Sd rotations
ROTATION OF $\left(x_{1 l}, z\right)$ AROUND THE AXIS $(\alpha, \beta, \gamma)$ OF AN angle $Q$

$$
\alpha^{2}+\beta^{2}+\alpha^{2}=1
$$

$$
\begin{aligned}
& (x, y, z) \quad \square D\left(\begin{array}{cc}
i z & x+i y \\
-x+i y & -i z
\end{array}\right) \\
& \text { PAULI MATRiCES } \\
& \left(\begin{array}{cc}
i z & \bar{Z}+i \underline{Z} \\
-X+i Z & -i Z
\end{array}\right)=M\left(\frac{8}{2}\right)\left(\begin{array}{cc}
i z & x+i y \\
-x+i y & -i z
\end{array}\right) M\left(-\frac{0}{2}\right) \\
& \text { where } M(\theta)=\left(\begin{array}{ll}
\cos \frac{\theta}{2}+i \gamma \sin \frac{\theta}{2} & (\alpha+i \beta) \sin \frac{\theta}{2} \\
(-\alpha+i \beta) \sin \frac{\theta}{2} & \cos \frac{\theta}{2}-i \gamma \sin \frac{\theta}{2}
\end{array}\right)
\end{aligned}
$$

EXAMPLE

$$
\begin{aligned}
& (x, 4, z)=(1,0,0) \quad \theta=\pi \quad(\alpha, \beta, \gamma)=(0,0,1) \\
& M\left(\frac{\pi}{2}\right)=\left(\begin{array}{cc}
i & 0 \\
0 & -i
\end{array}\right) \quad M(-\pi / 2)=\left(\begin{array}{cc}
-i & 0 \\
0 & i
\end{array}\right) \\
& \left(\begin{array}{cc}
i Z & X+i Z \\
-X+i Z & -i Z
\end{array}\right)=\left(\begin{array}{cc}
i & 0 \\
0 & -i
\end{array}\right)\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right)\left(\begin{array}{cc}
-i & 0 \\
0 & i
\end{array}\right) \\
& =\left(\begin{array}{ll}
0 & i \\
i & 0
\end{array}\right)\left(\begin{array}{cc}
-i & 0 \\
0 & i
\end{array}\right)=\left(\begin{array}{cc}
0 & -1 \\
1 & 0
\end{array}\right) \\
& \Rightarrow(7, Y, Z)=(-1,0,0) \\
& \theta=\frac{\pi}{2} \quad \cos \frac{\pi}{4}=\sin \frac{\pi}{4}=\frac{1}{\sqrt{2}} \\
& M\left(\frac{\pi}{4}\right)=\left(\begin{array}{cc}
1+i & 0 \\
0 & 1-i
\end{array}\right) / \sqrt{2} \\
& M\left(-\frac{\pi}{4}\right)=\left(\begin{array}{cc}
1-i & 0 \\
0 & 1+i
\end{array}\right) / \sqrt{2} \\
& \left(\begin{array}{cc}
i z & X+i 工 \\
-X+i Y & -i z
\end{array}\right)=\frac{1}{\sqrt{2}}\left(\begin{array}{cc}
1+i & 0 \\
0 & 1-i
\end{array}\right)\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right) \frac{1}{\sqrt{2}}\left(\begin{array}{cc}
1-i & 0 \\
0 & 1-i
\end{array}\right) \\
& =\frac{1}{2}\left(\begin{array}{cc}
0 & 1+i \\
-1+i & 0
\end{array}\right)\left(\begin{array}{cc}
1-i & 0 \\
0 & 1+i
\end{array}\right) \\
& =\frac{1}{2}\left(\begin{array}{cc}
0 & 2 i \\
2 i & 0
\end{array}\right)=\left(\begin{array}{ll}
0 & i \\
i & 0
\end{array}\right) \\
& (X, I, Z)=(0,1,0) \\
& \theta=\pi \quad(\alpha, \beta, \gamma)=(1,1,1) / \sqrt{3} \\
& \left(\begin{array}{cc}
i z & \bar{y}+i z \\
-\bar{Y}+i z & -i Z
\end{array}\right)=\frac{1}{\sqrt{3}}\left(\begin{array}{cc}
i & 1+i \\
-1+i & -i
\end{array}\right)\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right) \frac{1}{\sqrt{3}}\left(\begin{array}{cc}
-i & -1-i \\
1-i & i
\end{array}\right) \\
& =\frac{1}{3}\left(\begin{array}{cc}
-1-i & i \\
i & -1+i
\end{array}\right)\left(\begin{array}{cc}
-i & -1-i \\
1-i & i
\end{array}\right)=\frac{1}{3}\left(\begin{array}{cc}
2 i & -1+2 i \\
1+2 i & -2 i
\end{array}\right) \\
& \Rightarrow(X, \tau, z)=\frac{1}{3}(-1,2,2)
\end{aligned}
$$

