\tilde{C} = \left[ \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} -2 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \right]^{-1}$$

$$\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} - \begin{pmatrix} -3 & 2 \\ 2 & -1 \end{pmatrix} \rightarrow \tilde{C} = \begin{pmatrix} -1 & 0 \\ -1 & 2 \end{pmatrix}^{-1} = \frac{1}{2} \begin{pmatrix} -2 & 0 \\ -1 & 1 \end{pmatrix}$$

$$\tilde{C} = \begin{pmatrix} -1 & 0 \\ -1 & 2 \end{pmatrix}^{-1} = \frac{1}{2} \begin{pmatrix} -2 & 0 \\ -1 & 1 \end{pmatrix}$$



$$\tilde{D} = \left[ \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} - \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} \frac{1}{3} \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}}_{\frac{1}{3} \begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}} \right]^{-1}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} - \frac{1}{3} \begin{pmatrix} -1 & 1 \\ 0 & 3 \end{pmatrix} \rightarrow$$

$$\tilde{D} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}^{-1} = \frac{1}{2} \begin{pmatrix} 0 & 2 \\ 3 & -1 \end{pmatrix}$$

$$M^{-1} = \frac{1}{2} \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ -2 & 0 & 0 & 2 \\ -1 & 1 & 3 & -1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & \frac{1}{2} & 0 & 1 \\ 2 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \end{bmatrix}$$

~~M^{-1}~~

$$\begin{aligned} \tilde{A} &= [A - BD^{-1}C]^{-1} & \tilde{A} &= -\tilde{B}D\tilde{B}^{-1} \\ \tilde{B} &= [C - DB^{-1}A]^{-1} & \tilde{B} &= -\tilde{A}B\tilde{D}^{-1} \\ \tilde{C} &= [B - AC^{-1}D]^{-1} & \tilde{C} &= -\tilde{D}C\tilde{A}^{-1} \\ \tilde{D} &= [D - CA^{-1}B] & \tilde{D} &= -\tilde{C}A\tilde{C}^{-1} \end{aligned}$$

OU

TEMOS QUE ESCOLHER !!!

NÃO PODEMOS USAR POR EXEMPLO  $\tilde{D}$  (1ª COL) E  $\tilde{C}$  (2ª COL)

$$\tilde{A} = \left[ \begin{pmatrix} 1 & 1/2 \\ 2 & 1 \end{pmatrix} - \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}}_{\mathbb{1}} \right]^{-1} = \begin{pmatrix} 1 & -1/2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 2 & -2 \end{pmatrix}$$

$$\tilde{B} = \begin{pmatrix} -2 & 1 \\ -2 & 2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ -2 & 2 \end{pmatrix}$$

$$\tilde{C} = \left[ \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} - \underbrace{\begin{pmatrix} 1 & 1/2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} -2 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}}_{\begin{pmatrix} -3/2 & 1 \\ -3 & 2 \end{pmatrix}} \right]^{-1} = \left[ \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} - \begin{pmatrix} 1 & -1/2 \\ 2 & -1 \end{pmatrix} \right]^{-1} = \begin{pmatrix} -4 & 3 \\ -2 & 2 \end{pmatrix}$$

$$\tilde{D} = \begin{pmatrix} 4 & -3 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} -3/2 & 1 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 3 & -2 \end{pmatrix}$$

$$M^{-1} = \begin{pmatrix} 2 & -1 & -2 & 1 \\ 2 & -2 & -2 & 2 \\ -4 & 3 & 3 & -2 \\ -2 & 2 & 3 & -2 \end{pmatrix}$$

USANDO A MESMA TÉCNICA  
DEMONSTRAR QUE

$$\begin{bmatrix} 1 & \alpha & 0 & 1 \\ 2 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 2 & 1 & 1 \end{bmatrix}^{-1} = \frac{1}{\alpha} \begin{bmatrix} 1 & \alpha-1 & -1 & 1-\alpha \\ 1 & -1 & -1 & 1 \\ -2 & 2-\alpha & 2-\alpha & 2(\alpha-1) \\ -1 & 1 & 1+\alpha & -1 \end{bmatrix}$$

PARA  $\alpha = 0$

~~M^{-1}~~



MATRIZ 3x3

$$M = \left( \begin{array}{cc|c} 1 & 1 & 0 \\ 2 & 1 & 1 \\ \hline 1 & 1 & 1 \end{array} \right)$$

$$A^{-1} = \begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix}$$

$$D^{-1} = 1$$

$$\tilde{A} = \left[ \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \right]^{-1} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^{-1} = \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\tilde{B} = \left[ 1 - (1 \ 1) \begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right]^{-1} = \left[ 1 - (1 \ 1) \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right]^{-1} = 1$$

$$\begin{pmatrix} 0 & 1 & x \\ 1 & -1 & x \\ x & x & 1 \end{pmatrix}$$

$$\tilde{B} = - \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\tilde{C} = - (1 \ 1) \begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix} = (-1 \ 0)$$

$$M^{-1} = \begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$



$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1} = \begin{pmatrix} \tilde{A} & \tilde{B} \\ \tilde{C} & \tilde{D} \end{pmatrix}$$

