Escalonamento

$$\begin{cases} x + 2y + z = 2 \\ 3x + 8y + z = 12 \\ 4y + z = 2 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 3 & 8 & 1 & 12 \\ 0 & 4 & 1 & 2 \end{array}\right) \qquad \begin{array}{cccc} 1 & 2 & 1 & 2 \\ 0 & \times & \times & \times \\ 0 & \times & \times & \times \end{array}$$

$$\left(\begin{array}{ccc|ccc|c} 1 & 0 & 0 & 1 & 2 & 1 & 2 \\ -3 & 1 & 0 & 3 & 8 & 1 & 12 \\ 0 & 0 & 1 & 0 & 4 & 1 & 2 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 2 & -2 & 6 \\ 0 & 4 & 1 & 2 \end{array}\right)$$

0

$$\left(\begin{array}{ccc|ccc|c} 1 & 0 & 1 & 1 & 2 & 1 & 2 \\ 0 & 1 & 0 & 0 & 2 & -2 & 6 \\ 0 & -2 & 1 & 0 & 4 & 1 & 2 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 2 & -2 & 6 \\ 0 & 0 & 5 & -10 \end{array}\right)$$

$$\begin{cases} x + 2y + z = 2 \\ 2y - 2z = 6 \\ 5z = -10 \end{cases} \qquad \begin{array}{l} x + 2 - 2 = 2 \rightarrow x = 2 \\ y = 1 \\ \rightarrow z = -2 \end{array}$$

$\boxed{(2, 1, 2)}$

SOL: PONTO

Interpretação geométrica

$\pi_1, \pi_2 \rightarrow r_{12}$

$r_{12} \not\subset \pi_3$

$r_{12} \not\parallel \pi_3$

MUDAMOS AGORA O NOSSO SISTEMA

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 3 & 8 & 1 & 12 \\ 0 & 4 & -4 & 2 \end{array}\right)$$

PRIMEIRO PASSO ok!

SEGUNDO MUDA

$$\left(\begin{array}{ccc|ccc|c} 1 & 0 & 0 & 1 & 2 & 1 & 2 \\ 0 & 1 & 0 & 0 & 2 & -2 & 6 \\ 0 & -2 & 1 & 0 & 4 & -4 & 2 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 2 & -2 & 6 \\ 0 & 0 & 0 & -10 \end{array}\right)$$

$0 = -10$!!!

$$\begin{cases} x + 2y + z = 2 \\ 2y - 2z = 6 \end{cases} \rightarrow \begin{cases} x = 2 - r - 2y \\ y = 3 + r \end{cases}$$

$(x, y, z) = (-4, 3, 0) + r(-3, 1, 1)$

$\pi_3 : 4y - 4z = 2$ $\qquad r_{12} \parallel \pi_3$

MAS $r_{12} \not\subset \pi_3$

$(-4, 3, 0) \notin \pi_3$

$\boxed{\text{SEM SOLUÇÃO}}$

SE $(-4,3,0) \in \pi_3$ TEREMOS ∞ SOLUÇÕES

"RETA"

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 3 & 8 & 1 & 12 \\ 0 & 4 & -4 & 12 \end{array}\right]$$

1º PASSO

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 2 & -2 & 6 \\ 0 & 4 & -4 & 12 \end{array}\right]$$

2º PASSO

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 2 & -2 & 6 \\ \hline 0 & 0 & 0 & 0 \end{array}\right]$$

