

Forma quadraticas

$$Ax^2 + 2Bxy + C^x y^2 + \dots$$

$$(x \ y) \begin{pmatrix} A & B \\ B & C \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = Ax^2 + 2Bxy + C^x y^2$$

AUTOVALORES: $(A-\lambda)(C-\lambda) - B^2 = 0$

$$\lambda^2 - (A+C)\lambda + AC - B^2 = 0 \Rightarrow \lambda_{1,2} = \frac{A+C \pm \sqrt{(A+C)^2 - 4AC + B^2}}{2} = \frac{A+C \pm \sqrt{(A-C)^2 + B^2}}{2}$$

AUTOVALORES REAIS E DISTINTOS!

$$\begin{pmatrix} A & B \\ B & C \end{pmatrix} = S \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} S^{-1}$$

$$\begin{pmatrix} A & B \\ B & C \end{pmatrix} \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

$$\begin{pmatrix} \alpha & \gamma \\ \beta & \delta \end{pmatrix} \begin{pmatrix} A & B \\ B & C \end{pmatrix} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} \alpha & \gamma \\ \beta & \delta \end{pmatrix}$$

$$S^t M = D S^t$$

↓
S⁻¹

$$\Rightarrow (x \ y) \begin{pmatrix} A & B \\ B & C \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} =$$

$$(x \ y) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} \alpha & \gamma \\ \beta & \delta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} =$$

$$(X \ Y) \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}$$

APLICAMOS A TRANSPOSTA

$\lambda_1, \lambda_2 > 0$ ELIPSE

$\lambda_1, \lambda_2 = 0$ PARÁBOLA

$\lambda_1, \lambda_2 < 0$ HIPÉRBOLA

EX 1

$$3x^2 + 10xy + 3y^2 - 2x - 14y - 13 = 0$$

$$\begin{pmatrix} A & B \\ B & C \end{pmatrix} = \begin{pmatrix} 3 & 5 \\ 5 & 3 \end{pmatrix} \quad \begin{matrix} (3-\lambda)^2 - 25 = 0 \\ \lambda^2 - 6\lambda - 16 = 0 \end{matrix}$$

$$\lambda_{1,2} = 3 \pm \sqrt{9+16} < \begin{matrix} -2 \\ 8 \end{matrix}$$

HIPÉRBOLA

$$\begin{pmatrix} 3 & 5 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} -2 & 0 \\ 0 & 8 \end{pmatrix}$$

$$3\alpha + 5\gamma = -2\alpha$$

$$3\beta + 5\delta = 8\beta$$

$$5\alpha + 3\gamma = -2\gamma$$

$$5\beta + 3\delta = 8\delta$$

$$\alpha = -\gamma$$

$$\beta = \delta$$

$$\tan \theta = \frac{\gamma}{\alpha}, \frac{\delta}{\beta}$$

$$\begin{matrix} -1 & 1 \end{matrix}$$

$$\theta, \frac{\pi}{2} + \theta, \pi + \theta, \frac{3\pi}{2} + \theta$$

$$\tan\left(\frac{\pi}{2} + \theta\right) = -\frac{1}{\tan \theta}, \tan(\pi + \theta) = \tan \theta, \tan\left(\frac{3\pi}{2} + \theta\right) = -\frac{1}{\tan \theta}$$

EX. 2

$$25x^2 - 14xy + 25y^2 + 64x - 64y - 224 = 0$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 25 & -7 \\ -7 & 25 \end{pmatrix}$$

$$\begin{matrix} (25-\lambda)^2 = 49 \\ 25-\lambda = \pm 7 \end{matrix} \Rightarrow \lambda < \begin{matrix} 18 \\ 32 \end{matrix}$$