$$\tilde{A} = (A - BD^{-1}C)^{-1}$$

$$\tilde{B} = (C - DB^{-1}A)^{-1}$$

$$\tilde{C} = (B - AC^{-1}D)^{-1}$$

$$\tilde{D} = (D - CA^{-1}B)^{-1}$$

$$\tilde{A} = -\tilde{B}DB^{-1}$$

$$\tilde{B} = -\tilde{A}BD^{-1}$$

$$\tilde{C} = -\tilde{D}CA^{-1}$$

$$\tilde{D} = -\tilde{C}AC^{-1}$$

por exemplo se  $A = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}$   ~~$\alpha = 1/2$~~   $\rightarrow \begin{bmatrix} 1 & \alpha \\ 2 & 1 \end{bmatrix}$  e depois fazer  $\alpha \rightarrow 1/2$

$$\begin{pmatrix} 1 & \alpha & 0 & 1 \\ 2 & 1 & 1 & 1 \\ \beta & 1 & 0 & 1 \\ 1 & 2 & 1 & 1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{1-2\alpha} \begin{pmatrix} 1 & -\alpha \\ -2 & 1 \end{pmatrix}$$

$$B^{-1} = D^{-1} = \begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix}$$

$$C^{-1} = \frac{1}{2\beta-1} \begin{pmatrix} 2 & -1 \\ -1 & \beta \end{pmatrix}$$

$$\tilde{A} = (A - C)^{-1} = \begin{pmatrix} 1-\beta & \alpha-1 \\ 1 & -1 \end{pmatrix}^{-1} = \frac{1}{\alpha-\beta} \begin{pmatrix} 1 & \alpha-1 \\ 1 & \beta-1 \end{pmatrix} = \tilde{A}$$

$$\tilde{B} = -\tilde{A}$$

$$\tilde{C} = \left[ \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} - \begin{pmatrix} 1 & \alpha \\ 2 & 1 \end{pmatrix} \frac{1}{2\beta-1} \begin{pmatrix} 2 & -1 \\ -1 & \beta \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \right]^{-1}$$

$$= (2\beta-1) \left[ \begin{pmatrix} 0 & 2\beta-1 \\ 2\beta-1 & 2\beta-1 \end{pmatrix} - \begin{pmatrix} 2-\alpha & \alpha\beta-1 \\ 3 & \beta-2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \right]^{-1}$$

$$= (2\beta-1) \left[ \begin{pmatrix} 0 & 2\beta-1 \\ 2\beta-1 & 2\beta-1 \end{pmatrix} - \begin{pmatrix} \alpha\beta-1 & 1-\alpha+\alpha\beta \\ \beta-2 & 1+\beta \end{pmatrix} \right]^{-1}$$

$$= (2\beta-1) \begin{pmatrix} 1-\alpha\beta & 2\beta-2+\alpha-\alpha\beta \\ \beta+1 & \beta-2 \end{pmatrix}^{-1}$$

$$= (2\beta-1) \begin{pmatrix} 1-\alpha\beta & (2-\alpha)(\beta-1) \\ \beta+1 & \beta-2 \end{pmatrix}^{-1}$$

$$= \frac{1}{(1-\alpha\beta)(\beta-2) - (\beta^2-1)(2-\alpha)} \begin{bmatrix} \beta-2 & (\alpha-2)(\beta-1) \\ -(\beta+1) & 1-\alpha\beta \end{bmatrix}$$

$$\beta - \alpha\beta^2 + 2\alpha\beta - 2\beta^2 + \alpha\beta^2 - \alpha$$

$$\beta(1-2\beta) + \alpha(2\beta-1)$$

$$(2\beta-1)(\alpha-\beta)$$

$$\tilde{C} = \frac{1}{\alpha-\beta} \begin{bmatrix} \beta-2 & (\alpha-2)(\beta-1) \\ -(\beta+1) & 1-\alpha\beta \end{bmatrix}$$

$$\tilde{D} = -\tilde{C}AC^{-1} = \frac{1}{\beta-\alpha} \begin{pmatrix} \beta-2 & (\alpha-2)(\beta-1) \\ -(\beta+1) & 1-\alpha\beta \end{pmatrix} \begin{pmatrix} 1 & \alpha \\ 2 & 1 \end{pmatrix} \frac{1}{2\beta-1} \begin{pmatrix} 2 & -1 \\ -1 & \beta \end{pmatrix}$$

$$= \frac{1}{(\beta-\alpha)(2\beta-1)} \begin{bmatrix} \beta-2 & (\alpha-2)(\beta-1) \\ -(\beta+1) & 1-\alpha\beta \end{bmatrix} \begin{bmatrix} 2-\alpha & \alpha\beta-1 \\ 3 & \beta-2 \end{bmatrix}$$

$$= \frac{1}{(\beta-\alpha)(2\beta-1)} \begin{bmatrix} (\alpha-2)(2\beta-1) & (\beta-2)(\alpha-1)(2\beta-1) \\ (1-2\beta)(\alpha+1) & (1-\alpha\beta)(2\beta-1) \end{bmatrix}$$

$$\tilde{D} = \frac{1}{\alpha-\beta} \begin{bmatrix} 2-\alpha & (2-\beta)(\alpha-1) \\ \alpha+1 & \alpha\beta-1 \end{bmatrix}$$



$$\frac{1}{\alpha-\beta} \begin{bmatrix} 1 & \alpha-1 & -1 & 1-\alpha \\ 1 & \beta-1 & -1 & 1-\beta \\ \beta-2 & (\alpha-2)(\beta-1) & 2-\alpha & (2-\beta)(\alpha-1) \\ -(\beta+1) & 1-\alpha\beta & \alpha+1 & \alpha\beta-1 \end{bmatrix}$$

entradas por exemplo

$$\beta=0 \quad \alpha=1$$