$\longrightarrow$ infinitas soluções

$$(1,-2,1)+s(1,1,1)+t(-1,1,0) = (3,2,1)+u(2,1,1)+v(1,1,2)$$

Escalonamento

$$\begin{array}{cccc|c} s & t & u & v & \\ 1 & -1 & -2 & -1 & 2 \\ \boxed{1} & 1 & -1 & -1 & 4 \\ \boxed{1} & 0 & -1 & -2 & 0 \end{array} \longrightarrow \left|\begin{array}{cccc|c} 1 & \boxed{-1} & -2 & -1 & 2 \\ 0 & 2 & 1 & 0 & 2 \\ 0 & \boxed{1} & 1 & -1 & -2 \end{array}\right.$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 0 & -4 & -6 \\ 0 & 2 & 0 & 2 & 8 \\ 0 & 0 & 1/2 & -1 & -3 \end{array}\right. \longleftarrow \left|\begin{array}{cccc|c} 1 & 0 & \boxed{-3/2} & -1 & 3 \\ 0 & 2 & \boxed{1} & 0 & 2 \\ 0 & 0 & 1/2 & -1 & -3 \end{array}\right.$$

$$\begin{aligned} s &= 4v-6 \\ 2t &= 8-2v \rightarrow t = 4-v \\ \frac{u}{2} &= v-3 \rightarrow u = 2v-6 \end{aligned}$$

$$\boxed{(s,t,u) = (-6,4,-6)+v(4,-1,2)}$$

$\pi_2: (3,2,1)+u(2,1,1)+v(1,1,2)$

$(3,2,1)+-6(2,1,1)+2v(2,1,1)+v(1,1,2)$

$$\boxed{(-9,-4,-5)+v(5,3,4)}$$

RETA $\pi_1 \cap \pi_2$

$$\begin{array}{ccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \hline 1 & 2 & -1 & 0 & 1 & 3 \\ 2 & 1 & 0 & -1 & -2 & 0 \\ 3 & 1 & -2 & 1 & 1 & 2 \\ 1 & 1 & 1 & -2 & 1 & 3 \\ 0 & 0 & 1 & -1 & -1 & 0 \end{array} \longrightarrow \begin{array}{ccccc|c} 1 & 2 & -1 & 0 & 1 & 3 \\ 0 & -3 & 2 & -1 & -4 & -6 \\ 0 & -5 & 1 & 1 & -2 & -7 \\ 0 & -1 & 2 & -2 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 & 0 \end{array}$$

α α

$$\begin{array}{ccccc|c} 7 & 0 & 0 & -2 & -7 & -4 \\ 0 & -21/6 & 0 & 3/2 & 0 & -4 \\ 0 & 0 & -7/3 & 8/3 & 14/3 & 3 \\ 0 & 0 & 0 & -1/4 & 7 & 13/2 \\ 0 & 0 & 0 & 1/3 & 7/3 & 3 \end{array} \longleftarrow \boxed{\begin{array}{ccccc|c} 1 & 0 & 1/3 & -2/3 & -5/3 & -1 \\ 0 & -3 & 2 & -1 & -4 & -6 \\ 0 & 0 & -7/3 & 8/3 & 14/3 & 3 \\ 0 & 0 & 4/3 & -5/3 & 4/3 & 2 \\ 0 & 0 & 1 & -1 & -1 & 0 \end{array}}$$

$3+\frac{7}{3}\alpha$ α

$$\left[\begin{array}{ccccc|l}
-7/8 & 0 & 0 & 0 & \cancel{63/8} & 7 - 63/8\, x_5 \\
0 & -7/12 & 0 & 0 & \cancel{7} & 35/6 - 7\, x_5 \\
0 & 0 & -7/32 & 0 & \cancel{\frac{238}{32}} & 279/32 - 238/32\, x_5 \\
0 & 0 & 0 & -\frac{1}{4} & \cancel{7} & 13/2 - 7\, x_5 \\
0 & 0 & 0 & 0 & \frac{35}{4} & 35/4
\end{array}\right]$$