ELIPSE

$$\begin{matrix} 1 & \tan \theta \\ & -1 \end{matrix}$$

$$\begin{pmatrix} 25 & -7 \\ -7 & 25 \end{pmatrix} \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} 18 & 0 \\ 0 & 32 \end{pmatrix}$$

$$\begin{matrix} 25\alpha - 7\gamma = 18\alpha \\ -7\gamma + 25\alpha = 18\alpha \end{matrix}$$

$$\begin{matrix} 25\beta - 7\delta = 32\beta \\ -7\delta + 25\beta = 32\delta \end{matrix}$$

EX. 3

$$4xy + 3y^2 + 16x + 12y - 36 = 0$$

$$\begin{pmatrix} A & B \\ B & C \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 2 & 3 \end{pmatrix}$$

$$\begin{aligned} (3-2)(-2) - 4 &= 0 \\ \lambda^2 - 3\lambda - 4 &= 0 \end{aligned} \Rightarrow$$

$$\lambda_{1,2} = \frac{3 \pm \sqrt{9+16}}{2} \begin{matrix} -1 \\ 4 \end{matrix}$$

HIPÉRBOLA

$$\begin{pmatrix} 0 & 2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 4 \end{pmatrix}$$

$$\begin{aligned} 2\gamma &= -\alpha \\ 2\alpha + 3\gamma &= -\gamma \end{aligned}$$

$$\begin{aligned} 2\delta &= 4\beta \\ 2\beta + 3\delta &= 4\delta \end{aligned}$$

$$\gamma = -\frac{\alpha}{2}$$

$$\delta = 2\beta$$

$$\tan \theta = -\frac{1}{2}, 2$$

EX. 4

$$9x^2 - 24xy + 16y^2 - 20x + 110y - 50 = 0$$

$$\begin{pmatrix} A & B \\ B & C \end{pmatrix} = \begin{pmatrix} 9 & -12 \\ -12 & 16 \end{pmatrix}$$

$$\begin{aligned} (9-2)(16-2) - 144 &= 0 \\ \lambda^2 - 25\lambda + 144 - 144 &= 0 \end{aligned} \Rightarrow$$

$$\lambda = 0 \quad \lambda = 25$$

PARÁBOLA

$$\begin{pmatrix} 9 & -12 \\ -12 & 16 \end{pmatrix} \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 25 \end{pmatrix}$$

$$\begin{aligned} 9\alpha - 12\gamma &= 0 \\ -12\alpha + 16\gamma &= 0 \end{aligned}$$

$$\begin{aligned} 9\beta - 12\delta &= 25\beta \\ -12\beta + 16\delta &= 25\delta \end{aligned}$$



$$\gamma = \frac{3}{4}\alpha$$

$$\delta = -\frac{4}{3}\beta$$

$$\tan \theta = \frac{3}{4}, -\frac{4}{3}$$

EX. 5

$$\frac{x}{2} - \frac{y}{\sqrt{3}} + \frac{7x^2}{48} - \frac{y}{3} - \frac{\sqrt{3}y}{2} - \frac{5xy}{24\sqrt{3}} + \frac{31y^2}{144} = -1$$

$$\begin{pmatrix} A & B \\ B & C \end{pmatrix} = \begin{pmatrix} \frac{7}{48} & -\frac{5}{48\sqrt{3}} \\ -\frac{5}{48\sqrt{3}} & \frac{31}{144} \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 21 & -5\sqrt{3} \\ -5\sqrt{3} & 31 \end{pmatrix}$$

$$\begin{aligned} (21-\lambda)(31-\lambda) - 75 &= 0 \\ \lambda^2 - 52\lambda + 576 &= 0 \end{aligned}$$

$$\lambda = 26 \pm 10 \begin{matrix} 16 \\ 36 \end{matrix}$$

ELIPSE

$$\begin{pmatrix} 21 & -5\sqrt{3} \\ -5\sqrt{3} & 31 \end{pmatrix} \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} 16 & 0 \\ 0 & 36 \end{pmatrix}$$