> (INITIAL VECTOR)
> (1, 1, 1 \}
> (MATRIX FORM OF THE INITIAL VECTOR)
> $\left\{\left(\begin{array}{cc}\mathrm{i} & 1+\mathrm{i} \\ -1+\mathrm{i} & -\mathrm{i}\end{array}\right)\right\}$
> (ROTATION ANGLE\}
> (180)
> (ROTATION AXIS
> $\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right\}$
> (ROTATION MATRIX
> $\left\{\left(\begin{array}{cc}0 & \frac{1+i}{\sqrt{2}} \\ -\frac{1-i}{\sqrt{2}} & 0\end{array}\right)\right\}$
> (MATRIX FORM OF THE ROTATED VECTOR)
> $\begin{aligned}\left\{\left(\begin{array}{cc}0 & \frac{1+i}{\sqrt{2}} \\ -\frac{1-i}{\sqrt{2}} & 0\end{array}\right)\right. & \left.\cdot\left(\begin{array}{cc}\mathrm{i} & 1+\mathrm{i} \\ -1+\mathrm{i} & -\mathrm{i}\end{array}\right) \cdot\left(\begin{array}{cc}0 & -\frac{1+i}{\sqrt{2}} \\ \frac{1-i}{\sqrt{2}} & 0\end{array}\right)\right\} \\ & \left\{\left(\begin{array}{cc}-\mathrm{i} & 1+\mathrm{i} \\ -1+\mathrm{i} & \mathrm{i}\end{array}\right)\right\}\end{aligned}$
(INITIAL VECTOR)
(1, 1, 1)
(MATRIX FORM OF THE INITIAL VECTOR)
$\left\{\left(\begin{array}{cc}\mathrm{i} & 1+\mathrm{i} \\ -1+\mathrm{i} & -\mathrm{i}\end{array}\right)\right\}$
\{ROTATION ANGLE
(180)
(ROTATION AXIS)
$\left\{\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}, 0\right\}$
(ROTATION MATRIX)

$$
\left\{\left(\begin{array}{cc}
0 & \frac{1-i}{\sqrt{2}} \\
-\frac{1+i}{\sqrt{2}} & 0
\end{array}\right)\right\}
$$

(MATRIX FORM OF THE ROTATED VECTOR)

$$
\begin{aligned}
\left\{\left(\begin{array}{cc}
0 & \frac{1-i}{\sqrt{2}} \\
-\frac{1+i}{\sqrt{2}} & 0
\end{array}\right)\right. & \left.\cdot\left(\begin{array}{cc}
i & 1+\mathrm{i} \\
-1+\mathrm{i} & -\mathrm{i}
\end{array}\right) \cdot\left(\begin{array}{cc}
0 & -\frac{1-i}{\sqrt{2}} \\
\frac{1+i}{\sqrt{2}} & 0
\end{array}\right)\right\} \\
& \left\{\left(\begin{array}{cc}
-\mathrm{i} & -1-\mathrm{i} \\
1-\mathrm{i} & \mathrm{i}
\end{array}\right)\right\} \\
& \{\text { ROTATED VECTOR }\} \\
& (\{-\mathbf{1},-\mathbf{1},-1)\}
\end{aligned}
$$