$\longrightarrow x_1 = 1$

$\longrightarrow x_2 = 2$

$\longrightarrow x_3 = 3$

$\longrightarrow x_4 = 2$

$\longrightarrow x_5 = 1$

$$\frac{35}{4} + \frac{7}{4}\alpha \longrightarrow x_5 = 1 + \frac{\alpha}{5} - \frac{2}{5}\beta$$

$$x_1 = 1 - \frac{8}{7}\left(-\frac{63}{8}\,\frac{\alpha}{5}\right) = 1 + \frac{9}{5}\alpha \quad - \frac{3}{5}\beta$$

$$x_2 = 2 - \frac{12}{7}\left(-7\,\frac{\alpha}{5}\right) = 2 + \frac{12}{5}\alpha \quad - \frac{4}{5}\beta$$

$$x_3 = 3 - \frac{32}{7}\left(-\frac{238}{32}\,\frac{\alpha}{5}\right) = 3 + \frac{34}{5}\alpha \quad - \frac{13}{5}\beta$$

$$x_4 = 2 - 4\left(-7\,\frac{\alpha}{5}\right) = 2 + \frac{28}{5}\alpha \quad - \frac{11}{5}\beta$$

x_1	x_2	x_3	x_4	x_5	
1	2	-1	0	1	3
2	1	0	-1	-2	0
3	1	-2	1	1	2
1	1	1	-2	1	3
0	0	1	-1	-1	0

α

controle !!! $\boxed{\{1,2,3,2,1\} + \frac{\alpha}{5}\{9,12,34,28,1\}}$

1) $x_1 + 2x_2 - x_3 + x_5 = 3$

$1 + 4 - 3 + 1 = 3$ ✓

$9 + 24 - 34 + 1 = 0$ ✓

2) $2x_1 + x_2 - x_4 - 2x_5 = 0$

$2 + 2 - 2 - 2 = 0$ ✓

$18 + 12 - 28 - 2 = 0$ ✓

3) $3x_1 + x_2 - 2x_3 + x_4 + x_5 = 2$

$3 + 2 - 6 + 2 + 1 = 2$ ✓

$27 + 12 - 68 + 28 + 1 = 0$ ✓

4) $x_1 + x_2 + x_3 - 2x_4 + x_1 = 3$

$1 + 2 + 3 - 4 + 1 = 3$ ✓

$9 + 12 + 34 - 56 + 1 = 0$ ✓

5) $x_3 - x_4 - x_5 = \alpha$

$3 - 2 - 1 = 0$ ✓

$\frac{1}{5}(34 - 28 - 1) = 1$ ✓

Incluir na segunda equação o parámetro beta

$$2x_1 + x_2 - x_4 - 2x_5 = \beta$$

Melhorando a técnica de escalonamento

Ex 1

$$\begin{aligned} x + 2y + z &= 2 \\ 3x + 8y + z &= 12 \\ 4y + z &= 2 \end{aligned} \Rightarrow \begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 3 & 8 & 1 & 12 \\ 0 & 4 & 1 & 2 \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 1 \\ 0 & x & x \\ 0 & 0 & x \end{pmatrix}$$

21 $\quad a+3=0 \quad\quad a=-3$

31 $\quad b+3c=0 \quad\quad b=-3c \quad\quad b=6$

32 $\quad 2b+8c+4=0 \quad\quad c=-2$

$$\begin{array}{ccc|ccc|c} 1 & 0 & 0 & 1 & 2 & 1 & 2 \\ -3 & 1 & 0 & 3 & 8 & 1 & 12 \\ 6 & -2 & 1 & 0 & 4 & 1 & 2 \end{array}$$

$\boxed{2, 1, -2}$

$$\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 2 & -2 & 6 \\ 0 & 0 & 5 & -10 \end{array}$$

$x = -2y - z + 2 \longrightarrow 2$

$2y = 2z + 6 \rightarrow y = 1$

$\longrightarrow z = -2$

Ex 2

$$\begin{array}{ccc|c} 1 & -1 & -2 & 2+v \\ 1 & 1 & -1 & 4+v \\ 1 & 0 & -1 & 2v \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 & -2 \\ 1 & 1 & -1 \\ 1 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & -2 \\ 0 & x & x \\ 0 & 0 & x \end{pmatrix}$$