$$21\alpha - 5\sqrt{3}\gamma = 16\alpha$$

$$21\beta - 5\sqrt{3}\delta = 36\beta$$

$$\alpha = \sqrt{3}\gamma$$

$$-\sqrt{3}\delta = 3\beta$$

$$-5\sqrt{3}\alpha + 31\gamma = 16\gamma$$

$$-5\sqrt{3}\beta + 31\delta = 36\delta$$

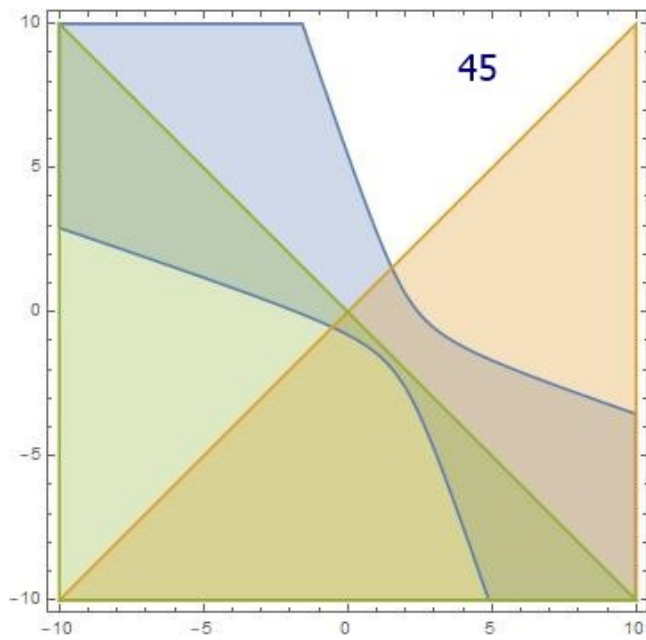
$$3\gamma = \sqrt{3}\alpha$$

$$-\sqrt{3}\beta = \delta$$

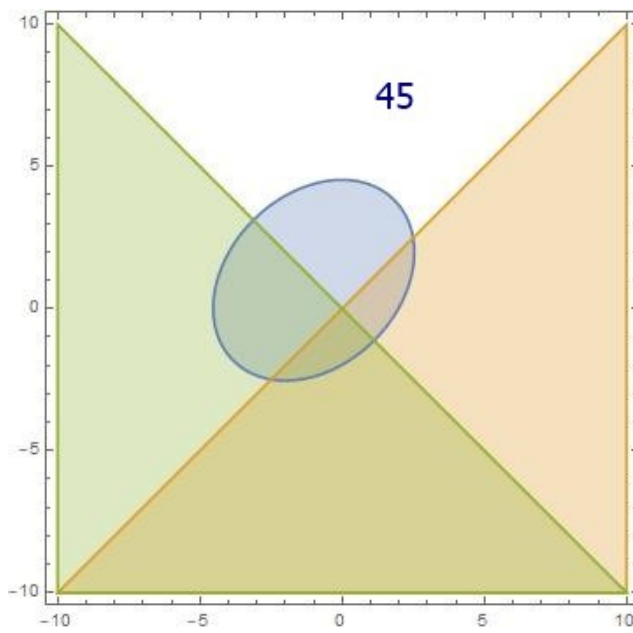
$$\frac{\gamma}{\alpha} = \frac{1}{\sqrt{3}}$$

$$\frac{\delta}{\beta} = -\sqrt{3}$$

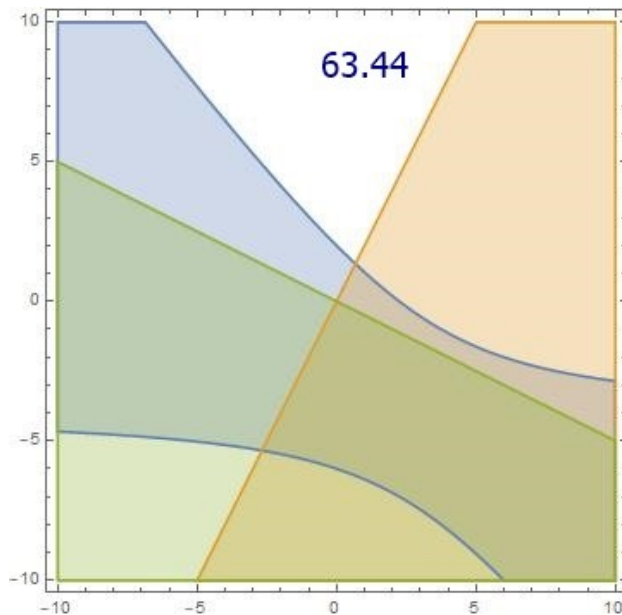
$$\tan \theta = \frac{1}{\sqrt{3}}, -\sqrt{3}$$