$$
\begin{aligned}
& \text { \{INITIAL VECTOR\} } \\
& (1,1,1\} \\
& \text { [MATRIX FORM OF THE INITIAL VECTOR) } \\
& \left\{\left(\begin{array}{ccc}
i & 1+i \\
-1+i & -i
\end{array}\right)\right\} \\
& \text { \{ROTATION ANGLE\} } \\
& \text { \{90\} } \\
& \text { (ROTATION AXIS) } \\
& \{1,0,0\} \\
& \text { (ROTATION MATRIX) } \\
& \left\{\left(\begin{array}{cc}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{array}\right)\right\} \\
& \left.\begin{array}{c}
\text { \{MATRIX FORM OF THE ROTATED VECTOR\} } \\
\left\{\left(\begin{array}{cc}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{array}\right) \cdot\left(\begin{array}{cc}
i & 1+i \\
-1+i & -i
\end{array}\right) \cdot\left(\begin{array}{cc}
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{array}\right)\right\} \\
\left\{\left(\begin{array}{cc}
i & 1-i \\
-1-i & -i
\end{array}\right)\right\} \\
\\
\text { \{ROTATED VECTOR\}}
\end{array}\right\} \\
& \text { (INITIAL VECTOR ) } \\
& (1,-1,0) \\
& \text { \{MATRIX FORM OF THE INITIAL VECTOR\} } \\
& \left\{\left(\begin{array}{cc}
0 & 1-i \\
-1-i & 0
\end{array}\right)\right\} \\
& \text { (ROTATION ANGLE) } \\
& \text { (90) } \\
& \text { (ROTATION AXIS) } \\
& \left\{-\frac{1}{\sqrt{6}},-\frac{1}{\sqrt{6}}, \sqrt{\frac{2}{3}}\right\} \\
& \text { (ROTATION MATRIX) } \\
& \left\{\left(\begin{array}{cc}
\frac{i \sqrt{2} \cdot \sqrt{3}}{\sqrt{6}} & -\frac{1}{2}, \frac{1}{2} \\
\frac{1}{3} \\
\frac{1}{2} \cdot \frac{1}{2} & \frac{-\sqrt{2}, \sqrt{3}}{\sqrt{3}}
\end{array}\right)\right\} \\
& \text { (MATRIX FORM OF THE ROTATED VECTOR\} } \\
& \left\{\left(\begin{array}{cc}
\frac{i \sqrt{2}+\sqrt{3}}{\sqrt{6}} & -\frac{1}{2}+\frac{1}{2} \\
\sqrt{3} \\
\frac{1}{2}-\frac{1}{2} & \frac{-i \sqrt{2}+\sqrt{3}}{\sqrt{3}}
\end{array}\right) \cdot\left(\begin{array}{cc}
0 & 1-i \\
-1-i & 0
\end{array}\right) \cdot\left(\begin{array}{cc}
\frac{-i \sqrt{2} \cdot \sqrt{3}}{\sqrt{6}} & \frac{1}{2}+\frac{i}{2} \\
\sqrt{3} \\
-\frac{1}{2} \cdot \frac{1}{2} & \frac{i \sqrt{2}+\sqrt{3}}{\sqrt{3}}
\end{array}\right)\right\} \\
& \left\{\begin{array}{r}
\frac{1}{6}(i \sqrt{2}(-i \sqrt{2}+\sqrt{3})+i \sqrt{2}(i \sqrt{2}+\sqrt{3})) \\
\frac{1}{6}\left((-1+i)+(1-i)(i \sqrt{2}+\sqrt{3})^{2}\right) \\
\frac{1}{6}\left((1+i)-(1+i)(-i \sqrt{2}+\sqrt{3})^{2}\right)
\end{array} \frac{1}{6}(-i \sqrt{2}(-i \sqrt{2}+\sqrt{3})-i \sqrt{2}(i \sqrt{2}+\sqrt{3})) .\right\} \\
& \text { (ROTATED VECTOR) } \\
& \left\{\left\{\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}\right\}\right\}
\end{aligned}
$$