21: $\quad a+1=0 \quad\quad a=-1$

31: $\quad b+c+1=0 \longrightarrow c=-1/2$

32: $\quad -b+c=0 \quad\quad b=c$

$$\begin{array}{ccc|ccc|c} 1 & 0 & 0 & 1 & -1 & -2 & 2+v \\ -1 & 1 & 0 & 1 & 1 & -1 & 4+v \\ -1/2 & -1/2 & 1 & 1 & 0 & -1 & 2v \end{array} \Rightarrow \begin{array}{ccc|c} 1 & -1 & -2 & 2+v \\ 0 & 2 & 1 & 2 \\ 0 & 0 & \frac{1}{2} & -3+v \end{array}$$

$s = t + 2u + 2 + v = 4 - v - 12 + 4v + 2 + v = -6 + 4v$

$2t = -u + 2 \longrightarrow t = 4 - v$

$\frac{1}{2}u = -3 + v \longrightarrow u = -6 + 2v$

$$\boxed{\text{SOLUÇÃO} \quad (-6, 4, -6) + v(4, -1, 2)}$$

EX 3

$$\begin{aligned} x + 2y + z + 4w &= 2 \\ 2x \quad + z + 3w &= 1 \\ 3y + 2z + w &= 1 \\ x + y + z - w &= 2 \end{aligned}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ a & 1 & 0 & 0 \\ b & c & 1 & 0 \\ e & f & g & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 & 4 \\ 2 & 0 & 1 & 3 \\ 0 & 3 & 2 & 1 \\ 1 & 1 & 1 & -1 \end{pmatrix}$$

21) $a + 2 = 0$

31) $b + 2c = 0$

32) $2b + 3 = 0$

41) $e + 2f + 1 = 0$

42) $2e + 3g + 1 = 0$

43) $e + f + 2g + 1 = 0$

$$\boxed{a = -2, \; b = -\frac{3}{2}, \; c = \frac{3}{4}}$$

41-43) $f = 2g$

41) $e + 4g + 1 = 0$

42) $2e + 3g + 1 = 0$

2×41-42) $5g = -1$

$$\boxed{e = -\frac{1}{5}, \; f = -\frac{2}{5}, \; g = -\frac{1}{5}}$$

$$\left(\begin{array}{cccc|cccc|c} 1 & 0 & 0 & 0 & 1 & 2 & 1 & 4 & 2 \\ -2 & 1 & 0 & 0 & 2 & 0 & 1 & 3 & 1 \\ -3/2 & 3/4 & 1 & 0 & 0 & 3 & 2 & 1 & 1 \\ -1/5 & -2/5 & -1/5 & 1 & 1 & 1 & 1 & -1 & 2 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 1 & 4 & 2 \\ 0 & -4 & -1 & -5 & -3 \\ 0 & 0 & 5/4 & -\frac{11}{4} & -5/4 \\ 0 & 0 & 0 & -\frac{16}{5} & 1 \end{array}\right)$$

$$\boxed{w = -\frac{5}{16}}$$

$\frac{5}{4}z = -\frac{11}{4}\frac{5}{16} - \frac{5}{4} \Rightarrow z = -\frac{11}{16} - 1 \Rightarrow$

$$\boxed{z = -\frac{27}{16}}$$

$4y = 3 - z - 5w = 3 + \frac{27}{16} + \frac{25}{16} = \frac{100}{16}$

$$\boxed{y = \frac{25}{16}}$$

$x = -2y - z - 4w + 2 = -\frac{50}{16} + \frac{27}{16} + \frac{20}{16} + 2$

$$\boxed{x = \frac{29}{16}}$$

$$\boxed{\frac{1}{16}(29, 25, -27, -5)}$$

$29 + 50 - 27 - 20 = 32$ ✓

$58 - 27 - 15 = 16$ ✓

$75 - 54 - 5 = 16$ ✓

$29 + 25 - 27 + 5 = 32$ ✓

Aplicando a técnica de escalonamento

4 vetores em $\mathbb{R}^3$ serão sempre L.D. (Linearmente Dependentes)

dados $\begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}2\\1\\2\end{pmatrix}, \begin{pmatrix}3\\0\\1\end{pmatrix}, \begin{pmatrix}4\\1\\1\end{pmatrix}$ encontrar a combinação linear que leva ao vetor nulo