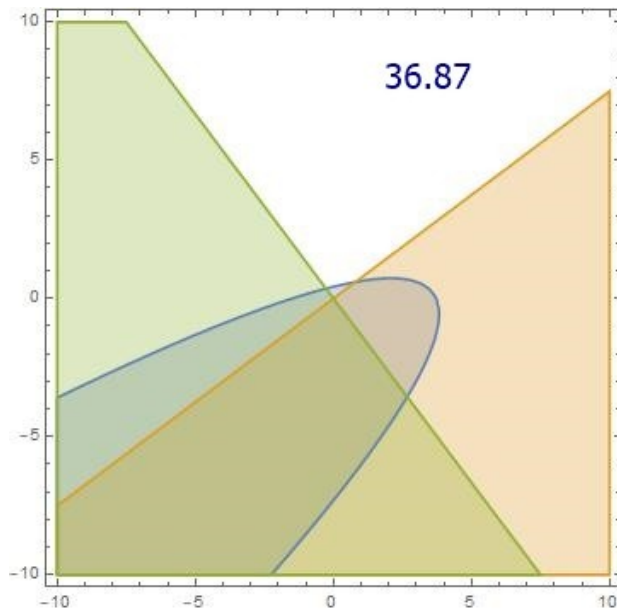
- $3x^2 + 10xy + 3y^2 - 2x - 14y - 13$
- $y - x$
- $y + x$



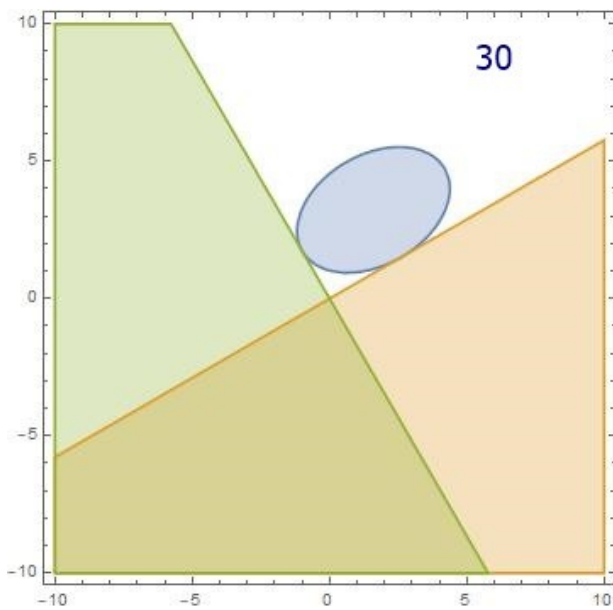
- $25x^2 - 14xy + 25y^2 + 64x - 64y - 224$
- $y - x$
- $y + x$



- $4xy + 3y^2 + 16x + 12y - 36$
- $y - 2x$
- $y + \frac{x}{2}$



- $9x^2 - 24xy + 16y^2 - 20x + 110y - 50$
- $y - \frac{3x}{4}$
- $y + \frac{4x}{3}$



- $\frac{x}{2} - \frac{x}{\sqrt{3}} + \frac{7x^2}{48} - \frac{y}{3} - \frac{\sqrt{3}y}{2} - \frac{5xy}{24\sqrt{3}} + \frac{31y^2}{144} + 1$
- $y - \frac{x}{\sqrt{3}}$
- $y + \sqrt{3}x$