# Forma de Jordan

consideramos o caso $\alpha=\beta$ neste caso não podemos reduzir a matriz à forma diagonal

$$\boxed{M = S J S^{-1} \qquad J = \begin{pmatrix} \alpha & 1 \\ 0 & \alpha \end{pmatrix}}$$

$$J = \begin{pmatrix} \alpha & 1 \\ 0 & \alpha \end{pmatrix} \quad J^2 = \begin{pmatrix} \alpha^2 & 2\alpha \\ 0 & \alpha^2 \end{pmatrix} \quad J^3 = \begin{pmatrix} \alpha^3 & 3\alpha^2 \\ 0 & \alpha^3 \end{pmatrix} \quad \dots \quad \sum_0^\infty {}_n \frac{J^n}{n!} = \begin{pmatrix} e^\alpha & e^\alpha \\ 0 & e^\alpha \end{pmatrix}$$

$$\boxed{\exp[M] = S \begin{pmatrix} e^\alpha & e^\alpha \\ 0 & e^\alpha \end{pmatrix} S^{-1}}$$

$$M = \begin{pmatrix} 4 & -1 \\ 4 & 0 \end{pmatrix} \qquad (4-\lambda)(-\lambda)+4=0 \rightarrow \lambda^2-4\lambda+4=0 \rightarrow (\lambda-2)^2=0$$

$$\boxed{\lambda=2}$$

$$\begin{pmatrix} 4 & -1 \\ 4 & 0 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$$

$$\begin{aligned} 4a - c &= 2a \; ✓ \\ 4b - d &= a + 2b \\ 4a &= 2c \\ 4b &= c + 2d \end{aligned}$$

$a=1 \rightarrow c=2$

$$\begin{aligned} 4b - d &= 1 + 2b \\ 4b &= 2 + 2d \end{aligned} \rightarrow 1 + d = 2b$$

$d=1 \rightarrow b=1$

$b=0 \rightarrow d=-1$

$$S = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \qquad S = \begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}$$

DUAS POSSIBILIDADES ?

$$\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} e^2 & e^2 \\ 0 & e^2 \end{pmatrix}\left[-\begin{pmatrix} 1 & -1 \\ -2 & 1 \end{pmatrix}\right] = e^2 \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix} = e^2 \begin{pmatrix} 3 & -1 \\ 4 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} e^2 & e^2 \\ 0 & e^2 \end{pmatrix}\left[-\begin{pmatrix} -1 & 0 \\ -2 & 1 \end{pmatrix}\right] = e^2 \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix} = e^2 \begin{pmatrix} 3 & -1 \\ 4 & -1 \end{pmatrix}$$

ok!

$$\boxed{\exp\left[\begin{pmatrix} 4 & -1 \\ 4 & 0 \end{pmatrix}\right] = \begin{pmatrix} 3 & -1 \\ 4 & -1 \end{pmatrix} e^2}$$

PREPARANDO EXERCÍCIOS

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 6 & 1 \\ 0 & 6 \end{pmatrix}\left[-\frac{1}{2}\begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}\right] = \frac{1}{2}\begin{pmatrix} 6 & 7 \\ 6 & -5 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 13 & -1 \\ 1 & 11 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}\left[\frac{1}{4}\begin{pmatrix} 1 & 1 \\ -3 & 1 \end{pmatrix}\right] = \frac{1}{4}\begin{pmatrix} 2 & -1 \\ 6 & 5 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -3 & 1 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 5 & 1 \\ -9 & 11 \end{pmatrix}$$

CONTROLE

$$\left(\frac{13}{2}-\alpha\right)\left(\frac{11}{2}-\alpha\right)+\frac{1}{4} = \alpha^2 - 12\alpha + 36 = (\alpha-6)^2 \quad \text{ok!}$$

$$\left(\frac{5}{4}-\alpha\right)\left(\frac{11}{4}-\alpha\right)+\frac{9}{16} = \alpha^2 - 4\alpha + 4 = (\alpha-2)^2 \quad \text{ok!}$$