$$\alpha\begin{pmatrix}1\\1\\0\end{pmatrix} + \beta\begin{pmatrix}2\\1\\2\end{pmatrix} + \gamma\begin{pmatrix}3\\0\\1\end{pmatrix} + \delta\begin{pmatrix}4\\1\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix} \Rightarrow \underbrace{\begin{pmatrix}1&2&3&4\\0&1&1&0\\0&2&1&1\end{pmatrix}}_{3\times4}\underbrace{\begin{pmatrix}\alpha\\\beta\\\gamma\\\delta\end{pmatrix}}_{4\times1} = \underbrace{\begin{pmatrix}0\\0\\0\end{pmatrix}}_{3\times1}$$

ESCALONAMENTO

$$\begin{matrix}1&2&3&4\\1&1&0&1\\0&2&1&1\end{matrix} \Rightarrow \begin{matrix}1&2&3&4\\0&1&3&3\\0&2&1&1\end{matrix} \Rightarrow \begin{matrix}1&2&3&4\\0&1&3&3\\0&0&5&5\end{matrix} \left|\begin{matrix}\alpha\\\beta\\\gamma\\\delta\end{matrix}\right. = \begin{matrix}0\\0\\0\end{matrix}$$

$\boxed{\text{solução} \quad \delta\{-1, 0, -1, 1\}}$

$\alpha + 2\beta + 3\gamma + 4\delta = 0 \rightarrow \alpha = -\delta$

$\beta + 3\gamma + 3\delta = 0 \rightarrow \beta = 0$

$5\gamma + 5\delta = 0 \rightarrow \gamma = -\delta$

$$-1\cdot\begin{pmatrix}1\\1\\0\end{pmatrix} + 0\cdot\begin{pmatrix}2\\1\\2\end{pmatrix} - 1\cdot\begin{pmatrix}3\\0\\1\end{pmatrix} + 1\cdot\begin{pmatrix}4\\1\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix} \checkmark$$

Determinar se os vetores $\begin{pmatrix}1\\1\\0\\1\end{pmatrix}, \begin{pmatrix}2\\1\\2\\1\end{pmatrix}, \begin{pmatrix}3\\0\\1\\1\end{pmatrix}$ são L.D. ou L.I., se forem L.D. calcular a combinação linear que leva ao vetor nulo

$$\begin{matrix}1&2&3\\1&1&0\\0&2&1\\1&1&1\end{matrix} \Rightarrow \begin{matrix}1&2&3\\0&1&3\\0&2&1\\1&1&1\end{matrix} \Rightarrow \begin{matrix}1&2&3\\0&1&3\\0&0&5\\1&1&1\end{matrix} \begin{matrix}\rightarrow \alpha=0\\\rightarrow \beta=0\\\rightarrow \gamma=0\\ \end{matrix}$$

$\boxed{\text{vetores L.I.}}$

Repetir o exercício anterior com os vetores $\begin{pmatrix}1\\1\\0\\2\end{pmatrix}, \begin{pmatrix}2\\1\\1\\3\end{pmatrix}, \begin{pmatrix}3\\0\\3\\3\end{pmatrix}$

$$\begin{matrix}1&2&3\\1&1&0\\0&1&3\\2&3&3\end{matrix} \Rightarrow \begin{matrix}1&2&3\\0&1&3\\0&1&3\\2&3&3\end{matrix} \Rightarrow \begin{matrix}1&2&3\\0&1&3\\0&0&0\\2&3&3\end{matrix} \Rightarrow \begin{matrix}1&2&3\\0&1&3\\0&0&0\\0&1&3\end{matrix} \rightarrow 0\ 0\ 0$$

DUAS LINHAS NULA GARANTEM A LINEAR DEPENDÊNCIA

4 EQUAÇÕES E 3 PARAMETROS α, β, γ

PRECISAMOS DE 2 EQUAÇÕES E 3 PARAMETROS

$\boxed{\text{REGRA} \quad n\times m \quad m\times 1 \quad n-m+1 \text{ LINHAS NULAS}}$

solução $\begin{matrix}\alpha + 2\beta + 3\gamma = 0\\ \beta + 3\gamma = 0\end{matrix} \Rightarrow \boxed{\gamma = \{3, -3, 1\}}$

Exemplos de vetores L.D. em $\mathbb{R}^3$

1) $\left\{\begin{pmatrix}1\\1\\2\end{pmatrix},\begin{pmatrix}2\\2\\5\end{pmatrix},\begin{pmatrix}3\\3\\7\end{pmatrix}\right\}$ 2) $\left\{\begin{pmatrix}1\\1\\2\end{pmatrix},\begin{pmatrix}2\\2\\5\end{pmatrix},\begin{pmatrix}3\\3\\8\end{pmatrix}\right\}$

encontrar a combinação linear que leva o vetor nulo

Melhor resolver usando a matriz

$$\begin{matrix}1&2&3\\1&2&3\\2&5&a\end{matrix} \Rightarrow \begin{matrix}1&2&3\\0&0&0\\0&1&a-6\end{matrix} \Rightarrow \begin{matrix}\alpha+2\beta+3\gamma=0\\ \beta+(a-6)\gamma=0\end{matrix}$$

$$\beta=(6-a)\gamma \quad , \quad \alpha+2(6-a)\gamma+3\gamma=0 \Rightarrow \boxed{\gamma\{2a-15,6-a,1\}}$$

$a=7$ $\gamma\{-1,-1,1\}$ $\begin{pmatrix}-1-2+3\\-1-2+3\\-2-5+7\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$

$a=8$ $\gamma\{1,-2,1\}$ $\begin{pmatrix}1-4+3\\1-4+3\\2-10+8\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$

DADO O SISTEMA

$$\begin{aligned}x+2y+3z&=2\\2x+y+z&=2\\3x+3y+4z&=b\end{aligned}$$

DETERMINAR PARA QUAL VALOR DE b O SISTEMA NÃO TEM SOLUÇÃO

$$\begin{matrix}1&2&3\\+&+&+\\2&1&1\\=&=&=\\3&3&4\end{matrix}$$

A MATRIZ NÃO TEM INVERSA

$$\left(\begin{array}{ccc|c}1&2&3&2\\2&1&1&2\\3&3&4&b\end{array}\right) \Rightarrow \left(\begin{array}{ccc|c}1&2&3&2\\0&3&5&2\\3&3&4&b\end{array}\right) \Rightarrow \left(\begin{array}{ccc|c}1&2&3&2\\0&3&5&2\\0&3&5&6-b\end{array}\right) \Rightarrow \left(\begin{array}{ccc|c}1&2&3&2\\0&3&5&2\\0&0&0&4-b\end{array}\right)$$

O SISTEMA TERÁ SOLUÇÃO SOMENTE QUANDO b FOR IGUAL A 4

PARA b=4 A SOLUÇÃO SERÁ DADA POR

$$\begin{cases}x+2y+3z=2 \longrightarrow x=\dfrac{2+z}{3}\\ 3y+5z=2 \longrightarrow y=\dfrac{2-5z}{3}\end{cases}$$

SOLUÇÃO $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}2/3\\2/3\\0\end{pmatrix}+z\begin{pmatrix}1/3\\-5/3\\1\end{pmatrix}$

CONTROLE!

$$\begin{pmatrix}1&2&3\\2&1&1\\3&3&4\end{pmatrix}\begin{pmatrix}2/3\\2/3\\0\end{pmatrix}=\begin{pmatrix}2\\2\\4\end{pmatrix} \checkmark$$

$$\begin{pmatrix}1&2&3\\2&1&1\\3&3&4\end{pmatrix}\begin{pmatrix}1/3\\-5/3\\1\end{pmatrix}=\begin{pmatrix}1/3-10/3+3\\2/3-5/3+1\\1-5+4\end{pmatrix} \checkmark$$